15. R. E. MEYEROTT & G. BREIT, "Small differential analyzer with ball carriage integrators and selsyn coupling," *Rev. Sci. Instruments*, v. 20, 1949, p. 874–876.

This article describes a small differential analyzer built at Yale University around three ball carriage type mechanical integrators. The advantages of this type of integrator are that it has a good output torque, the input and output shafts are mechanically convenient, and the units are readily available as war surplus equipment. Coupling between integrators and input or output devices is by means of selsyns instead of shaft couplings, which makes the equipment very flexible and minimizes the set up time for new problems. The selsyns rotate at high speed (2500 to 1 to the integrator) to eliminate selsyn error. The analyzer also includes three input or output devices (called "function units") and an "adder" consisting of two selsyns coupled to a mechanical differential.

A test problem set in to generate a pure sine wave gave an increase of amplitude of 0.2 per cent per cycle, which compares favorably with results obtained with more elaborate machines. The device has proved satisfactory and convenient for the solution of problems with a nominal accuracy of one per cent.

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16. CYRIL STANLEY SMITH, "Blowing bubbles for a dynamic crystal model," *Jn. Applied Physics*, v. 20, 1949, p. 631.

This communication deals with the development of a certain type of analogue solution for problems in crystal structure and describes an improved method, using a nozzle, for obtaining the bubble rafts used by BRAGG and NYE for metal crystal study. (*Cf.* R. Soc. London, *Proc.*, v. 190 A, 1947, p. 474–481.)

F. J. M.

NOTES

116. ON FINDING THE SQUARE ROOT OF A COMPLEX NUMBER.—There are two classical methods used for finding x and y in

$$(a + bi)^{\frac{1}{2}} = x + yi. \tag{1}$$

The first of these is based on DEMOIVRE's theorem. One computes in succession

$$R = (a^2 + b^2)^{\frac{1}{2}} \qquad \phi = \arctan (b/a)$$
$$x = R \cos \tfrac{1}{2}(\phi + 2K\pi) \qquad y = R \sin \tfrac{1}{2}(\phi + 2K\pi), \qquad (K = 0, 1).$$

The second method is derived from purely algebraic considerations. From (1) it follows that

$$x^2 - y^2 = a, \qquad 2xy = b \tag{2}$$
$$R = x^2 + y^2 = (a^2 + b^2)^{\frac{1}{2}}$$
$$x = \pm ((R + a)/2)^{\frac{1}{2}}$$
$$y = \pm ((R - a)/2)^{\frac{1}{2}}. \tag{3}$$

The signs are determined in accordance with (2). The second method avoids the use of trigonometric lookups. In addition, the first method is reducible to the second. Hence, the second method is the better one to use. However, it, too, has some serious shortcomings. When b is small, either x or y suffers from a loss in significance. How does one prevent such a loss?

GARRETT BIRKHOFF has suggested a third procedure. Define

$$t = \tfrac{1}{2}(R + |a|). \tag{4}$$

Then x and y are determined as follows:

If	$a > 0$,	$x = t^{\frac{1}{2}}$,	$y = \frac{1}{2}bt^{-\frac{1}{2}}$.
If	$a < 0$,	$x = \frac{1}{2}bt^{-\frac{1}{2}}$,	$y = t^{\frac{1}{2}}$.
If	$a = 0$,	$x = y = (b/2)^{\frac{1}{2}}$.	

Of course in all cases, the signs of x and y are determined in accordance with (2).

The numerical analyst should prefer the third method for two reasons: (a) accuracy is preserved, (b) the square root in (3) is replaced by a division, a great time-saving advantage.

Example: Find the square root of $-7 + .03i$.

The second method gives only

$$x = .00566\ 95 \qquad y = 2.64575\ 7386$$

while the third method gives

$$x = .00566\ 94540\ 7 \qquad y = 2.64575\ 7386.$$

Should any of the readers find an exposition of this technique, I should be glad to know about it. Birkhoff and I believe it to be new.

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117. TABLE FOR SOLVING CUBIC EQUATIONS.—Judging by the title of the following 75-page pamphlet published more than sixty years ago one would never suspect that in it might be any table of mathematical interest: JOHN BORDEN, *Some Higher Plane Curves*, Chicago, E. T. Decker, 1888. Curves of the third, fourth, and fifth degrees are considered; but on pages 55–75 is a table of the roots of $m^3 - 3m + C = 0$ for values of C between $+2$ and -2. Each column headed respectively 1, 2, 3, contains the values of a root in numerical order. The value of C next to the right of the column of roots has its proper value for C directly opposite and to the right of it. In the first column headed 3, the values are successively $\sqrt{3}$, 1.733(.001)2, and corresponding to each of these values is a 9D value for C. In the column headed 1, the values range 0(.001)1; in the column headed 2, the range is $\sqrt{3}$, 1.731($-$.001)1. The author remarks: "The value of each root has its proper value for C directly opposite and to the right of it." "When the value of C is positive, roots nos. 1 and 2 are positive and no. 3 is negative: and when C is negative nos. 1 and 2 are negative and no. 3 is positive." Corresponding to the root 3 = 1.949, there are the corresponding root 1 = .945 and the root 2 = 1.055, followed by 54 other pairs .946(.001).999 and

1.054(−.001)1.001. To each of these 109 roots is given the corresponding value of C to 9D. Then comes the final entry in the table: root $3 = 2$, $C = 2$; root $1 = 1$, $C = 2$; root $2 = 1$, $C = 2$.

R. C. A.

118. Note on the paper "On a definite integral" by R. H. Ritchie [*MTAC*, v. 4, p. 75–77].—The value given by Ritchie for the integral

$$f(x) = \int_0^\infty (u + x)^{-1} \exp(-u^2)du$$

is not free of ambiguity, since the definition of Ei(y) given in the paper does not make sense for $y = x^2 > 0$. The difficulty arises over the inversion of the Laplace transform (L.T.) $p^{-1} \ln (p - 1)$.

Moreover, $f(x)$ can be evaluated by a routine process, since it is simply[1] the repeated L.T. of $\exp(-u^2)$. The L.T. of $\exp(-u^2)$ is[2]

$$\tfrac{1}{2}\pi^{\frac{1}{2}} \exp(\tfrac{1}{4}t^2) \operatorname{erfc}(\tfrac{1}{2}t)$$

where

$$\operatorname{erfc} z = 2\pi^{-\frac{1}{2}} \int_z^\infty \exp(-u^2)du.$$

Next, the L.T. of this can be obtained by means of Campbell & Foster,[2] formula 959.5. The result is incorrectly rendered in Cossar & Erdélyi;[3] the corrected form is

$$-\tfrac{1}{2}\exp(-x^2)\operatorname{Ei}(-x^2e^{i\pi}) + \tfrac{1}{2}i\pi \exp(-x^2) \operatorname{erfc}(xe^{i\pi/2}).$$

This can be expressed in terms of real valued functions if we use the function[4]

$$\overline{\operatorname{Ei}}(z) = \tfrac{1}{2}\operatorname{Ei}(-ze^{i\pi}) + \tfrac{1}{2}\operatorname{Ei}(-ze^{-i\pi}).$$

The result is

$$f(x) = \exp(-x^2)\left[\pi^{\frac{1}{2}} \int_0^x \exp(u^2)du - \tfrac{1}{2}\overline{\operatorname{Ei}}(x^2)\right].$$

A. E.

[1] D. V. Widder, *The Laplace Transform*, Princeton, 1941, Chapter VIII, 1.
[2] G. A. Campbell & R. M. Foster, *Fourier Integrals for Practical Applications*, New York, 1948, no. 903.0.
[3] J. Cossar & A. Erdélyi, *Dictionary of Laplace Transforms*, London, 1944–1946, p. VI, 76. In entry 3, read $(2\alpha i\pi^{\frac{1}{2}})^{-1}$ instead of $i\pi^{\frac{1}{2}}(2\alpha)^{-1}$; in entry 4 read erfc $[p/(2\alpha)]$ in place of erfc $(p\alpha^{\frac{1}{2}})$, and π^{-1}Ei in place of Ei.
[4] E. Jahnke & F. Emde, *Tables of Functions*, Leipzig, 1938, p. 2.

119. Harry Bateman Bibliography.—In *MTAC*, v. 3, p. 141–142 we presented a complete summary of published biographical sketches of Bateman, and lists of his publications, as well as a supplement to such lists of material published in the *Educational Times* (*E.T.*) and *Educational Times Reprint* (*E.T.R.*). Miss C. Brudno, in the Applied Mathematics Branch, Mechanics Division, of the Naval Research Laboratory, Washington, has noted the following five additions to this latter list of problems proposed: 14943, *E.T.*, v. 54, 1901, p. 328; sols. by Bateman and another, *E.T.R.*, v. 1, 1902, p. 98–100. 14975, *E.T.*, v. 54, 1901, p. 423; sols. *E.T.R.*, n.s. , v. 2, 1902, p. 111, v. 3, 1903, p. 29. 15119, *E.T.*, v. 55, 1902, p. 233; sol. v. 55, 1902,

p. 516, also *E.T.R.*, n.s., v. 3, 1903, p. 110–111. 15221, *E.T.*, v. 55, 1902, p. 438; sol. *E.T.R.*, n.s., v. 4, 1903, p. 88. 16009, *E.T.*, v. 59, 1906, p. 270; sol. *E.T.R.*, n.s., v. 11, 1907, p. 57–61.

R. C. A.

QUERY

35. W. THIELE'S TABLE.—According to KAYSER'S VOLLSTÄNDIGES BÜCHERLEXICON and HINRICH'S *Katalog, Tafel der Wolfram'schen hyperbolishen 48 stelligen Logarithmen. Bearbeitet und erweitert von W. Thiele,* was published in two printings, by different companies, 118 p., at Dessau: (1) 1905; (2) 1908. This publication appears to be extremely rare. There are copies of (2) in libraries of Harvard University, of Mr. C. R. COSENS, Cambridge, England, of Brown University (film), and in the John Crerar Library, Chicago. It was Cosens who, in 1939, directed my attention to (1). In what libraries is a copy of (1) located?

R. C. A.

QUERIES—REPLIES

45. THE INTEGRAL $\int_0^x e^{-A\sec\theta}d\theta$ (Q **19**, v. 2, p. 196).—Attention is called to a manuscript table of this integral in UMT **103**.

46. PITISCUS TABLES (Q **29**, v. 3, p. 398; QR **40**, p. 498–499, **42**, p. 562–563).—For nearly 40 years I've had in my library BASIL ANDERSON & R. T. RICHARDSON, *Catalogue of the Books and Tracts on Pure Mathematics in the Central Library*, Newcastle-upon-Tyne, Newcastle-upon-Tyne, 1901. This library contains many old valuable books, but I did not earlier think to check for its possible Pitiscus items. On p. 33 it is indicated that the library owns copies of both 19966a (the *Canon*) and 19967 (the English *Trigonometry*, 1614), which therefore supplements the information we assembled [*MTAC*, v. 3, p. 499]. The library has also a copy of the 1612 German edition of the *Trigonometry* **6** [v. 3, p. 391].

R. C. A.

CORRIGENDA

V. 3, p. 54, l. 3, *for* 4–17, *read* 3–16.

V. 3, p. 406, l. 10 and 11, *interchange* cos $\frac{1}{4}k\pi(k=1, 2, 3)$ and cos $\frac{1}{8}(2k+1)\pi(k=0, 1, 2, 3)$.

V. 4, p. 21, l. 1 and 2, *for* Asymptotic distribution of range from that of reduced range *read* The distribution of the range. l. 7 *for* 193–196 *read* 395–396. l. 9 *delete* .1(.1).9(.01). l. −21, *for* 4 ∞ D *read* ∞.

V. 4, p. 23, l. −19, *for* 43–126 *read* 113–126.

V. 4, p. 29, l. −13, *for* p. xx *read* p. 11–15. l. −12, *for* Jardan *read* Jarden.

V. 4, p. 59, l. −1, *for* $a_{i1}a_{1j}/a_{11}$ *read* $a_{ij} - a_{i1}(a_{1j}/a_{11})$.

V. 4, p. 78, l. 25, *for* 1938 *read* 1935.

V. 4, p. 82, l. 14, *for* 548–553 *read* 948–953.

V. 4, p. 84, l. 5, *for* 5D *read* 4D.

V. 4, p. 91, l. 7, *for* U *read* u.

V. 4, p. 96, l.−23, *for* 37–41 *read* 33–41.

V. 4, p. 99, l.−23, *for* 332–371 *read* 322–371.