25. G. B. WALKER, "Factors influencing the design of a rubber model," Inst. Elec. Engrs., *Proc.*, v. 96, part II, 1949, p. 319–324. Discussion of this paper v. 97, part II, 1950, p. 439–444.

In the design of vacuum tubes a rubber membrane is often used to give a gravitational reproduction of the potential field in the tube and steel balls are used to reproduce the electrons. The author discusses the errors due to the fact that the surface of the membrane does not exactly satisfy the Laplace equation, the error due to spin of the ball around the axis normal to the surface and frictional forces. The surface error depends on the maximum gradient and is shown to be negligible in certain special cases. The effect of the "spin" terms is shown to be dependent on the scale factor. Frictional losses are the most important, and a method of measuring these in a special set up is described. The author concludes that the model should be as small as possible and that the error in the ball's kinetic energy (due to friction) can be kept less than 2 per cent of the maximum potential differences between points of the boundary.

In connection with the discussion on this paper the electrolytic tank of BOOTHROYD, CHERRY and MAKAR [cf. *MTAC*, v. 33, p. 49–50] and the resistance networks of E. E. HUTCHINGS and of G. LIEBMANN [cf. *MTAC*, v. 35, p. 179] were demonstrated. The accuracies of the various systems were compared also in these discussions.

<div align="right">F. J. M.</div>

NOTES

129. ZEROS OF $I_{n+1}(x)J_n(x) + J_{n+1}(x)I_n(x)$. A table of the first ten zeros of $f_n(x) \equiv I_{n+1}(x)J_n(x) + J_{n+1}(x)I_n(x)$ for $n = 0, 1, 2,$ and 3, was published by AIREY[1]. This table is extended herewith to include all zeros ≤ 20. For the sake of completeness, Airey's values are reproduced here, with the kind permission of the editors of the *Proceedings*.

Airey's values were compared with those of CARRINGTON,[2] who gave all zeros ≤ 16. Corresponding to $n = 0$, Airey gave the first zero as 3.1955, whereas Carrington gave 3.1961. This entry was recomputed; the true value to five decimals is 3.19622. Other entries in Airey's differ from Carrington's by at most a unit in the third decimal place, where both authors give the same zeros. Differences of Airey's values show no obvious errors, but his entries were not otherwise verified by us.

G. FRANKE[3] published the first two zeros for $n = 4$ and the first zero for $n = 5, 6,$ and 7, to one, two, or three decimals. Comparison of his entries with those published here shows that his last place is not correct.

The entries given here were computed by inverse interpolation in $[\exp(-x)]f_n(x)$, with the aid of values of $I_n(x)$ which were made available to us in manuscript form by J. C. P. MILLER. The extensive tables of $J_n(x)$ of the Harvard Computation Laboratory provided the other required tabular values. GLADYS FRANKLIN of the NBSINA performed the computations. Entries for $n > 3$ are correct to within $\pm.00002$. The work was carried out with the aid of funds provided by the ONR, in connection with an eigenvalue problem investigated by N. ARONSZAJN.

Zeros of $f_n(x)$

s	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
1	3.19622	4.611	5.906	7.144	8.34661
2	6.3064	7.799	9.197	10.536	11.83672
3	9.4395	10.958	12.402	13.795	15.14987
4	12.5771	14.109	15.579	17.005	18.39596
5	15.7164	17.256	18.745	20.192	
6	18.8565	20.401	21.901	23.366	
7	21.9971	23.545	25.055	26.532	
8	25.1379	26.689	28.205	29.693	
9	28.2790	29.832	31.354	32.849	
10	31.4200	32.975	34.502	36.003	

s	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$
1	9.52570	10.68703	11.83453	12.97091	14.09809
2	13.10736	14.35516	15.58455	16.79874	18.00010
3	16.47508	17.77643	19.05806		
4	19.75828				

s	$n = 10$	$n = 11$	$n = 12$	$n = 13$	$n = 14$
1	15.21753	16.33031	17.43732	18.53925	19.63669
2	19.19045				

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[1] J. R. Airey, "The vibrations of circular plates and their relation to Bessel functions." Phys. Soc., London, *Proc.*, v. 23, Dec. 1910–Aug. 1911, p. 225–232.

[2] H. Carrington, "The frequencies of vibration of flat circular plates fixed at the circumference," *Phil. Mag.*, s. 6, v. 50, 1925, p. 1261–1264.

[3] Georg Franke, "Erzwungene Schwingungen einer eigenspannten kreisförmigen Platte." *Annalen der Physik*, s. 5, v. 2, 1929, p. 649–675.

130. A Method of Factorisation Using a High-Speed Computer. The usual process of finding the factors or establishing the primality of a large number N involves the determination of the remainders r_n in the equation

(1) $$N = f_n q_n + r_n, \quad 0 \leqq r_n < f_n.$$

Only prime numbers need be taken for f_n, but in practice we take all integers less than $N^{\frac{1}{2}}$ except multiples of small primes 2, 3, 5, etc.

If N is a large prime the work involved is considerable and the time for the complete operation depends mainly on the speed of division of the computer.

This note describes a method of factorisation which replaces the division routine at least for the range of f_n from $2N^{\frac{1}{3}}$ to $N^{\frac{1}{2}}$ by operations which are more rapid on some machines. Since for large N this range includes most of the possible f_n a considerable saving of time is effected on these machines.

The method uses the known relation between the selected f_n to determine from equation (1) a relation between successive r_n from which r_n can be obtained by a few additions and subtractions and duplications.

Suppose, for simplicity, that $f_{n+1} = f_n + 2$. Then if ∇ denotes the backward difference operator we can easily show that

(2) $$\nabla^2 r_{n+1} = -4\nabla q_n - (f_n + 2)\nabla^2 q_{n+1}.$$

Now
$$(N/f_n) - 1 < q_n \leqq (N/f_n),$$
so that

(3)
$$\nabla^2(N/f_n) - 2 < \nabla^2 q_n < \nabla^2(N/f_n) + 2.$$

also

(4)
$$\nabla^2(N/f_n) = 16/\{f_n(f_n - 2)(f_n - 4)\}.$$

For $f_n \geqq 2N^{\frac{1}{2}} + 1$, we have from (4)
$$\nabla^2(N/f_{n+1}) = \frac{8N}{(2N^{\frac{1}{2}} + 3)(2N^{\frac{1}{2}} + 1)(2N^{\frac{1}{2}} - 1)} < 1.$$

It follows from (3) that $\nabla^2 q_{n+1}$ can have only the values $-1, 0, 1$ or 2.

The process of testing whether f_{n+1} is a factor of N then consists of the following steps, starting from the point at which, for some n, the quantities $r_{n-1}, r_n, f_n, \nabla q_n$ are stored in memory positions.

A   Replace f_n by $f_{n+1} = f_n + 2$.
B   Replace r_n by $r_{n+1} = 2r_n - r_{n-1} - 4\nabla q_n - (f_n + 2) X$, where X is one of $-1, 0, 1, 2$, to be chosen uniquely so that $0 \leq r_{n+1} < f_n + 2$.
C   Replace r_{n-1} by r_n.
D   Replace ∇q_n by $\nabla q_{n+1} = \nabla q_n + X$.
E   Test for $r_{n+1} = 0$.
F   Test for $f_{n+1} \geqq N^{\frac{1}{2}}$.

This process is manifestly simple. On the pilot model of the ACE we have used it to establish the primality or discover a factor of a twelve decimal digit number in less than 15 minutes, each step in the above process taking 1 millisecond.

For $f_n < 2N^{\frac{1}{2}} + 1$ some of the advantage of the process is lost since X is no longer necessarily $-1, 0, 1$ or 2 but has to be found at step (B) above by dividing $2r_n - r_n - 1 - 4\nabla q_n$ by $f_n + 2$. By using this process throughout, however, a powerful check is provided. Each r_n depends on the previous remainders so that if the machine finds no factor and if the last remainder found is correct, as can be verified by direct division by the last trial factor, considerable confidence can be placed in the result that the particular number is prime.

Modifications of the above process make possible the treatment of cases in which f_n is an arithmetical progression such as $4n + 1$.

The work described above has been carried out as part of the research program of the National Physical Laboratory and this article is published by permission of the Director of the Laboratory.

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[Editorial Note: Roselyn Lipkis reports that, as applied to the SWAC, the above method speeds up the factoring process by a factor of 8. All six steps A–F are performed in approximately 1.3 milliseconds.]

131. RECENT DISCOVERIES OF LARGE PRIMES. Ever since LUCAS announced the discovery of the prime $2^{127} - 1$ in 1876, many attempts have been made to discover larger primes. These attempts have succeeded only recently as follows:

(a) A. FERRIER[1] has identified $(2^{148} + 1)/17$ as a prime, using a method based on the converse of Fermat's theorem and a desk calculator.

(b) Using the same method and the EDSAC, WHEELER and MILLER[2,3] have proved the primality of $1 + k(2^{127} - 1)$ for $k = 114, 124, 388, 408, 498, 696, 738, 744, 780, 934, 978$, and finally $1 + 180(2^{127} - 1)^2$, a number of 79 decimal digits.

(c) Using the standard LUCAS test for Mersenne primes as programmed by R. M. ROBINSON, the SWAC has discovered the primes $2^{521} - 1$ and $2^{607} - 1$ on January 30, 1952. These lead to the 13th and 14th perfect numbers.

<div align="right">D. H. L.</div>

[1] Letter of July 14, 1951.
[2] J. C. P. MILLER & D. J. WHEELER, "Large prime numbers," *Nature*, v. 168, 1951, p. 838.
[3] J. C. P. MILLER, "Large primes," *Eureka*, 1951, no. 14, p. 10–11.

QUERIES

40. TABLE OF MULTIPLICATION.—Brown University has just acquired a copy of J. B. Oyon, *Tables de Multiplication, A l'Usage de MM. les Géomètres*. Second edition, Paris, 1812; quarto, 507 p., bound in two volumes. This work gives the product of all integer pairs up to 509×500. There is no indication of any author's name, but in the *Catalogue Général des Livres emprimés de la Bibliothèque National*, v. 128, we find the work listed under Oyon's name in a third edition, Paris, 1824; and also a fourth edition, v. 2, Lyon, 1864, which seems to continue the table to 509×1000. The Catalogue's first publication listed after Oyon's name is a 4-volume *Collection des Lois, Arrêtés, Instructions . . .* , Paris, 1804–1808.

Where may information concerning Oyon be found? When was the first edition of his *Tables* published and where may it be consulted? The third edition is also in the British Museum. What other libraries have the second and fourth editions?

<div align="right">R. C. ARCHIBALD</div>

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CORRIGENDA

V. 5, p. 67, eqn. (2), *for* $=$ *read* $\doteq$.
V. 5, p. 116, l. $-10, -9,$ *for* Column of Probabilities *read* Column of Expectations.
V. 5, p. 118, l. 20, *for* ten *read* $n + 2$.
V. 5, p. 119, l. $-8,$ *for* C to N *read* C to NX.
V. 5, p. 130, l. 17, *for* k_2/h *read* $k_2/2$.
V. 5, p. 163, l. 11, *for* K *read* $\bar{K}$.
V. 5, p. 167, l. $-15,$ *for* A. C. *read* C. A.
V. 5, p. 167, l. $-14,$ *for* Camb. Phil. Soc. *Proc., read* Phil. Mag.
V. 5, p. 258, l. $-3,$ *for* **49** *read* **39**.