Our experience involved a stable system of differential equations, but the use of frequency analysis is justified in most cases, including unstable cases in which a scale change occurs.

F. J. M.

[1] Harvard University, Computation Laboratory, *Annals*, v. 16, p. 176–187. [*MTAC*, v. 3, p. 437.]

[2] SHIH-NGE LIN, "Numerical solution of complex roots of quartic equations," *Jn. Math. Phys.*, v. 26, 1947, p. 279–283.

[3] This formula assumes that the computed and hence available $f$ is the function to be integrated. If one wishes to make an allowance for the distinction between this $f$ and the correct $f$ associated with the true solution, a difference equation must be solved. In case $\lambda$ is a characteristic root of $H$, the corrected value $\Gamma_0$ is obtained by dividing the above formula (24) by

$$1 + .5000u + .1667u^2 + .0417u^3 - .0972u^4 - .1424u^5 \cdots.$$

This correction is significant.

## RECENT MATHEMATICAL TABLES

**761[A].**—R. COUSTAL, "Calcul de $\sqrt{2}$, et réflexion sur une espérance mathématique," Acad. Sci. Paris, *Comptes Rendus*, v. 230, 1950, p. 431–432.

The first four terms of the binomial expansion of

$$\sqrt{2} = a(1 - 2x)^{-\frac{1}{2}}$$

where

$$a^2 = 2 - 4x$$

and $a$ is an approximation to $\sqrt{2}$, good to 333D, were used to obtain $\sqrt{2}$ to 1032D. Besides this value the author gives the distribution of digits in the 1033S values of $\sqrt{2}$ and $1/\sqrt{2}$. In the first 1000D in the $\sqrt{2}$, the digits 0–9 have the following frequencies

108, 98, 109, 82, 100, 104, 90, 104, 113, 92.

Such a distribution has a chi-square of 8.38. The probability of such a value from a normal distribution is almost exactly 1/2. For $1/\sqrt{2}$ the probability is merely .05. [Compare MTAC, v. 4, p. 109–111].

The author "reflects" on the paradox that if one takes the product of the first 1033 digits of the decimal expansion of a real number $x$ in the interval $0 < x < 1$, the expected value of the product is $(9/2)^{1033} > 10^{674}$, whereas the probability that it is exactly zero is $1 - (9/10)^{1033} > 1 - 10^{-47}$.

D. H. L.

**762[C].**—H. S. UHLER, "A mathematician's tribute to the state of Israel," *Scripta Mathematica*, v. 14, 1949, p. 281–283.

The author gives ln 173 and ln 5709 to 290D.

**763[D, H, L].**—C. N. DAVIES, "The sedimentation and diffusion of small particles," R. Soc. London, *Proc.*, v. 200, 1949, p. 100–113.

This paper contains a table of the first 16 positive roots of the equation

$$2\alpha x + \tan x = 0$$

for $\alpha = 2, 1/2, 1/3, 1/6, 1/30$. Values are to 5S, 6S and 7S. For $\alpha = 1/2$ the equation is $\tan x = -x$ and the first 11 roots given in this case agree with those of POOLER, *MTAC*, v. 3, p. 496.

**764[D].**—TOSHIZO MATSUMOTO, "On Hayami's turbulent tensor," Kyoto, Imperial University, College of Science, *Memoirs*, v. 24A, no. 2, 1944, p. 63–72.

On p. 66–69, is a table, computed by HIROSI NAKAHETA, of the functions: $z^{\frac{1}{2}} \cos \frac{1}{2}\pi z$, to 4D, $\delta$; $z^{\frac{1}{2}}$ to 8D; $\cos \frac{1}{2}\pi z$ to 7D, for $z = 0(.01)2$.

R. C. A.

**765[E].**—H. G. HOPKINS, "Elastic deformations of infinite strips," Cambridge Phil. Soc., *Proc.*, v. 46, 1950, p. 164–181.

The appendix (p. 181) is a 6D table of

$$g_1 = (16 \cosh^2 x)/(55 \cosh^2 x + 25x^2 + 9)$$

and

$$g_3 = (16 \cosh^2 x)/(39 \cosh^2 x + 9x^2 + 25)$$

for $x = 0(.1)6$. At $x = 6$ the functions are already near their limiting values 16/55 and 16/39.

**766[F].**—D. JARDEN & A. KATZ, "Additional page (477) to D. N. Lehmer's Factor Table," *Riveon Lematematika*, v. 3, 1949, p. 49 [English summary p. 52].

This is the same as UMT **85**[F], *MTAC*, v. 4, p. 29.

**767[F].**—A. KATZ, "Some more new factors of Fibonacci-numbers," *Riveon Lematematika*, v. 3, 1949, p. 14 [English summary, p. 54].

The author continues the factorization of the terms of the Fibonacci sequences $U_n$ and $V_n$ [see *MTAC*, v. 3, p. 299] giving the complete factorization of $U_{117} = 2 \cdot 233 \cdot 29717 \cdot 135721 \cdot 673024656781$, $V_{73} = 151549 \cdot 11899937029$ and $V_{108} = 2 \cdot 7 \cdot 23 \cdot 6263 \cdot 103681 \cdot 177962167367$. The factors 128621 and 119809 are given for $V_{109}$ and $V_{128}$ respectively. No further factors of the two series exist below $2 \cdot 10^5$ up to $n = 128$.

D. H. L.

**768[F].**—G. PALAMÀ, "Tabella delle posizioni iniziali relative al 'Neocribrum' di L. Poletti," Parma, Univ., *Rivista Mat.*, v. 1, 1950, p. 85–98.

The "Neocribrum" is a form of factor table devised by L. POLETTI [*MTAC*, v. 3, p. 532] and has for column headings the set of 6 numbers

(S) 1, 7, 11, 13, 17, 19, 23, 29

which are those prime to 30. Two cells of the table in the same column and adjacent lines correspond to numbers differing by 30. Once a prime $p > 5$ appears as a factor of a number in any one column it reappears $p$ lines farther down and continues to appear periodically. Hence to construct the factor table it is necessary to know the line number at which a given prime factor will first appear. The purpose of the paper is to supply this information for all primes $p$ for which $17 \leq p \leq 3547$ (the primes 7, 11 and 13 are already printed into the otherwise blank forms of the sieve). Hence the table gives for each of the 491 primes the solutions $x_i$ of the congruences

$$30(x_i - 1) + m_i \equiv 0 \pmod{p} \qquad (i = 1(1)6)$$

where $m_i$ are the numbers of (S). The awkward $x - 1$ is due to the fact that Poletti numbers his lines beginning with 1 rather than 0. The table gives also the least positive $h$ for which $h + 1001$ is divisible by $p$. This is to enable the application of the results to successive "cycles" of the table.

It is a little hard to see the need for publishing such a table. If one is going to construct a factor table of just this kind it is clearly indispensable. However, the table has other uses. The column headed 29 gives directly the value of 1/30 modulo $p$ and the first column is a convenient list of primes.

D. H. L.

**769**[G].—F. N. David & M. G. Kendall, "Tables of symmetric functions—part I," *Biometrika*, v. 36, 1949, p. 431–449.

The authors present tables of coefficients in the linear representation of the monomial symmetric function in terms of products of sums of like powers, and conversely the coefficients in the expansion of the latter in terms of the former. The tables extend to symmetric functions of weights $\leq 12$. Thus from the two tables of weight 4 we read, for example, that

$$2 \sum \alpha^2\beta\gamma = 2 \sum \alpha^4 - 2(\sum \alpha^3)(\sum \alpha) - (\sum \alpha^2)^2 + (\sum \alpha^2)(\sum \alpha)^2$$

and that

$$(\sum \alpha^2)(\sum \alpha)^2 = \sum \alpha^4 + 2 \sum \alpha^3\beta + 2 \sum \alpha^2\beta^2 + 2 \sum \alpha^2\beta\gamma.$$

In order to avoid fractions the monomial symmetric functions are multiplied by the product of factorials of the exponents in the partition of the weight represented by the monomial. The resulting functions are called "augmented." Thus $\sum \alpha^2\beta^2\gamma^2\delta\epsilon$ is multiplied by 3! 2!

The authors appear to be unaware of the tables of Sukhatme, Ziaud-Din, Kerawala and Kerawala & Hanafi [*MTAC*, v. 3, p. 24]. The first of these gives the same information for weights $\leq 8$. The others give only half as much information (i.e., the coefficients in the expression of the monomials in terms of sums of like powers) for weights 9, 10 and 11. The table of weight 12 under review is completely new.

The various checking procedures used by the authors should be sufficient to produce a set of tables completely free from error. Nevertheless it would have been wise to collate the tables with those previously given which are known to contain errata [*MTAC*, v. 3, p. 24]. The application of the tables to problems in statistics is treated briefly.

The tables are arranged beautifully in lexicographical order, the two triangular halves fitting into a perfect rectangular layout. The type, though rather small, is quite clear.

D. H. L.

**770[G].**—J. A. Todd, "The characters of a collineation group in five dimensions," R. Soc. London, *Proc.*, v. 200, 1949, p. 320–336, insert between p. 336–337.

The paper gives a table of characters of a primitive group of collineations of order $6531840 = 2^8 \cdot 3^6 \cdot 5 \cdot 7$ in five space and a table of characters of a subgroup of index 2. The group is described in another paper.[1]

[1] J. A. Todd, "The invariants of a finite collineation group in five dimensions," Cambridge Phil. Soc., *Proc.*, v. 46, 1950, p. 73–90.

**771[I].**—Anders Reiz, "On quadrature formulae," Cambridge Phil. Soc., *Proc.*, v. 46, 1950, p. 119–126.

The author considers the usual quadrature formula for an integral with weight function $w(x)$:

$$\int_a^b w(x)f(x)dx = \sum_{i=1}^{n} p_i f(x_i) + R_n.$$

As is well known, the best choice of $x_i$ is the set given by the Gauss' method corresponding to $w(x)$. However, these values are not rational and the computer is faced with the task of interpolating to find $f(x_i)$. If we choose for $x_i$ the Gaussian values rounded off to 2D and modify the $p_i$ accordingly we obtain a formula which is theoretically easier to use and which retains nearly the full force of the Gauss approximation.

Six tables of the $x_i$, $p_i$ are given for the six weight functions

$$1, \quad \pi^{-1}(1-x^2)^{-\frac{1}{2}}, \quad \pi^{-1}(1-x^2)^{\frac{1}{2}}, \quad \pi^{-1}(1-x)^{\frac{1}{2}}(1+x)^{-\frac{1}{2}},$$
$$e^{-x} \quad \text{and} \quad \pi^{-\frac{1}{2}}\exp(-x^2).$$

The order $n$ extends as far as 10, 8, 8, 5, 5, and 8 in these respective cases.

For the first four weight functions the interval $(a, b)$ of integration is either $(0, 1)$ or $(-1, 1)$ while in the last two cases it is $(0, \infty)$ and $(-\infty, \infty)$ respectively.

The $x_i$ are given to 2D, as mentioned above, and the coefficients $p_i$ to 7D.

Five examples are given comparing the present method with the strict Gauss formula and Weddle's rule. As might be expected, the modified Gauss method is only slightly inferior and considerably easier to use. It is much superior to Weddle's rule.

In "actual practice" the above avoidance of interpolation difficulties may be, in some cases, only apparent. In fact if the function to be integrated is observed or tabulated on an interval $(a, b)$ different from $(0, 1)$ or $(-1, 1)$, such as $(0, 2\pi)$, then a transformation of variable is required which may introduce interpolation after all.

D. H. L.

**772[I].**—Herbert E. Salzer, "Formulas for complex cartesian interpolation of higher degree," *Jn. Math. Phys.*, v. 28, 1949, p. 200–203.

The purpose of the present table is to provide the coefficients for a Lagrangean interpolation polynomial adaptable to interpolation over a square grid in the complex plane. The polynomial assumes the form

$$f(P) = \frac{\sum a_k F(z_k)}{\sum a_k},$$

where $z_k = k$ are the points of grid-configuration, $a_k = A_k/(P - k)$, and $P = p + qi$ is the variable point. For example, in the case of three-point interpolation the configuration consists of the three points 0, 1, $i$ and the values of $A_k$ are correspondingly $A_0 = -i, A_1 = \frac{1}{2}(1 + i), A_i = \frac{1}{2}(-1 + i)$. From this we find that $\sum a_k$ reduces to $1/[P(P - 1)(P - i)]$ and the interpolation polynomial becomes

$$f(P) = -i(P - 1)(P - i)f_0 + \tfrac{1}{2}(1 + i)P(P - i)f_1 + \tfrac{1}{2}(-1 + i)P(P - 1)f_i.$$

Beginning with five-point interpolation, the author gives alternative configurations "which have the property of making the location of $P$ (or $z$) considerably more central with regard to the points $k$ (or $z_k$), and hence would be expected to yield greater accuracy. The number of these more central configurations which are given are: for five-point—one, for six- and seven-point—two, for eight- and nine-point—three. Thus, the user has a great latitude of choice in available formulas for complex interpolation, for checking that interpolation, and for central choice of the argument $P$."

This paper is a continuation of the original work of A. N. LOWAN and the author[1] [*MTAC*, v. 1, p. 358–359] and a paper by the author.[2] The work makes use of a simplification introduced by W. J. TAYLOR[3] for real Lagrangean interpolation.

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[1] A. N. LOWAN & H. E. SALZER, "Coefficients for interpolation within a square grid in the complex plane," *Jn. Math. Phys.*, v. 23, 1944, p. 156–166.

[2] H. E. SALZER, "Coefficients for complex quartic, quintic and sextic interpolation within a square grid," *Jn. Math. Phys.*, v. 27, 1948, p. 136–156.

[3] W. J. TAYLOR, "Method of Lagrangian curvilinear interpolation," NBS, *Jn. Res.*, v. 35, 1945, p. 151–155.

**773[K].**—D. J. GREB & J. N. BERRETTONI, "AOQL single sampling plans from a single chart and table," Amer. Stat. Assn., *Jn.* v. 44, 1949, p. 62–76.

As the title states, a single chart and table are given in the paper to find the AOQL (Average Outgoing Quality Limit) single sampling inspection plan which will yield a "practical" minimum amount of total inspection. Total inspection is used here to mean the combined amount resulting from the single sample inspected from each and every lot and the 100% screening inspection of lots rejected under the single sampling plan. Given an AOQL value in % it is desired to maintain and the lot size, N, Chart I of the paper (p. 66) is entered to find the acceptance number, $c$. Table II of the paper (p. 65) is then entered along with the AOQL value to find the sample size $n$ for the single sampling plan $(c, n)$ with $c = 0(1)12$, AOQL = .1,.25(.25)1(.5)5(1)10.

The present paper and associated tables appear to eliminate the necessity of knowing the process average accurately, at least for many practical situations.

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**774[K].**—D. J. FINNEY, "The truncated binomial distribution," *Annals of Eugenics*, v. 14, 1949, p. 319–328.

A number $s$ of observations of binomial random variables, all with the same probability $p$ of success, are obtained under circumstances which make it possible to observe only values greater than zero. The problem is to obtain the maximum likelihood estimate of $p$. Referring to his earlier paper[1] for derivations, the author presents an iterative technique for solving the likelihood equation which is made simple by giving a 3D table of the weights of single observations and of the bias in weighted scores for $s = 2(1)20$ and $p = .01(.01).05(.05).95$. A similar technique is described for the case of doubly truncated binomial distributions in which neither the number of "no successes" nor the number of "no failures" can be observed, and a similar table of weights and biases, both to 3D, for $s = 3(1)20$ and $p = .01(.01).05(.05).50$ is given.

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[1] D. J. FINNEY, "The estimation of the frequency of recombinations. I. Matings of known phase," *Jn. Genetics*, v. 49, 1949, p. 159–175.

**775[K].**—N. L. JOHNSON, "Systems of frequency curves generated by methods of translation," *Biometrika*, v. 36, 1949, p. 149–176.

The author investigates the properties of probability functions, $p_z$, where $z = \gamma + \delta \ln f(y)$ is distributed normally with unit variance, $f(y) \geqq 0$. If $f(y) = y$ we obtain the well-known log-normal system, $S_L$; if $f(y) = y/(1 - y)$, $0 < y < 1$, a new system $S_B$; and if $f(y) = y + \sqrt{y^2 + 1}$, $-\infty < y < \infty$, the new system $S_U$. On page 157 is a chart in terms of the PEARSON measures of skewness and kurtosis, $\beta_1$ and $\beta_2$, showing the regions of $S_L$, $S_B$, and $S_U$. On page 164 is a nomogram which gives $\delta$ and $\gamma/\delta$ in terms of $\beta_1$ and $\beta_2$, $0 \leq \beta_1 \leq 1.3$, $3 \leq \beta_2 \leq 5$ for the system $S_U$. Table 8, page 174, gives the values of $\mu_1'$, $\sigma$, $\beta_1$, $\beta_2$ for $\delta = .5, 1, 2$, and $\gamma = 0(.5)2.5$ for $S_B$. The author applies his results to the graduation of observed frequency distributions and to the normalization of skewed distributions.

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**776[K].**—K. R. NAIR, "A further note on the mean deviation from the median," *Biometrika*, v. 36, 1949, p. 234–235.

Values of the coefficient of variation (2S) are given for the mean deviation from the mean and the mean deviation from the median for samples of size 2(1)10 from a normal population. They are the same within 1 figure in the second place. This implies that the mean deviation from the median is just as precise an estimate of dispersion as the mean deviation from the mean for samples from a normal distribution.

Actually there are more precise linear estimates of dispersion than either of these. Some examples are given in a text by DIXON and the reviewer.[1]

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[1] W. J. DIXON & F. J. MASSEY, *Introduction to Statistical Analysis*, lithograph ed. Eugene, 1949.

**777[K].**—E. G. OLDS, "The 5% significance levels for sums of squares of rank differences and a correction," *Annals Math. Stat.*, v. 20, 1949, p. 117–118.

In 1938 the author published[1] a series of tables having to do with the distribution of the rank correlation coefficient. Table V gave pairs of values between which $\sum d_i^2$ ($d_i$ being the rank difference for the $i^{\text{th}}$ individual) has a probability, $P$, of being included under the hypothesis that $\sum d^2 = (n^3 - n)/6$. This hypothesis is equivalent to the null hypothesis that the rank correlation coefficient is zero. The table gave pairs of values for $n = 11$ to $n = 30$ inclusive and for $P = .99, .98, .96, .90$ and $.80$. The table in the present paper extends the original table by giving pairs of 1D values corresponding to $P = .95$.

The correction is for the formula printed in the earlier publication for the variance of the normal deviate, $x$, used in calculating the values of the table.

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[1] E. G. OLDS, "Distribution of the sums of squares of rank differences for small numbers of individuals," *Annals Math. Stat.*, v. 9, 1938, p. 133–148.

**778[K].**—FRANCES SWINEFORD, "Further notes on differences between percentages," *Psychometrika*, v. 14, 1949, p. 183–187.

The two tables are designed to determine the least common size, $N$, for each of two samples for testing the hypothesis that the difference in the two population proportions, $p_1 - p_2$, is at least $d_t$. It is assumed that the sample proportions, $p_1'$ and $p_2'$, are distributed normally and, therefore, that the appropriate test is a one-tailed test of the hypothesis that $p_1 - p_2 = d_t$. Then $N = 5.4119(p_1q_1 + p_2q_2)/(d_0 - d_t)^2$ at the 1% point and approximately half as much (1/2.0003) at the 5% point, where $d_0 = p_1' - p_2'$. The tables give $N$ for the 1% points only.

Table 1 gives $N' = 10.8238pq/(d_0 - d_t)^2$, where $p = \frac{1}{2}(p_1' + p_2')$, to 0D for $p = .10(.05).90$ and $|d_0 - d_t| = .050(.002).080(.005).135$. Table 2 gives the correction terms $(p_1q_1 + p_2q_2)/2pq$ to 3D(3S) for $p = .10(.05).90$ and $d_t = .10(.05).50$.

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**779[K].**—J. E. WALSH, "On the power function of the 'best' $t$-test solution of the Behrens-Fisher problem," *Annals Math. Stat.*, v. 20, 1949, p. 616–618.

Let $m$ sample values be drawn from $N(a_1, \sigma_1^2)$ and $n$ sample values from $N(a_2, \sigma_2^2)$, $m \leq n$, (where $N(a, \sigma^2)$ represents a normal probability function with mean $a$ and variance $\sigma^2$), then SCHEFFÉ has shown that a "$t$" test solution of the BEHRENS-FISHER problem, $\sigma_1^2/\sigma_2^2$ not known, using in the numerator the difference of sample means, and the denominator based on the square root of a function of the sample values which has a $\chi^2$-distribution with $m - 1$ degrees of freedom has certain optimum properties. The purpose of the note is to compare the power function of this $t$ test with the power function of the correspondingly most powerful test for the case in which the ratio $\sigma_1^2/\sigma_2^2$ is known, for one-sided and two-sided symmetrical tests. This is done by finding the power efficiency. Two 3D tables are given for the power efficiency of Scheffé's test, $\sigma_1^2/\sigma_2^2$ not known, against the test $\sigma_1^2/\sigma_2^2$ known, on page 617 where $\alpha$ = significance level, for $\alpha = .05$, $m = 4, 6, 10, 15, 20, 30, 50, 100, \infty$ and the same range for $n$; also $\alpha = .01$, $m = 6, 8, 10, 15, 20, 30, 50, 100, \infty$, same range for $n$.

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**780[K].**—J. E. Walsh, "On the 'information' lost by using a $t$-test when the population variance is known," Amer. Stat. Assn., *Jn.*, v. 44, 1949, p. 122–125.

Table 1 gives the approximate number of sample values "wasted" if, when the population variance is known, one uses a $t$-test (estimating variance from sample) in place of the appropriate normal deviate test, when one is testing whether the population mean differs from a given constant value. 5%, 2.5%, 1% and .5% significance levels are tabulated to 2S for both the one-sided and symmetrical test. Sample values wasted is defined in terms of equal power functions.

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**781[K].**—J. E. Walsh, "Applications of some significance tests for the median which are valid under very general conditions," Amer. Stat. Assn., *Jn.*, v. 44, 1949, p. 342–355.

The tests considered are valid for samples of $n$ from one or more universes with a common median, which are symmetric and have continuous cumulative distributions. Table I lists one to five such tests for each $n = 4(1)15$ with their approximate one-sided and symmetric significance levels to 3D and their efficiencies to nearest .5 on the assumption that the universes sampled are normal. Table II lists further tests for $n = 4(1)9$ for which the bounds of these significance levels are given, as well as their significance levels and efficiencies to same precision as Table I for samples from normal. Table I was also published elsewhere.[1]

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[1] J. E. Walsh, "Some significance tests for the median which are valid under very general conditions," *Annals Math. Stat.*, v. 20, 1949, p. 64–81.

**782[L].**—S. Chandrasekhar, "On Heisenberg's elementary theory of turbulence," R. Soc. London, *Proc.*, v. 200A, 1949, p. 20–23.

The function $f(x)$ satisfies a certain nonlinear differential equation. Introducing the new variables

$$y = \int_0^x f(t)t^2dt \qquad \text{and} \qquad g = x^3f(x),$$

and denoting differentiation with respect to $y$ by primes, the differential equation becomes

$$g^{3/2}g'' + 2y(4 + g') + 2g^{1/2}(4 - g') - 8g = 0.$$

For small $y$,

$$g(y) = 4y + y^{3/2}(a + (4/3) \ln y) + \cdots$$

where $a$ is an arbitrary constant. Starting with this value, the differential equation was integrated numerically for $a = 1.8104739$, 1.81, 1.75, 1.5, 1.0, .5, 0. Four decimal tables of $f(x)$ are given for various ranges of $x$ and varying intervals.

A. E.

**783**[**L**].—L. HOWARTH, "Rayleigh's problem for a semi-infinite plate," Cambridge Phil. Soc., *Proc.*, v. 46, 1950, p. 127–140.

The work leads to the expression

$$\tau = \frac{\mu W}{(\nu t)^{1/2}} \left\{ \frac{1}{\pi^{1/2}} + \frac{1}{(2\pi)^{3/2}} \int_R^\infty e^{-(1/8)v^2} K_{1/4}(\tfrac{1}{8}v^2) \frac{dv}{v} \right\},$$

and the notation $\tau_R = (\pi\nu t)^{-1/2}\mu W$ is used. Table 1 gives values to 3 decimal places of $(W^2t/\nu)^{1/2}\tau/(\rho W^2)$ and of $(\mu W)^{-1} \int_0^r (\tau - \tau_R)dr$ for $R = (\nu t)^{-1/2}r = .01$, .05, .1, .2(.2)2(.5)4 and $\infty$.

A. E.

**784**[**L**].—M. K. KROGDAHL, "On broadening of hydrogen lines in stellar spectra—I," *Astrophys. Jn.*, v. 110, 1949, p. 355–374.

The function

$$-\frac{1}{15}\left(\frac{z}{2}\right)^4 \sin\frac{2}{z} + \frac{1}{15}\left(\frac{z}{2}\right)^3 \cos\frac{2}{z} - \frac{29}{30}\left(\frac{z}{2}\right)^2 \sin\frac{2}{z} - \frac{1}{6}\frac{z}{2}\cos\frac{2}{z} + \frac{4}{3}\frac{z}{2}\int_{2/z}^\infty \cos y \frac{dy}{y} + \frac{1}{6}\int_{2/z}^\infty \sin y \frac{dy}{y} + \frac{32}{15}\left(\frac{z}{2}\right)^{3/2}\int_0^{2/z} \cos y \frac{dy}{\sqrt{y}}$$

is tabulated to 3D for $z = 0(.1)1(.2)3(.5)5$, and to 2D for $z = 6(1)10$. The computations were apparently based on tables contained in JAHNKE-EMDE. [1933 ed. p. 6–9, 34–35].

A. E.

**785**[**L**].—MERBT, "Untersuchung zur Arbeit von H. G. Küssner 'Lösungen der klassischen Wellengleichung für bewegte Quellen' " Aerodynamische Versuchsanstalt, Göttingen e. V., Abteilung J 06, *Bericht* B 44/J/31, ZWB/AVA/Re/44/J/31, ZWB 10514, 1944. English translation "Wave propagation from moving sources" by O. W. Leibiger Research Laboratories, Petersburg, N. Y., ATI No. 32413, 8–8–701, 1948. 14 p.

The function

$$h_{mn}(\alpha, \sigma) = \frac{1}{2\pi}\int_0^{2\pi} (1 - 2\sigma\cos\chi)^{-1/2} \exp\{imn[-\chi - \alpha(1 - 2\sigma\cos\chi)^{1/2}]\}d\chi$$

occurs in an investigation[1] of sound propagation from a source moving in a circle. In the present paper the following notations are used:

$$\beta = imn\alpha, \qquad A = (1 - 2\sigma\cos\chi)^{\frac{1}{2}}, \qquad y = e^{\beta\sigma\cos\chi},$$
$$F(y) = A^{-1}e^{-\beta A}, \qquad f(\sigma) = e^{-\beta A}.$$

First the first 8 derivatives of $F(y)$ are given in terms of $\beta$ and $A$, then the values, in terms of $\beta$, of these derivatives when $y = 1$. From these the first 7 coefficients in the asymptotic expansion

$$h_{mn}(\alpha, \sigma) = i^{mn} \sum_{\nu=1}^{\infty} b_\nu J_{mn}(\nu mn\alpha\sigma)$$

are computed. Next, the first 6 derivatives of $f(\sigma)$ are given in terms of $\beta$ and $A$, and the values of these derivatives for $\sigma = 0$ in terms of $\beta$ and $\chi$. The Maclaurin series of $f(\sigma)$ leads to an expansion of $h$ in powers of $\sigma$ which is said to show satisfactory convergence, at any rate for small $m$, for all values of $\alpha$ and $\sigma$ which are of importance; in fact it is stated that a few terms of the series suffice for numerical computation. Certain polynomials which occur in the first 7 coefficients of the expansion of $h_{mn}$ in powers of $\sigma$ are given explicitly. There are 14 such polynomials, 7 to be used for even $m$ and 7 for odd $m$. Table I of the appendix gives the coefficients of these polynomials, and Table II certain auxiliary quantities which can be used for a rapid computation of the coefficients.

A. E.

[1] H. G. KÜSSNER, "Lösungen der klassischen Wellengleichung für bewegte Quellen," *Z. angew. Math. Mech.*, v. 24, 1944, p. 243–250.

**786[L].**—F. W. J. OLVER, "Transformation of certain series occurring in aerodynamic interference calculations," *Quart. Jn. Mech. Appl. Math.*, v. 2, 1949, p. 452–457.

$$k_s = 8\pi\mu^2 s \sum_{n=1}^{\infty} (-1)^{n-1} K_0(2\pi\mu sn) + 4\mu \sum_{n=1}^{\infty} (-1)^{n-1} \frac{K_1(2\pi\mu sn)}{n}$$

is given in Table 1 (p. 456) to 6 decimal places for $\mu = .5$, $s = 1(1)6$; $\mu = 1$, $s = 1, 2, 3$; and $\mu = 2$, $s = 1, 2$.

Table 2 (p. 457) is an auxiliary table giving values to various degrees of accuracy of $K_j(n\pi)$ and $I_j(n\pi)$ for $j = 0, 1$ and $n = 1(1)7$.

**787[L].**—F. RIEGELS, "Formeln und Tabellen für ein in der räumlichen Potentialtheorie auftretendes elliptisches Integral," *Archiv d. Math.*, v. 2, 1949–50, p. 117–125.

The integral in question is

$$G_n(k^2) = (-1)^n \int_0^{\frac{1}{2}\pi} \frac{\cos 2n\theta}{(1 - k^2\sin^2\theta)^{\frac{3}{2}}}\, d\theta,$$

and can be expressed in terms of complete elliptic integrals of the first and second kinds in the form

$$(1 - k^2)^{-1} k^{-2n} [f_n E(k^2) - g_n K(k^2)],$$

where $f_n$ and $g_n$ are polynomials of degree $n$ in $k'^2$. The exact values of the coefficients of these polynomials are given (as common fractions) for $n = 0(1)7$.

$(2/\pi)G_n(k^2)$ can be expanded as a power series in $k^2$, and 5 decimal values of the coefficients of this power series up to the coefficient of $k^{30}$ are given for $n = 0(1)7$.

$k'^2G_n(k^2)$ can also be expanded in the form: power series in $k'^2 + \ln(4/k')$ times a power series in $k'^2$. The coefficients occurring in this expansion up to (and including) the terms in $k'^{10}$ are tabulated to 5 or 6 significant places for $n = 0(1)7$.

The principal tables are 4 decimal tables of $k'^2$ and of $k'^2G_n(k^2)$ for $n = 0(1)3$, $k^2 = .00(.01).99$ and $k^2 = .900(.001).999$.

The computations were carried out on the Hollerith equipment of the Max Planck Gesellschaft of Göttingen. The original computations used the interval .001 throughout and included also $G_4$ to $G_7$. The unpublished parts of the tables are available through the author.

A. E.

**788[L].**—G. SCHWEIKERT, "Zur Theorie des Gasdrucks gegen eine bewegte Wand," *Zeit. angew. Math. Mech.*, v. 29, 1949, p. 289–300.

Let

$$F(x) = 2\pi^{-\frac{1}{2}}\int_x^\infty \exp(-t^2)dt$$

and

$$G(x) = \pi^{\frac{1}{2}}x \exp(x^2)F(x).$$

The article contains tables to 5D of

$$F(x), \qquad 2x^2F(x), \qquad (2 - F)(1 + 2x^2) + 2\pi^{-\frac{1}{2}}x \exp(-x^2)$$

and

$$[1 + 2x^2(1 - \theta^2)]F(x) - 2\pi^{-\frac{1}{2}}x \exp(-x^2)$$

for

$$\theta = 0(.1).4, 1 \text{ and for } x = 0(.1)1.5, 2, 2.5$$

and tables to 8D of $\log G(x)$, $1/G(x)$ and

$$1 + \tfrac{1}{2}x^{-2} - 1/G(x)$$

for $x = .05, .1(.1)2(.5)4.5$.

**789[L].**—GEOFFREY TAYLOR, "The formation of a blast wave by a very intense explosion. I. Theoretical discussion." R. Soc. London, *Proc.*, v. 201A, 1950, p. 159–174. "II. The atomic explosion of 1945," *ibid*, p. 175–186.

In the course of this work there appear the functions $f$, $\phi$, $\psi$ of $\eta$ which satisfy the system of nonlinear differential equations

$$(\eta - \phi)\phi' = \frac{1}{\gamma}\frac{f'}{\psi} - \tfrac{3}{2}\phi, \qquad \frac{\psi'}{\psi} = \frac{\phi' + 2\phi/\eta}{\eta - \phi}$$

$$3f + \eta f' + \frac{\gamma\psi'}{\psi}(-\eta + \phi)f - \phi f' = 0$$

in which $\gamma$ is a constant.

Table 1 of Part I (p. 164) contains 3 decimal values of $f$, $\phi$, $\psi$ for $\eta = 1(-.02).5$. These values were computed by step-by-step numerical integration of the differential equations with $\gamma = 1.4$ and the initial values $f(1) = 1.167$, $\phi(1) = .833$, $\psi(1) = 6.000$.

Approximate formulas for the functions in question are developed on p. 165.

Table 2 of Part I (p. 166) contains 3 decimal values of $f$, $\phi$ and 2 decimal values of $\psi$ for $\eta = 1, .95, .9, .8, .7, .5, 0$. These values were computed from the approximate formulas with $\gamma = 1.666$, $f(1) = 1.250$, $\phi(1) = .750$, $\psi(1) = 4.000$.

Table 2 of Part II (p. 178) contains 3 decimal values of $f$, $\phi$, $\psi$ for $\eta = 1(-.02).9(-.05).4$ obtained by approximate calculation for $\gamma = 1.30$, $f(1) = 1.130$, $\phi(1) = .869$, $\psi(1) = 7.667$. Some values of the temperature are added.

There are also other tables.

A. E.

**790[L].**—A. VAN WIJNGAARDEN & W. L. SCHEEN, "Table of Fresnel integrals," Akademie van Wetenschappen, Amsterdam, *Afd. Natuurkunde, Verhandelingen*, eerste sectie, v. 19, no. 4, 1949, 26 p. (Report R49 of the Computation Department of the Mathematical Centre at Amsterdam.) Price 2.50 guilders.

This is a table of

$$C(u) = \int_0^u \cos \tfrac{1}{2}\pi t^2 dt \qquad \text{and} \qquad S(u) = \int_0^u \sin \tfrac{1}{2}\pi t^2 dt$$

for $u = [0(.01)20; 5D]$ with modified second differences.

$$(1) \quad C(u) = \sum_{k=0}^{\infty} C_{4k+1} u^{4k+1}, \qquad C_{4k+1} = (-1)^k (\tfrac{1}{2}\pi)^{2k} / [(2k)!(4k+1)]$$

$$(2) \quad S(u) = \sum_{k=0}^{\infty} S_{4k+3} u^{4k+3}, \qquad S_{4k+3} = (-1)^k (\tfrac{1}{2}\pi)^{2k+1} / [(2k+1)!(4k+3)].$$

Tables are given for $k = 0(1)22$ or $21$, of $C_{4k+1}[C_1 = 1,\ C_5 = -2.467401100272340 \cdot 10^{-1}, \cdots, C_{89} = 2 \cdot 10^{-49}]$, and of $S_{4k+3}[S_3 = 5.235987755982989 \cdot 10^{-1}, \cdots, S_{87} = -5 \times 10^{-47}]$.

Asymptotic expansions are

$$(3) \quad C(u) \sim \tfrac{1}{2} + \sin \tfrac{1}{2}\pi u^2 \sum_{k=0}^{\infty} \gamma_{4k+1} u^{-(4k+1)} - \cos \tfrac{1}{2}\pi u^2 \sum_{k=0}^{\infty} \sigma_{4k+3} u^{-(4k+3)}$$

$$(4) \quad S(u) \sim \tfrac{1}{2} - \cos \tfrac{1}{2}\pi u^2 \sum_{k=0}^{\infty} \gamma_{4k+1} u^{-(4k+1)} - \sin \tfrac{1}{2}\pi u^2 \sum_{k=0}^{\infty} \sigma_{4k+3} u^{-(4k+3)}$$

where

$$(5) \qquad \gamma_{4k+1} = (-1)^k 2(4k)!(2\pi)^{-(2k+1)}/(2k)!$$

$$(6) \qquad \sigma_{4k+3} = (-1)^k 2(4k+2)!(2\pi)^{-(2k+2)}/(2k+1)!$$

There are tables of the coefficients (5) and (6) for $k = 0(1)14$ and $k = 0(1)13$ respectively.

In computing the main table 9D values were calculated by means of (1) and (2) for $u = 0(.5)2.5$ and the same from (3), (4) for $u = 2.5(.5)12$. In order to get this accuracy by means of asymptotic series for values of $u$ as low as 2.5, the technique described by GOODWIN & STATON[1] was employed. The next step was to prepare preliminary 7D tables of $C(u)$ and $S(u)$ for $u = 0(.01)12$ by numerical integration of 5D values of $\cos \frac{1}{2}\pi t^2$ and $\sin \frac{1}{2}\pi t^2$ with an interval $h = .01$. After most elaborate treatments and testings, the authors were led to their rounded-off 5D values, which are "guaranteed."

The values of the table for $u = 12(.01)20$ were calculated by means of the asymptotic series and checked by complete duplication of the computation. Modified second differences, $\delta^{2*} = \delta^2 - 0.184\delta^4$ were computed from the 7D tables, then rounded off to 5D and checked by differencing.

Linear interpolation will yield no larger error than $4 \times 10^{-5} \times u$. Full profit of the accuracy of the table is obtained by the use of EVERETT'S formula up to second modified differences:

$$f(x_0 + ph) = (1 - p)f_0 + pf_1 + E_0{}^2\delta_0{}^{2*} + E_1{}^2\delta_1{}^{2*}.$$

A small table of the interpolation polynomials $E_0{}^2$ and $E_1{}^2$ is given.

*Extracts from text.*

We have recently referred in *MTAC* to several other tables of $C(u)$ and $S(u)$: C. M. Sparrow (v. 3, p. 479), for $u = 0(.005)8$; 4D; D. L. ARENBURG & D. LEVIN (v. 3, p. 479), for $u = 0(.1)20$, and $u = 8(.02)16$; U. S. NAVY, RES. LAB., Boston (v. 3, p. 417), for $u = 0(.1)20$; 4D or 4S; R. T. BIRGE (v. 4, p. 30), for $u = 0(.05)12.05$; 4D.

A comparison of the first 100 values of $C(u)$ and $S(u)$ of tables under review with the corresponding entries in the Sparrow tables indicated the following 29 apparent unit-errors in the fourth places of Sparrow: $S(.15)$, $C(.23)$, $S(.31)$, $C(.33)$, $S(.33)$, $C(.45)$, $C(.47)$, $C(.49)$, $S(.57)$, $S(.59)$, $S(.63)$, $S(.65)$, $S(.69)$, $C(.72)$, $C(.73)$, $C(.74)$, $C(.75)$, $C(.76)$, $S(.77)$, $C(.79)$, $S(.79)$, $C(.84)$, $S(.85)$, $C(.91)$, $C(.93)$, $S(.93)$, $S(.94)$, $C(.95)$, $C(.98)$. There are also 6 2-unit errors at $C(.77)$, $C(.78)$, $C(.80)$, $C(.81)$, $C(.82)$, $C(.83)$. These results suggest that tables based on Sparrow are likely to be not without error.

Wijngaarden & Scheen make no reference to any earlier table of Fresnel integrals. A misprint for $C(4.95)$, 0.45404 has been corrected by hand to 0.54504.

R. C. A.

[1] E. T. GOODWIN & J. STATON, "Table of $\int_0^\infty e^{-u^2}du/(u + x)$," *Quart. Jn. Mech. Appl. Math.*, Oxford, v. 1, 1948, p. 319–326. See *MTAC*, v. 3, p. 483.

**791[L].**—D. V. ZAGREBIN, "K voprosu o tochnosti formuly Stoksa" [Concerning the accuracy of Stokes' formula], Akad. Nauk. SSSR, Inst. Teoret. Astr., *Bull.*, v. 4, no. 3 (56), 1949, p. 134–141.

Table 1, p. 137, is a 3D table of six very special functions which are combinations of trigonometric and logarithmic functions with complete elliptic integrals. There are also tables to 3,4D of 15 definite integrals of products of these functions by even powers of the sine and cosine.

**792[M].**—Roy C. Spencer, Pauline Austin, Elizabeth Chisholm, Ellen Fine, & Jeane Schwartz, *Tables of Fourier Transforms of Fourier Series, Power Series, and Polynomials. Report S-58, July 10, 1945.* Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Mass., ii, 29 p. 21.4 × 27.9 cm.

The tables, occupying p. 6–27, are as follows:

T. I, p. 6–11, Fourier transform of a constant, $g_0(\phi) = \phi^{-1} \sin \phi$, $g_0{}^2$, its derivative $Dg_0$, $(Dg_0)^2$, $\phi$ = [0(5°)1080°; 8D].
T. II, p. 12–13, $D^n g_0$ for $n$ = 1(1)8, $\phi$ = [0(30°)1080°; 8D].
T. III, p. 14, Fourier transforms $T$ of $\cos n\pi x$ and $\sin n\pi x$ for $n$ = 1(1)3. Tables of $T \cos n\pi x$, $iT \sin n\pi x$, $n$ = 1(1)3, $\phi$ = [0(30°)720°; 7D].
T. IV, p. 15, Fourier transforms of $\cos \frac{1}{2}n\pi x$, $\sin \frac{1}{2}n\pi x$, $n$ = 1, 3. Tables of $T \cos \frac{1}{2}n\pi x$, $iT \sin \frac{1}{2}n\pi x$, $n$ = 1, 3, $\phi$ = [0(30°)720°; 7D].
T. V, p. 16, $T(1 - x^2)^n$, $n$ = 1(1)3, $\phi$ = [0(30°)720°; 7D].
T. VI, p. 17, Fourier transforms, $TP_n(x) = i^n j_n(\phi)$, $n$ = 2(1)4. Tables of $j_n(\phi)$, $n$ = 2(1)4, $\phi$ = [0(30°)720°; 7D].
T. VII, p. 18–25, Fourier transforms of $i^n x^n$, $g_0$, $D^n g_0$, $n$ = 1(1)8, $\phi$ = [0(.1)20; 8D].
T. VIII, p. 26–27, Fourier transforms of $(1 - x^2)^n$, $(1 + D^2)^n g_0$, $n$ = 1(1)4, $\phi$ = [0(.1)10; 8D].

There are a number of disagreements with values given in the tables of RMT **726** [*MTAC*, v. 4, p. 80–81].

R. C. A.

**793[U].**—Pierre Hugon, *Nouvelles Tables pour le calcul de la droite de hauteur à partir du point estime*, Paris, Girard, Barrère and Thomais, 1947, xiv, 92 p. 15.0 × 21.2 cm. + 1 chart 25 × 42.4 cm.

The tables are two in number; the principal table was designed for use on shipboard in calculating the altitude for the dead reckoning position by logarithms and haversines. It consists of 90 pages of five-place values of the natural haversine, log haversine, and log cohaversine, with argument 1°(1′)179°; each page contains an interpolation table for each of the three functions for tenths of a minute of arc. Preceding the principal table are two pages of corrections for refraction, height of eye and semidiameter to be applied to observed altitudes of the lower limb of the sun and of stars. The arguments are observed altitude 6°(1°)16°(2°)20°(5°)50°(10°)90° and height of eye 3(1)10(2)26 meters.

The foreword and explanation are presented first in French and again in English. The formulas used are:

$$\begin{aligned} \text{hav}\,(90° - h) &= A + B \\ \log A &= \log \text{cohav}\, t + \log \text{hav}\,(d - L) \\ \log B &= \log \text{hav}\, t + \log \text{cohav}\,(d + L) \end{aligned}$$

where $t$, $d$, and $h$ are the local hour angle, declination and altitude of the celestial body and $L$ is the dead reckoning latitude. Since the haversine and the cohaversine are always positive and between zero and one, the rule of signs is no longer needed. Also since corresponding values of the natural haversine and log haversine are given side by side, no separate table of logarithms of numbers is given.

The azimuth is determined by the use of a nomogram folded inside the back cover; it is based on the formula:

$$\cos h \cos Z = \sin (d - L) \operatorname{cohav} t + \sin (d + L) \operatorname{hav} t$$

where $Z$ is the azimuth angle of the celestial body. It is intended that the azimuth shall be determined only to the nearest degree which is generally adequate for ordinary navigational purposes.

The author suggests that the accuracy of his method as compared to the classical FRIOCOURT method is as follows:

| | $h = 60°$ | $h = 75°$ | $h = 84°$ |
|---|---|---|---|
| Friocourt | $+0'.5$ | $+0'.9$ | $+2'.2$ |
| Hugon | $+0'.4$ | $+0'.7$ | $+1'.7$ |

and hence the claim is for greater speed and ease of use rather than greater accuracy with the same number of decimals.

The printing of the tables is rather poor on the whole, but a part of the trouble may be blamed on the quality of the paper which is mediocre. It is to be hoped that the proofreading of the tables has been done with greater care than that of the foreword and explanation. In the English explanation, $X - Y = 1$ should obviously be $X + Y = 1$, and in the expression for $X$ in both the French and English explanations, cohaversine $(D + \phi)$ should be cohaversine $P$.

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## MATHEMATICAL TABLES—ERRATA

In this issue references have been made to errata in RMT **790** (Wijngaarden & Scheen), **792** (Spencer *et al.*), **793** (Hugon), and Note **118**.

**173.**—*Polnoe Sobranie Sochinenii P. L. Chebysheva* [Complete Collection of Works by P. L. Chebyshev]. Volume 1, *Teoriia Chisel* [Theory of Numbers], Moscow and Leningrad, Academy of Sciences, 1946. 342 p. + portrait frontispiece. 15 × 23 cm. 20 roubles paper; 23 roubles bound. Edition (Second, stereotyped) of 3,000 copies.

Previous editions of this volume have been reviewed in *MTAC*, v. 1, p. 440–441. The present volume not only reproduces the errata of the 1944 edition but adds many new misprints both in the text and in the tables, p. 311–339. These latter are as follows:

| page | | |
|---|---|---|
| 311 | | line −20 *for* 2372 *read* 2237 |
| 314, | $p = 13$, | $N = 12$ *read* $I = 6$ |
| | $p = 19$, | *insert* $N = 1$ |
| | $p = 23$, | *insert* 4 between 17 and 5 |
| 317, | $p = 61$, | $N$ table, *for* line 1 *read* 1 10 39 24 57 21 27 26 16 38 |
| | $p = 67$, | $I = 47$ *for* $N = 38$ *read* $N = 18$ |
| 318, | $p = 71$, | $N = 16$ *for* $I = 15$ *read* $I = 22$ |
| | | $N = 26$ *for* $I = 22$ *read* $I = 15$ |
| 319, | $p = 89$, | insert the primitive root 35 |