Most of the articles are devoted to the mathematical derivation of parameters for design of four-bar linkages and harmonic transformer mechanisms which will best approximate the desired motion. Also included is a description of a computing machine (p. 158–160) with a continuous movement of the components rather than movement with discrete intervals such as used by common mechanical digital computers. The continuous motion is obtained by means of planetary gears.

The volume concludes with an article by I. I. ARTOBOLEVSKIĬ & N. I. LEVITSKIĬ on models of Chebyshev's mechanisms, which are preserved in the Leningrad Academy of Sciences. There are 25 of these mechanisms which are described and illustrated. The illustration of Chebyshev's "arithmometer" is disappointingly inadequate.

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15. J. G. L. MICHAEL, "A nomogram for calculating extended terms," Institute of Actuaries Students' Soc., *Jn.* v., 8, 1948, p. 147–159.

Two nomograms are given for the calculation of endowment insurance.

16. F. J. MURRAY, "Linear Equation Solvers," *Quart. Applied Math.*, v. 7, 1949, p. 263–274.

17. G. H. ORCUTT, "A New Regression Analyzer," R. Stat. Soc., *Jn.*, Sec. A, v. 111, 1948, p. 54–70.

The analyzer described in this paper is based on units, each of which consists of a card reader and commutator. By means of this combination a time sequence of voltages $X_1, X_2, X_3, \cdots, X_n$ corresponding to the two-digit quantities punched on the cards is obtained. When a number of units are combined with suitable output circuits it is possible to obtain a variety of second degree expressions, for instance $\sum (X_i - \bar{X})\cdot(Y_i - \bar{Y})$ or $\sum X_i \cdot Y_{i+k}$, where k is a shift subject to the operator's control. The author discusses in detail the application of these expressions to statistical problems including those in which varying time lags between sequences are to be considered. The advantages of the use of the commutator over parallel operation consists in the simplicity of the associated circuits and the fact that the various sequences of voltages can be shown immediately on an oscillograph.

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NOTES

110. NEW FACTORIZATIONS OF $2^n \pm 1$.—In *MTAC*, v. 3, p. 496–7 we gave a proof of the primality of $(2^{92} + 1)/17$. Using the same methods we have now established the primality of

$$N_1 = (2^{79} + 1)/3 = 2014\ 87636\ 60243\ 81957\ 84363$$

and

$$N_2 = (2^{85} + 1)/(3\cdot 11\cdot 43691) = 26831\ 42303\ 60653\ 52611$$
$$N_3 = (2^{83} - 1)/167 = 579\ 12614\ 11327\ 56490\ 87721.$$

In the case of N_1, which is the 6-th largest known prime, we find that for $y = 2^{77}$,

$$3^y \equiv 534\ 45942\ 48656\ 40551\ 54581 \equiv W \pmod{N_1}$$

and that

$$W^2 \equiv -3 \pmod{N_1}. \quad \text{Hence } 27^{N_1} \equiv 27 \pmod{N_1}.$$

The largest prime factor of $N_1 - 1$ is $p = 22366891 = (N_1 - 1)/m$. Furthermore

$$3^m - 1 \equiv 591\ 99625\ 59867\ 68206\ 78419 \pmod{N_1},$$

which is prime to N_1. From this it follows, by LEHMER's theorem, that all the prime factors of N_1 are of the form $px + 1$. Combining this with the fact that they are also of the form $158x + 1$ we obtain

$$dx + 1 = 3533968778x + 1.$$

Now if N_1 were composite we could write $N_1 = (dm + 1)(dn + 1)$ with $mn \neq 0$. Using the reasoning of the footnote on p. 497 it follows that the remainder on division of N_1 by d^2 would be less than 10^{14}, whereas it is greater than $3.6 \cdot 10^{18}$. Hence N_1 is not composite.

In the case of N_2, the proof was more difficult. In the first place it was found that if $q = 3 \cdot 11 \cdot 43691$ and if $Q = 3^q$, then

$$Q^{N_2} \equiv Q \pmod{N_2},$$

so that N_2 behaves like a prime. Next it was found that

$$N_2 - 1 = 2 \cdot 3 \cdot 5 \cdot 17 \cdot 257M = F \cdot M,$$

where

$$M = 20471\ 06358\ 13423.$$

The number M was in turn tested for primality using the facts that $3^M \equiv 3 \pmod{M}$ and that $M - 1$ is divisible by the prime 10235291.

The primality of N_2 now follows as before from the primality of M and the fact that

$$3^F - 1 \equiv 24823\ 70333\ 63136\ 14240 \pmod{N_2},$$

is prime to N_2.

With the exceptions of $2^{71} + 1$ and $2^{89} + 1$, the first 100 numbers of the form $2^n + 1$ are now completely factored.

The primality of N_3 follows in a similar way. First of all N_3 behaves like a prime, since it was found that if $a = 2^{82}$, then

$$3^a \equiv -3^{84} \pmod{N_3}.$$

Next $N_3 - 1$ is divisible by 383 and 4049. By using Lehmer's theorem it was proved that the possible prime factors of N_3 are all of the forms $383y + 1$, $4049z + 1$, $166w + 1$, and hence of the form

$$257427322x + 1.$$

The same argument as applied to N_1 finally establishes the primality of N_3. This makes the 15-th composite MERSENNE number to be completely factored.

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111.—ELECTRONIC COMPUTERS AND THE ANALYSIS OF STOCHASTIC PROCESSES.—HARTREE [1] has remarked on the impact of modern calculating machines upon mathematical analysis: by suitably recasting the mathematical treatment of a problem we may profit from the capacity and speed of these machines to get quick solutions of long or otherwise intractable enquiries. Stochastic processes supply a noteworthy illustration of the mathematician's need to think in terms of numerical methods.

LESLIE [2] considers the deterministic growth of animal populations; and by formal mathematics he reduces the problem of specifying the growth-rate, knowing the birth and death mechanism, to the determination of the dominant latent root of a matrix. This solves the problem analytically; but to solve it numerically we must calculate this latent root. DUNCAN & COLLAR [3] give the appropriate computing technique. To study their method is instructive: for it is identical with Leslie's analysis—that is, they take a given population (represented by a vector), subject it to a given birth and death process (represented by a matrix), and deduce the value of the dominant latent root from the observed growth-rate of the vector. This exemplifies the following common situation. There is a practical problem A, stated in terms of practical data B. To solve A, we set up C, the mathematical model of A; and by formal analysis deduce D, the solution of C. To compute D, however, for the given conditions B, we set up E, the numerical model of A conforming to B; and solve E by numerical methods. Evidently the construction of C and its reduction to D are unnecessary steps if we merely wish a solution D. This is not to stigmatize C as useless; for a formal symbolic solution often affords insight into the structure of a problem, and sometimes is more tractable than an entirely numerical resolution of E.

Situations of this kind will probably occur in abundance in the analysis of stochastic processes. To take one of the simplest instances, corresponding to the deterministic description of population growth, there is a more realistic stochastic description with a formal solution in terms of integral equations; and the position here is more extreme than in the example previously cited, because the numerical solution of these equations is more elaborate than the evaluation of a dominant latent root.

Stochastic processes are a relative innovation still certainly in their infancy: but it is clear from three fundamental papers [BARTLETT,[4] KENDALL,[5] and MOYAL [6]] that these processes, ranging as they do over a huge field of applications from epidemiology to atomic physics, will prove singularly important in the near future. It is proper to ask how far mathematical methods, and particularly numerical methods, are girded up to meet the coming demands from this quarter.

From a very general aspect, the problems comprise a set of arithmetical operations applied to random space-time functions, yielding answers whose complete specification involves statistical distributions. A possible direct attack—direct in the same sense as the Duncan-Collar method is direct when applied to Leslie's problem—is to feed in as data a sample from the random function aggregate, subject each member of the sample to the relevant arithmetical operations, and enumerate the results. This has the merit of simplicity; and the apparent drawback, that a large sample is probably needed to give an adequate representation of the output distributions, can be countered by the electronic computer's distinctive facility of performing routine arithmetic operations upon large masses of data, provided that the rate of supply of data is commensurate with the operating speed of the computer. Is it then feasible to generate random function samples electronically within the computer? At any rate, at first sight, this idea seems promising. The noise and shot effects of a thermionic valve furnish random functions. A random pulse/blank train in a computer's binary decimal sequence generates a rectangular distribution, and can (at least theoretically) lead to an integrated Fourier series whose coefficients are distributed normally and independently over the complex domain [PALEY & WIENER [7]].

This is not however the place to enter into details, even were they less speculative: but it is a matter for consideration whether stochastic processes could be analyzed in the direct fashion suggested on an electronic computer, and, if so, whether they will be pervasive enough to warrant building a special unit into the computer to generate random functions; and this note will have served its purpose if it provokes research on this issue at the present opportune juncture, when a number of electronic computers are projected or under construction or in their developmental stages in various parts of the world.

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[2] P. H. LESLIE, "On the use of matrices in population mathematics," *Biometrika*, v. 33, 1945, p. 183–212. "Some further notes on the use of matrices in population mathematics," *ibid.*, v. 35, 1948, p. 213–245. "Distribution in time of the births in successive generations," R. Stat. Soc. *Jn.*, s. A, v. 111, 1948, p. 44–53.

[3] W. J. DUNCAN & A. R. COLLAR, "A method for the solution of oscillation problems by matrices," *Phil. Mag.*, s. 7, v. 17, 1934, p. 865–909.

[4] M. S. BARTLETT, "Some evolutionary stochastic processes," R. Stat. Soc., *Jn.*, s. B, v. 11 (in press).

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[6] J. E. MOYAL, "Stochastic processes and statistical physics," R. Stat. Soc., *Jn.*, s. B, v. 11 (in press).

[7] R. E. A. C. PALEY & N. WIENER, *Fourier Transforms in the Complex Domain*, Amer. Math. Soc., *Colloq. Pub.* no. 19, New York, 1934.

QUERY

33. LENHART TABLES.—As a supplement to the final number, 6, Nov. 1838, of *The Mathematical Miscellany*, v. 1, edited by CHARLES GILL (1805–1855), is a 16-page pamphlet, with its own title-page, as follows: *Useful Tables relating to Cube Numbers, Calculated and arranged* by WILLIAM LENHART, *York, Penn. Designed to accompany his general investigation of the equation* $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$, *published in the Mathematical Miscellany, vol. 1, page 114; and by him through his friend, Professor C. Gill, presented to the Library of St. Paul's College, Flushing, Long Island, May 4th, 1837.* As indicated in D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, 1941, p. 64, this "rare table" "gives, for more than 2500 integers $A < 100\,000$, solutions of $x^3 + y^3 = Az^3$ in positive integers." On the back of the title page of this pamphlet is the following: "Besides the tables given