

24. HENRY WALLMAN, "An electronic integral transform computer and the practical solution of integral equations," *Franklin Inst., Jn.*, v. 250, 1950, p. 45-61.

A proposed device is described for presenting

$$\int_a^b K(x, t)f(t)dt$$

in the form of a graph of a function of x on a cathode ray tube. $K(x, t)$ is to be obtained by scanning a photographic plate whose opacity corresponds to the value of K at the point x, t . The multiplication of K and f and the value of $f(t)$ itself are to be obtained by using the components due to MACNEE.¹ The author shows how the persistence of the image on the screen of the cathode ray tube can be utilized to construct an iterate of the transform and describes also how non-linear transforms in the form

$$\int_a^b K(x, t)k(t, f(t))dt$$

can be obtained.

The paper also lists a variety of applications of such a device. The case in which $K(x, t) = \cos xt$ yields the impulse response of an electrical network. Such a device would readily yield the coefficients of the orthogonal expansion of a function, the Hilbert transform and the convolution integral.

Another set of applications is concerned with the solution of the integral equations. Special cases treated include simultaneous linear algebraic equations, the Volterra equation, the use of Liouville-Neumann series, Fredholm's integral equation of the second kind, the Dirichlet problem for a plane potential and a non-linear problem for a pendulum.

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¹ A. B. MACNEE, "A high speed electronic differential analyzer," *I.R.E. Proc.*, v. 37, 1948, p. 1315-1324 [*MTAC*, v. 4, p. 119-120].

NOTES

124. LESLIE JOHN COMRIE (1893-1950).—This great table maker and pioneer in the art of mechanical computation was born in New Zealand in 1893. He received his early training and a M.A. degree at the University of New Zealand. He saw active service during the first world war with the New Zealand Expeditionary Forces and after the armistice went to University College, London and Cambridge University, where he received his Ph.D. in Astronomy in 1923. After 3 years teaching in the United States at Swarthmore College and Northwestern University he returned to England and the Royal Greenwich Observatory as Deputy Superintendent of H.M. Nautical Almanac Office. He became Superintendent in 1930 and held that post until 1936. Here he introduced modern computing methods which did much to increase the efficiency and productivity of the office. It is this work which brought out his genius for the organization and keen analysis of computing and table preparation for which he later became so famous. He also served brilliantly as secretary of the BAASMTC during 1929-36 and was much concerned with the production of the committee's first six volumes. In 1937 he left the Observatory to devote his entire energy to the development of the Scientific Computing Service, the first enterprise of its kind. The history of this organization is one of lasting achievement and pioneering

effort. Its contributions to the war effort in the 1940's was the source of much justifiable pride to Comrie. He served as cooperative editor of MTAC from 1944-49, and in 1950 was elected Fellow of the Royal Society. His death, Dec. 11, 1950, came as a relief from a lingering affliction.

The reader is referred to *Mathematical Table Makers*¹ for a complete account of Comrie's writings and tables. He is perhaps known most widely as the editor of the third (1930) and later editions of *Barlow's Tables*. His most recent work was the preparation of the monumental two volume edition of *Chambers's Tables*. He delighted in the minute analysis of a computing problem with respect to a given machine and took great pleasure in finding scientific uses for a particular feature of a machine which had been intended by the manufacturer for some trivial commercial application. This exploitation of the commercially available equipment with its built-in, mass-production precision and service was always uppermost in his mind. He had no great enthusiasm for the unreliable specially made computing device. I can recall many friendly arguments with him beginning in 1932 on these two opposing points of view, my side of the argument being, in those days, rather difficult to maintain. His knowledge of numerical analysis was profound and at the same time severely practical. He will be remembered for his "throw back" method of modified differences. On his last visit to America in 1946 he was impressed by what he saw of new computing techniques, but also somewhat dismayed to see to what uses they were put. At one point of the inspection he took me aside to remark: "These people have a terrific amount to learn about computing." This remark is as applicable today as it was when he must have made it (to himself) as a young man at Greenwich. It seems to have been the keynote of this crusading calculator.

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¹ R. C. ARCHIBALD, *Mathematical Table Makers* (The Scripta Mathematica Series no. 3), New York, 1948 [*MTAC*, v. 3, p. 143]. In this work will be found two portraits of Comrie.

125. THE d^2 TEST OF RANDOM DIGITS.—The testing of digits for local randomness, when the digits are to be used in a Monte Carlo method, seems to require a different type of test from the four proposed by KENDALL and BABINGTON-SMITH.¹ Kendall's tests (frequency, serial, gap, poker) apply to random digits as used normally in random sampling, etc.

In a Monte Carlo method, random digits are used to select a random point in the unit square, the digits thus representing the coordinates of a point between (0, 0) and (1, 1).

For two such points, the probability that the square of the distance between them be less than α^2 is given by:²

$$P = \pi\alpha^2 - \frac{8\alpha^3}{3} + \frac{\alpha^4}{2},$$

for $\alpha^2 = 0.0, 0.1, 0.2, \dots, 0.9$

$$P = \frac{1}{3} + (\pi - 2)\alpha^2 + 4(\alpha^2 - 1)^{\frac{1}{2}} + \frac{8}{3}(\alpha^2 - 1)^{\frac{3}{2}} - \frac{\alpha^4}{2} - 4\alpha^2 \sec^{-1} \alpha$$

for $\alpha^2 = 1.0, 1.1, 1.2, \dots, 2.0$.

P is, of course, a continuous function. Discrete values of α^2 are selected for convenience in computation.

These probabilities are, in order:

α^2	P	α^2	P
0.1	.234832	1.1	.985703
0.2	.409805	1.2	.992048
0.3	.549300	1.3	.995788
0.4	.662018	1.4	.997926
0.5	.752987	1.5	.999080
0.6	.825601	1.6	.999652
0.7	.882349	1.7	.999898
0.8	.925163	1.8	.999982
0.9	.955593	1.9	.999999
1.0	.974926	2.0	1.000000

In applying the test, sets of 3 digits are sufficient to represent one coordinate; i.e., 12 digits are selected to give both coordinates of two points.

In a limited test, no significant difference was found when using two-digit coordinates; in other words, the third digit has only a minor effect on the square of the distance. The square of the distance is computed and punched to one decimal place. The distribution of the results is compared to the theoretical by the Chi-squared test. The calculation can be readily carried out on any of the IBM calculators (602, 602A, 604). The distribution of the results can be made in one run on a tabulator equipped with two digit selectors.

This test was carried out on the random digit table of the University of Wisconsin Computing Service (10,000 cards, each bearing 40 digits), making six calculations of d^2 on each card, selecting the columns to be read at random. Results were tabulated for each 1000 cards (6000 d^2 's); the χ^2 analysis giving:

Thousand	χ^2	p
1	17.085	.26
2	11.847	.62
3	18.585	.19
4	7.529	.91
5	7.762	.91
6	11.340	.66
7	14.162	.44
8	19.088	.17
9	8.370	.86
10	9.747	.81

using 14 degrees of freedom, since the last 6 classes were lumped together.

Wiring diagrams for the d^2 calculation on the 602A calculating punch and the distribution of results on the 416 or 405 tabulator are available on request.

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¹ M. G. KENDALL & B. BABINGTON-SMITH, *R. Stat. Soc., Jn.*, v. 101, 1938, p. 157, and *Supplement*, v. 6, 1939, p. 51.

² BENJAMIN WILLIAMSON, *Integral Calculus*. 6th ed., London, 1891, p. 390.

126. ON THE CALCULATION OF THE SQUARE ROOT BY AUTOMATIC COMPUTING MACHINES.—The calculation of the square root of a number on an automatic computing machine not equipped with an internally controlled square root order requires the application of some standard sub-program. It is generally convenient to employ a well-known iterative procedure based upon the formula

$$(1) \quad R^{(n+1)} = \frac{1}{2}[R^{(n)} + N/R^{(n)}],$$

where $R^{(n)}$ is the n th approximation to the square root $R = N^{\frac{1}{2}}$. The efficient application of this procedure requires a systematic method of selecting a zeroth approximation $R^{(0)}$ to the square root R , and the number of iterations required to calculate R to a specified number of significant figures depends largely upon the accuracy of the zeroth approximation.

Let N be a decimal number in the range $1 \leq N < 100$. Define ξ by

$$R = \xi + N/10,$$

and let the zeroth approximation $R^{(0)}$ be given by

$$R^{(0)} = \bar{\xi} + N/10,$$

where $\bar{\xi}$ is an average value of ξ . If the number N is distributed uniformly on a logarithmic scale,¹ we may take

$$\bar{\xi} = (\log 100)^{-1} \int_1^{100} (N^{\frac{1}{2}} - N/10) d \log N = 1.759 \sim 2,$$

the one-digit approximation $\bar{\xi} = 2$ being sufficiently accurate. Therefore, we let

$$(2) \quad R^{(0)} = 2 + N/10.$$

With (1), the first approximation $R^{(1)}$ may be written

$$R^{(1)} = \frac{1}{2} \left[\frac{N}{10} + 2 + \frac{10N}{N+20} \right].$$

This approximation differs from the root R by an amount $\Delta = R^{(1)} - R$, where

$$\Delta = 1 + \frac{R^2}{20} + \frac{5R^2}{R^2 + 20} - R.$$

In the range $1 \leq R \leq 10$, Δ is positive. It is equal to 0.25 at $R = 1$, decreases to a minimum of 0.038 at $R = 1.957 \dots$ and is equal to 0.166 \dots at $R = 10$. Therefore, the choice, (2), of a zeroth approximation leads to a first approximation with at least one correct significant figure. The calculation of the square root by the application of (1) correct to eight significant figures requires a maximum of five iterations and to sixteen significant figures a maximum of six iterations.

If N^* is any positive number such that $N^* = 10^{2k}N$, $1 \leq N < 100$, the zeroth approximation $R^{(0)*}$ to the root $R^* = (N^*)^{\frac{1}{2}}$ is

$$R^{(0)*} = 10^k(2 + N/10).$$

This method has been applied in this laboratory to the formulation of a square-root order on control panels for 10-digit arithmetic utilizing the IBM Card-Controlled Electronic Calculator.

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¹ It is reasonable to assume that dimensional numbers will be uniformly distributed on a logarithmic scale if the choice of dimensional units is random. However, this point is unimportant, since this result is comparatively insensitive to the nature of the averaging process.

QUERIES

37. THE SQUARE ROOT METHOD FOR LINEAR EQUATIONS.—In a letter dated 7 Feb. 1950 Mr. H. F. RAINSFORD of Colonial Surveys, Bushy Park, Teddington, England, commenting on the article entitled "The square root method for solving simultaneous linear equations" by J. LADERMAN in *MTAC*, v. 3, p. 13–16, points out that this method was not "probably first discovered by Banachiewicz in 1938" but goes back at least to CHOLESKY whose treatment of the problem was described by BENOÎT¹ in 1924. Can any reader supply an earlier reference to this method?

¹ BENOÎT, "Note sur une méthode de résolution des équations normales provenant de l'application de la méthode des moindres carrés à un système d'équations lineaires en nombre inférieur à celui des inconnues. Application de la méthode a la résolution d'un système define d'equations linéaires," International Geodetic and Geophysical Union, Association of Geodesy, *Bulletin Géodésique*, no. 2, 1924, p. 67–77. An English translation of this article has been kindly supplied by Mr. Rainsford and is available in the UMT FILE.

QUERIES—REPLIES

47. RUSSIAN BESSEL FUNCTION TABLES (Q 25, v. 3, p. 66).—The volume referred to in this Query, namely: *Tablitsy Znacheniy Funktsiy Bessela ot Mnimogo Argumenta*, was not published until 1950. [It will be reviewed in the next issue of *MTAC*.]

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