# MATHEMATICAL TABLES—ERRATA

In this issue references have been made to Errata in RMT **939** (SELMER), **946** (SICHEL), and in Note **129**.

**198.**—R. T. BIRGE, "Least squares' fitting of data by means of polynomials," *Revs. Mod. Phys.*, v. 19, 1947, p. 298–347.

P. 341, Table XII, $n = 8$, *for* $S_{16} = 16.3635416^*$ *read* $32.727083^*$.

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**199.**—R. A. FISHER & F. YATES, *Statistical Tables for Biological, Agricultural and Medical Research*. 3rd ed. New York, 1948.

These tables include five significance levels for the distribution of the variance ratio, $F$, and of $z = \frac{1}{2} \ln F$, as follows: $p = 0.2$ (calculated by H. W. NORTON), $p = 0.1$ "based on the tables of the incomplete Beta function of CATHERINE M. THOMPSON,[1] for which we are indebted to Professor E. S. PEARSON and Dr. V. G. PANSE," $p = 0.05$ and $0.01$ (calculated by R. A. FISHER), and $p = 0.001$ (attributed to C. G. COLCORD & L. S. DEMING[2]). The last is seriously infested with error, some being carried into the table of the $t$ distribution, and there are a few errors in the others, as follows:

| Page | $p$ | $n_1$ | $n_2$ | Function | *For* | *Read* |
|---|---|---|---|---|---|---|
| 34–35 | 0.2 | 4 | 1 | $z$ | 1.3097 | 1.3067 |
| | | | | $F$ | 13.73 | 13.64 |
| | | 8 | 1 | $z$ | 1.3400 | 1.3397 |
| | | | | $F$ | 14.59 | 14.58 |
| | | 24 | 2 | $z$ | 0.7452 | 0.7453 |
| | | 24 | 21 | $z$ | 0.1829 | 0.1831 |
| 36 | 0.1 | ∞ | 120 | $z$ | 0.0081 | 0.0881 |
| 41 | 0.01 | 1 | 2 | $F$ | 98.49 | 98.50 |
| | | 8 | 2 | $F$ | 99.36 | 99.37 |
| 42 | 0.001 | 1 | 3 | $z$ | 2.5604 | 2.5591 |
| 43 | | | | $F$ | 167.5 | 167.0 |
| 32 | | | | $t$ | 12.941 | 12.924 |
| 42 | | 1 | 5 | $z$ | 1.9255 | 1.9270 |
| 43 | | | | $F$ | 47.04 | 47.18 |
| 32 | | | | $t$ | 6.859 | 6.869 |
| 42 | | 1 | 7 | $z$ | 1.6874 | 1.6879 |
| 43 | | | | $F$ | 29.22 | 29.25 |
| 32 | | | | $t$ | 5.405 | 5.408 |
| 42 | | 1 | 40 | $z$ | 1.2674 | 1.2672 |
| | | 1 | 120 | $z$ | 1.2158 | 1.2159 |
| | | 2 | 5 | $z$ | 1.8002 | 1.8071 |
| 43 | | | | $F$ | 36.61 | 37.12 |
| 42 | | 2 | 29 | $z$ | 1.0903 | 1.0901 |
| | | 2 | 60 | $z$ | 1.0248 | 1.0250 |
| | | 2 | 120 | $z$ | 0.9952 | 0.9954 |
| 43 | | | | $F$ | 7.31 | 7.32 |

| Page | $p$ | $n_1$ | $n_2$ | Function | *For* | *Read* |
|---|---|---|---|---|---|---|
| 42 | 0.001 | 3 | 1 | $z$ | 6.5966 | 6.6000 |
| | (cont.) | 3 | 40 | $z$ | 0.9435 | 0.9431 |
| | | 4 | 6 | $z$ | 1.5433 | 1.5438 |
| 43 | | | | $F$ | 21.90 | 21.92 |
| 42 | | 4 | 7 | $z$ | 1.4221 | 1.4224 |
| | | 4 | 8 | $z$ | 1.3332 | 1.3333 |
| | | 5 | 6 | $z$ | 1.5177 | 1.5175 |
| | | 5 | 19 | $z$ | 0.9442 | 0.9452 |
| 43 | | | | $F$ | 6.61 | 6.62 |
| 42 | | 8 | 5 | $z$ | 1.6596 | 1.6598 |
| | | 8 | 29 | $z$ | 0.7679 | 0.7669 |
| | | 12 | 21 | $z$ | 0.7735 | 0.7734 |
| | | 24 | 5 | $z$ | 1.6123 | 1.6121 |
| | | 24 | 6 | $z$ | 1.4134 | 1.4136 |
| | | 24 | 8 | $z$ | 1.1662 | 1.1659 |
| | | 24 | 19 | $z$ | 0.7277 | 0.7279 |
| | | 24 | 21 | $z$ | 0.6964 | 0.6965 |
| | | 24 | 120 | $z$ | 0.4380 | 0.4381 |
| | | $\infty$ | 3 | $z$ | 2.4081 | 2.4080 |
| 42 | | $\infty$ | 60 | $z$ | 0.3198 | 0.3184 |
| | | $\infty$ | 120 | $z$ | 0.2199 | 0.2170 |
| 43 | | | | $F$ | 1.56 | 1.54 |

These errors were found chiefly by differencing, using comparison with other tables and recalculation where differencing was inadequate or inconclusive. It is probable that no other tabular value differs from the true value (as distinct from the "correct" tabular value) by more than unity in the last figure. However, there are a number of rounding errors, that is, tabular entries which differ from the correct tabular value by unity in the last figure, the true value lying between the two. This means that the amount rounded off is between 0.5 and 1.0 in absolute value, rather than between 0 and 0.5.

In addition, among the values of chi-square derived from Colcord and Deming's table of $z$, there are the following three errata:

| Page | $p$ | $n$ | *For* | *Read* |
|---|---|---|---|---|
| 33 | .001 | 3 | 16.268 | 16.266 |
| | | 4 | 18.465 | 18.467 |
| | | 5 | 20.517 | 20.515 |

The first of these involves a rounding error in $z$. The second and third arise because 4D in $z$ is here insufficient for 5S in chi-square.

Some explanation and comment as to the origin of these tables may prove helpful. As the tables for $p = 0.2$ were first published in F & Y, it is appropriate to remark that the 339 entries were calculated variously, 118 by interpolation in KARL PEARSON's "Tables of the Incomplete Beta Function" and "Tables of the Incomplete Gamma Function," 25 from R. A. Fisher's values of chi-square, 88 by direct calculation, and 108 (corresponding to most values of $n_2$ greater than 12) by harmonic interpolation in $z$.

Tables for $p = 0.1$ have been given for F by M. MERRINGTON & C. M. THOMPSON[3] and for $z$ and $F$ by V. G. PANSE & G. R. AYACHIT.[4] FISHER &

YATES mention both these sources. There are 41 entries in which $z$ according to F & Y differs by unity from that given by P & A. Examination of these cases shows that F & Y derived their values of $z$ from M & T. This partly explains the disagreement, because P & A derived their values of $z$ from Thompson's table, and inevitable rounding errors in the two source tables sometimes lead to different 4D values of $z$. In addition, the disagreement is explained by the inadequacy of Thompson's table for 4D in $z$, and by the occurrence of a number of rounding errors in deriving P & A's table. No rounding error was discovered in F & Y. The single gross error listed above is obviously typographical.

The statement by F & Y that their table is "based on the tables [by] Thompson" is thus shown to be misleading. In fact, contrary to Egon Pearson's statement in the introductory note thereto, the M & T table of $F$ cannot be derived from Thompson's table, as Thompson's values are sometimes insufficient for the accuracy to which M & T give F. For the same reason, F & Y cannot have derived their values of $z$ from Thompson's table. Surely also F & Y took their values of $F$ from M & T: there is no discrepancy between the two, and F & Y retain an even figure in the second decimal in every one of the five cases in which M & T give 50 in the third and fourth decimals.

The tables for $p = 0.05$ and 0.01 are entirely free of gross error for both $z$ and $F$, but there are a few rounding errors.

The table of $z$ for $p = 0.001$ as given by F & Y disagrees with that originally published by COLCORD & DEMING in ten entries. For $n_1 = n_2 = 1$, C & D gave 6.4577. When calculating the table for $p = 0.2$, I discovered this error and communicated it to Fisher. Thus F & Y give the correct value 6.4562. However, it may be noted that Fisher's "Statistical Methods for Research Workers," which included this table for the first time in the sixth edition, has repeated this error in all subsequent editions, all of which have appeared since this error was known. A second error occurred at $n_1 = 12$, $n_2 = 2$, for which C & D gave 3.4537. F & Y correctly give 3.4536, but Fisher's "Statistical Methods . . ." has always given the incorrect value.

The remaining eight differences all involve values for $n_2 = 120$, as follows:

| $n_1$ | C & D | F & Y | Correct |
|---|---|---|---|
| 1 | 1.2159 | 1.2158 | 1.2159,37 |
| 2 | 0.9948 | 0.9952 | 0.9953,81 |
| 3 | 0.8783 | 0.8773 | 0.8773,20 |
| 5 | 0.7425 | 0.7426 | 0.7425,80 |
| 6 | 0.6983 | 0.6986 | 0.6985,86 |
| 8 | 0.6329 | 0.6338 | 0.6337,39 |
| 24 | 0.4369 | 0.4380 | 0.4381,28 |
| $\infty$ | 0.2224 | 0.2199 | 0.2169,64 |

It appears that F & Y gave correct values for $n_1 = 3$, 5, and 6, improved values for $n_1 = 2$, 8, 24, and $\infty$, and replaced a correct value by an erroneous one for $n_1 = 1$. The source of F & Y's values is unknown (Yates, private correspondence). As Fisher's "Statistical Methods . . ." gives $n_2 = 1$ (1) 30, 60, $\infty$, it seems reasonably certain that the values for $n_2 = 40$ and 120 were filled in (when F & Y decided to include them) without reference to C & D.

I have tried to reproduce F & Y's values by several schemes of interpolation and approximation without success.

The table of $F$ for $p = 0.001$ was prepared from the table of $z$ by W. L. STEVENS and incorporates some of its errors. However, many of the errors in $z$ are too small to affect the values of $F$, generally to 2D. Also, the large error at $n_1 = 3$, $n_2 = 1$, is not found in the table of $F$ because Stevens calculated *de novo* all the values of $F$ for $n_2 = 1$ to 6S. This seemed the most satisfactory way of handling the problem of significant figures for these values, the 4D values of $z$ being here sufficient for only 4S in F.

A further observation is that the approximation formulas for use when $n_1$ and $n_2$ are both large are due to F & Y, not to the calculator(s) of the table to which any such formula is appended.

Lastly, the tables of $F$ by MERRINGTON & THOMPSON have received some scrutiny, though no thorough test has been made. Therefore the situation is not entirely clear, but it should not be thought that M & T are uniformly dependable in the last decimal tabulated, there being the following three errata, all for $p = 0.01$:

| $n_1$ | $n_2$ | *For* | *Read* |
|---|---|---|---|
| 3 | 120 | 3.9493 | 3.9491 |
| 8 | 1 | 5981.6 | 5981.1 |
| $\infty$ | 2 | 99.501 | 99.499 |

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[1] C. M. THOMPSON, "Tables of percentage points of the incomplete beta-function and of the chi-square distribution," *Biometrika*, v. 32, 1941, p. 151–181, 187–191.
[2] C. G. COLCORD & L. S. DEMING, "The one-tenth percent level of '$z$,'" *Sankhya*, v. 2, 1935, p. 423–424.
[3] M. MERRINGTON & C. M. THOMPSON, "Tables of percentage points of the inverted Beta (F) distribution," *Biometrika*, v. 33, 1943, p. 73–88.
[4] V. G. PANSE & G. R. AYACHIT, "Ten percent probability of $z$ and the variance ratio," *Indian Jn. Agricultural Science*, v. 14, 1944, p. 244–247.

**200.**—G. INGHIRAMI, *Table des Nombres Premiers et de la Décomposition des Nombres de* 1 *á* 100000. 1919.

Supplementary to the list of errata in D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, 1941, pp. 150–151, the following may be noted:

For 4 primes a dot (.) is not clearly printed: 69467, 69473, 69481, 69557.
For 8 numbers on p. 25 corrections are here noted:

| Number | *For* | *Read* | Number | *For* | *Read* |
|---|---|---|---|---|---|
| 67069 | 4 | 47 | 68593 | | 7 |
| 68359 | 19 | 197 | 68773 | 9 | 97 |
| 68363 | 13 | 137 | 68873 | | 7 |
| 68573 | | 47 | 69169 | 2 3 | 263 |

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**201.**—K. L. NIELSEN & L. GOLDSTEIN, "An algorithm for least squares," *Jn. Math. Phys.*, v. 26, 1947, p. 120–132.

*P*. 123, $m = 35$ *for* $A_{55} = 488447.843200$ *read* 488447.843265
*P*. 124, $m = 85$ *for* $A_{55} = 102214274.780204$ *read* 102214274.782041
*P*. 124, $m = 90$ *for* $A_{55} = 144092594.780204$ *read* 144092594.782041

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**202.**—I. M. VINOGRADOV & N. G. CHETAEV, *Tabli͡tsy Znacheniĭ Funk͡tsiĭ Bessel͡ia ot mnimogo Argumenta.* Moscow, Leningrad, 1950.

On pages III, V, 203–403, and on the spine, there are 408 errors in statements as to functions tabulated, namely: $J_{\frac{1}{3}}(ix)$ and $J_{-\frac{1}{3}}(ix)$. The correct functions are $i^{-\frac{1}{3}}J_{\frac{1}{3}}(ix) = I_{\frac{1}{3}}(x)$ and $i^{\frac{1}{3}}J_{-\frac{1}{3}}(ix) = I_{-\frac{1}{3}}(x)$.

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## UNPUBLISHED MATHEMATICAL TABLES

**136[F].**—A. FERRIER. *Factorization of* $n! \pm \alpha$. Photocopy of 4 manuscript pages. Deposited in the UMT FILE.

Two pages of tables give the complete decomposition of $n! \pm \alpha$ for $n = 7(1)15$, $\alpha = 2(1)20$ together with 13 other miscellaneous examples.

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**137[F].**—A. FERRIER. *Table of Factors of* $2^n - 1$. Photocopy of 5 manuscript pages. Deposited in the UMT FILE.

Two pages of tables give the latest information on the factors of $2^n - 1$, $n = 3(2)499$.

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**138[F].**—R. F. JOHNSON. *Tables of Products of Powers of Small Primes.* Tabulated from punched cards. Deposited in the UMT FILE.

There are two tables of

$$N = 2^\alpha 3^\beta 5^\gamma 7^\delta$$

for $\alpha = 0(1)11$; $\beta = 0(1)8$; $\gamma = 0(1)5$; $\delta = 0(1)4$. The first table is arranged lexicographically by $\alpha$, $\beta$, $\gamma$, $\delta$. The second is arranged in increasing order of $N$. Each table contains 3240 values of $N$ range between 1 and 100818950400000. The table is intended to facilitate the design of gear trains.

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