temperature. Application of such boundary conductances seems impossible with the present technique. Secondly, in many steady-state problems two or more materials of different conductivity enter the picture. Application of the membrane technique to such cases does not seem possible. The authors mention the desirability of investigating the membrane analogy in the realm of transient-heat conduction problems. There is no indication either on a mathematical or experimental ground that such approach is possible.

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NOTES

127. AN ITERATIVE PROCESS.—In a recent paper HARTREE[1] calls attention to the need for the development of a general method of obtaining iteration formulas of second and higher orders for the solution of any "algebraic" (i.e., not differential) equation. If $y = Y$ is the true solution of $F(y) = 0$, and $y_n = Y + \eta_n$ is an approximation to Y, the formula $y_{n+1} = G(y_n)$ is an iterative process. If $\eta_{n+1} = O(\eta_n{}^r)$, Hartree calls the process r-th order. The number of correct figures in y_{n+1} is approximately r times the number in y_n, so it is obvious why a high-order process is desired.

The NEWTON-RAPHSON process, based on

$$G(y) = y - F(y)/F'(y)$$

is second order, as Hartree points out. In effect, this process draws a tangent to the curve $Z = F(y)$ at $y = y_n$, and takes y_{n+1} at the intersection of this tangent with the y-axis. The primary source of error is the curvature of F, and any operation that reduces the curvature will improve the convergence of the iteration. If we write

$$H(y) = F(y)\{F'(y)\}^{-\frac{1}{2}}$$

we find

$$H''(y) = -\tfrac{1}{2}F(y)\{F_0''(y)(F'(y))^{-3/2}\}'.$$

The function $H(y)$ consequently has the same roots as F, and zero curvature at each of them. If we apply the Newton-Raphson process to it we get

$$G(y) = y - 2FF'/(2F'^2 - FF''),$$

and it is easily verified that this process is third-order.

As an example, if $F(y) = y^2 - a$ we get

$$y_{n+1} = y_n(y_n{}^2 + 3a)/(3y_n{}^2 + a)$$

as a third-order process for computing $a^{\frac{1}{2}}$. For $a = 10$, $y_0 = 3$, we find $y_1 = 3.16216$, $y_2 = 3.162277660168341$, which is in error by 4 units in the fifteenth figure.

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[1] D. R. HARTREE, "Notes on iterative processes," Camb. Phil. Soc., *Proc.*, v. 45, 1949, p. 230.