

1600 when Otho was about 50 years old DeMorgan refers to him in no. 11 as "then an old man."

<sup>22</sup> As DEMORGAN pointed out, the 86 pages of 1 are easily distinguishable by the inferiority of paper and type; the same is true of 2; also the corrected copies may be distinguished from the uncorrected ones in a moment as follows: look at the bottom of page 7, at the running titles of the columns. The uncorrected copy will have as it ought to have Basis Differentia Hypothenusa. But the *corrected* copy will have, as it ought not to have, Hypothenusa Differentia Basis.

### QUERY

32. FRENCH AND RUSSIAN TRANSLATIONS OF A VEGA-BREMIKER TABLE. In RMT 635, reference was made to the Carl Bremiker edition of Vega's *Logarithmisch-Trigonometrisches Handbuch*, first published in 1856. It was also noted that in 1857 an English translation of this edition by W. L. F. FISCHER, and an Italian translation by LUIGI CREMONA, were published. Who were the authors of the French translation of 1857 and of the Russian translation of 1858 also mentioned? The first two translations, as well as the 1857 (second Bremiker) German edition are in the Library of Brown University. In what library may the French or Russian translations be found?

R. C. A.

### QUERIES—REPLIES

41. GIRARD AND SNELL TABLES (Q30, v. 3, p. 451).—ALBERT GIRARD, *Tables des Sinus, Tangentes, & Secantes, selon le Raïd de 100 000 Parties. Avec un traité succinct de la Trigonometrie tant des triangles plans, que spheriques*. The Hague, 1626, 240 pages (unnumbered). 6.8 × 11.8 cm.

An examination of the copy of this book in the Library of Congress revealed that the main table contains natural sines, tangents, and secants in units of  $10^{-5}$  for every sexagesimal minute of the first quadrant, so arranged that the functions of complementary angles appear on facing pages. Each page contains functions for a range of half a degree.

Following this principal table is a section devoted to the statement and illustration of rules for the solution of the four standard cases of oblique plane triangles. Although Girard frequently resorts to the device of dissecting oblique triangles into right triangles, he does state and use the Law of Sines. In addition, he gives in the form of rules both the Law of Tangents and the Law of Cosines,—the latter in a form involving the versed sine.

A brief section dealing with some general theorems relating to plane polygons is followed by a treatment of both right and oblique spherical triangles. The discussion of the solution of oblique spherical triangles is limited to three cases: (i) all angles given; (ii) all sides given; and (iii) two sides and their included angle given.

The author then gives to five significant figures the length of a side of each of the five regular polyhedra when inscribed in a sphere of diameter 100 000. His results for the regular tetrahedron and regular octahedron contain rounding errors.

A more extensive table is included showing the lengths to the nearest integer of the sides of regular  $n$ -gons [ $n = 3(1)24$ ] inscribed in a circle of diameter 200 000. Careful examination showed this table to be entirely free from error.

After a brief section containing the statement of geometrical problems involving the solution of equations such as  $\tan \theta = 2\theta$ ,  $\sin \theta = \theta/\sqrt{2}$ ,  $\theta - \sin \theta = \pi/2$ ,  $\tan 2\theta - \tan \theta = 1$ , in modern notation, Girard reproduces Ludolph van Ceulen's 36-figure approximation to  $\pi$ , and presents a tabulation of arc length to radius  $10^{10}$  for the following central angles:  $1^\circ(1^\circ)10''(5^\circ)100''$ ,  $150^\circ$ ,  $180^\circ$ ,  $1'(1')10'(5')55'$ ,  $1''(1'')10''(5'')55''$ . This table contains ten rounding errors.

Two rules are stated for the calculation of close approximations to an acute angle of a right triangle in terms of the three sides. In modern trigonometric notation these rules are equivalent to the inequalities:

$$(1) \quad 3 \sin \theta / (2 + \cos \theta) < \theta < \frac{1}{3}(2 \sin \theta + \tan \theta).$$

The book concludes with miscellaneous information relative to triangles, including a schematic array showing the manner of dependence of the area of a plane triangle on various geometric magnitudes associated with a triangle.

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EDITORIAL NOTE: The inquiring reader will find it of value to read the remarks concerning this work in A. VON BRAUNMÜHL, *Vorlesungen über Geschichte der Trigonometrie*, v. 1, 1900, p. 237-238. It may be remarked that Dr. Wrench's description of Girard's main table is exactly equivalent to that of PITISCUS, 1600 (*MTAC*, v. 3, p. 391).

The problem leading to the equation  $\tan \theta = 2\theta$  is as follows (compare N104, LOTHAR COLLATZ, *MTAC*, v. 3, p. 496): What is the arc which equals half its tangent?

A relation differing from (1) only in that the right member is  $2 \sin \frac{1}{3}\theta + \tan \frac{1}{3}\theta$  was first given by WILLEBRORD SNELL, in his *Cyclometricus*, Leyden, 1621, prop. XXIX, p. 43 f. The first published proof of this right-hand member relation was given by HUYGENS in his *De Circuli Magnitudine inventa*, 1654. If  $3y = \theta$  the relation  $\theta < 2 \sin \frac{1}{3}\theta + \tan \frac{1}{3}\theta$  becomes  $y < \frac{1}{3}(2 \sin y + \tan y)$ , as given by Girard. The single approximation  $\theta \approx 3 \sin \theta / (2 + \cos \theta)$ , was first surmised by NICHOLAS DE CUSA (1401-1464); see his *Opera*, 1514 or 1565, p. 1120-1154. Compare K. T. VAHLEN, *Konstruktionen und Approximationen in systematischer Darstellung*. Leipzig, 1911, p. 188-190.

42. PITISCUS TABLES (Q29, v. 3, p. 398).—In 1927 I purchased for \$180, from Wheldon & Wesley, London, a copy of RHETICUS-PITISCUS, *Thesaurus Mathematicus*, Frankfurt, 1613, bound with the 1607 volume (86 p.) described in *MTAC*, v. 3, p. 395. I have also acquired (1924-37) a fine copy of the two volumes (6 parts) of the original 1596 RHETICUS-OTHO, *Opus Palatinum de Triangulis*. With reference to the quality of paper and printing of the 1607 Pitiscus item the inferiority is notably in evidence.

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EDITORIAL NOTE: Among the many other valuable volumes of mathematical and astronomical tables in Mr. Morgan's Library are the following: 1. Ptolemy's Table of Chords, first published in the Liechtenstein edition of the *Almagest*, 1515; also the George of Trebizond edition of 1528, and the Greek edition of Symon Grynæus, 1538. 2. Regiomontanus, *Tabulae Directionum*, 1552; also Wittenberg, 1606 edition. These volumes contain a table of sines at interval  $1'$  and calculated for a radius of 60 000. 3. VIETA, *Canon Mathematicus*. Paris, 1579, formerly the property of CHARLES HUTTON, purchased in 1930 from Sotheran for \$200. The great scarcity of this work (of which there is a copy at Brown University) may possibly be due to the fact that on account of errors in the volume, Vieta bought up and destroyed all copies which it was possible for him to locate. 4. B. PITISCUS, *Trigonometry*, 1612 edition (*MTAC*, v. 3, p. 391). 5. H. BRIGGS, *Arithmetica Logarithmica*. London, 1624 (*MTAC*, v. 1, p. 97-98, 170). 6. A. VLACQ, *Arithmetica Logarithmica*. 1628. 7. H. BRIGGS, *Trigonometria Britannica*. London, 1633. 8. H. SHERWIN, *Mathematical Tables*, first edition.

London, 1717. 9. J. DODSON, *Anti-Logarithmic Canon*. London, 1742. 10. G. VON VEGA, *Thesaurus Logarithmorum Completus*. Leipzig, 1794. 11. Astronomical Tables of King Alphonse X of Castile (1203–1284), first printed edition, Venice, 1483; also editions of 1492, 1518, 1524, and Paris, 1545, 1553. 12. E. HALLEY, *Tabulae Astronomicae*. London, 1749. 13. J. CASSINI, *Tables Astronomiques*. Paris, 1740. 14. Mr. Morgan is also the fortunate owner of what appears to be a fifteenth century vellum manuscript of the extraordinarily popular Toledo Planetary Tables, which originated with the astronomer Ibn al-Zarqāla (1029?–1087?). This manuscript consists of 124 leaves measuring  $17 \times 25$  cm. (See G. SARTON, *Introduction to the History of Science*, v. 1, Washington, 1927, p. 758–759.)

## CORRIGENDA

V. 1, p. 466, for Burden, J. A., read Burdon, J. A.

V. 2, p. 384, for e see Napierian Base, read e 54–55, 454; II 15, 68–69, 300; p. 387, delete Napierian Base e 54–55, 454; II 15, 68–69; p. 389, for Weiertrass, read Weierstrass.

V. 3, p. 385, for M. R. Hestenis, read M. R. Hestenes; p. 466, l. 2, for (1.570780, read 1.570780; p. 475, l. –12, for  $Re(S)$ , read  $Re(s)$ ; p. 496, l. 2, for  $10^{-5}$ , read  $<10^{-5}$ .