and

$$E(v_{ik}^2) = \frac{1 + r_{kk}}{1 - r_{kk}} \frac{r_{ik}^{1-\delta_{ik}}}{1 - r_{kk}}. \tag{8}$$

Thus, a comparison of (3) and (4) yields

$$E(s_{ik}^2) < E(v_{ik}^2) \quad \text{if and only if} \quad \frac{1}{p_{k0}} < 2q^{kk} - 1 \tag{9}$$

and similarly, (7) and (8) give

$$E(s_{ik}^2) < E(v_{ik}^2) \quad \text{if and only if} \quad \frac{1}{p_{k0}} < \frac{1 + r_{kk}}{1 - r_{kk}}. \tag{10}$$

This, of course, is Wasow's result expressed in terms of r_{kk} rather than λ_{kk}. Since r_{kk} is usually unknown, but $r_{kk} \geqslant p_{kk}$; (10) implies

$$E(s_{ik}^2) < E(v_{ik}^2) \quad \text{if} \quad \frac{1}{p_{k0}} < \frac{1 + p_{kk}}{1 - p_{kk}}. \tag{11}$$

These recurrent events associated with this stochastic process have permitted the derivation of further results which will be treated in a subsequent paper.

H. P. Edmundson

University of California
at Los Angeles

[1] W. R. Wasow, "A note on the inversion of matrices by random walks," *MTAC*, v. 6, 1952, p. 78–81.
[2] W. Feller, *An Introduction to Probability Theory and its Applications*, v. 1, New York, 1950.
[3] G. E. Forsythe & R. A. Leibler, "Matrix inversion by a Monte Carlo method," *MTAC*, v. 4, 1950, p. 127–129.
[4] J. H. Curtiss, "Sampling methods applied to differential and difference equations," IBM *Seminar on Scientific Computations*, November 1949.

RECENT MATHEMATICAL TABLES

1042[A].—C. E. Fröberg, *Hexadecimal Conversion Tables*. Lund, 1952, 20 p., 21.7 × 14.6 cm.

These tables are designed to assist in the coding of problems for binary computers. There are four tables. Table 1 is a table of the integers $1(1)2^{10}(2^4)2^{12}$ in both decimal and hexadecimal notation. Table 2 gives the hexadecimal equivalents of $n \cdot 10^{-2k}$, $n = 1(1)100$, $k = 1(1)8$, correct to 13 hexadecimals. Table 3 gives the hexadecimal equivalent of 50 frequently used constants correct to 16 hexadecimals. Finally Table 4 gives the decimal equivalents of numbers of the form

$$n \cdot 16^{-k} \quad n = 1(1)15, k = 1(1)13.$$

Results are given to 16D. The letters A B C D E F are used as hexadecimal notation for the numbers ten through fifteen. A few examples of the use of the tables are given in the introduction. This handy booklet should find its way into many a coding room.

D. H. L.

1043[F].—M. Kraitchik, *Introduction à la Théorie des Nombres.* Paris, 1952, viii + 202 p., 16.5 × 25.4 cm.

The material for this work is taken largely from the author's *Théorie des Nombres,*[1] v. 1 and 2, and *Recherches,*[2] v. 1. There are numerous small tables which may be reported as follows:

(1) p. 2. Factorization of $\pm 1 + p_1p_2\cdots p_n$, where p_n is the n-th prime, for $n \leq 16$.
(2) p. 4. Factorization of $2^{2^n} + 1$ as far as known in 1947.
(3) p. 5. Factorization of $\pm 1 + n!$ for $n \leq 22$.
(4) p. 12, 13. Complete factorization of $2^n - 1$ for n odd and $n = 1(2)99$, 105, 107, 111(2)117, 123, 127, 135.
(5) p. 38, 39. Factorization of $2^n + 1$ for odd $n \leq 135$ and for even $n \leq 148$ with some gaps.
(6) p. 40, 41. Factorization of $10^n \pm 1$ for $n = 1(2)31$ and $n = 2(2)20$, 24, 30, 36, 50.
(7) p. 55–62. Tables of power residues. These give the least positive or absolutely least residues of k-th powers for prime moduli $< L$ for the following values of k and L

$k =$	2,	3,	4,	5,	6,	7,	11
$L =$	200,	200,	200,	422,	242,	632,	1000

(8) p. 124–129. Tables of linear divisors of $x^2 - Dy^2$. These tables are in three parts. The first part gives the arithmetical progressions in which p must lie if D is to be a quadratic residue of p for all square free numbers D less than 35 in absolute value. The second part gives this information for $0 < D = 4k + 1 \leq 241$. The third part is for $0 > D = 4k + 3 \geq -239$. These contain two errata, carried over from the corresponding tables in *Théorie des Nombres,* v. 1:
p. 125 $D = 157$ *for* 107 *read* 109
p. 126 $D = 193$ *for* 155 *read* 129.
(9) p. 165–177. Tables of x (mod p) in $x^2 - y^2 = N$. These are for all primes $p \leq 67$ and are taken from *Théorie des Nombres,* v. 2, p. 156–166.
(10) p. 182. Table of a cyclotomic quadratic form. This table gives the coefficients of the polynomials X, Y, such that

$$X_n^2(x) - nxY_n^2(x) = \begin{cases} Q_n(x) & n = 4k+1 \\ Q_{2n}(x) & n = 4k+3 \end{cases}$$

for all square free $n \leq 35$ and also 39, 42, and 51. It is taken from *Recherches,*[2] v. 1, p. 88, with two errata. The coefficients of X_{29} should be 1, 15, 33, 13, 15, $\cdots$, 15, 13, 33, 15, 1.
(11) p. 184–186. Factorization of $2^{4n+2} + 1$. The table extends to all $4n + 2 \leq 162$ as well as 170, 174, 182, 186, 190, 198, 210, 222, 234, 250, 258, 270, and 330, with some results partially incomplete. For $n = 94$, 114, and 150, results are put in the wrong columns of the table.

D. H. L.

[1] M. Kraitchik, *Théorie des Nombres.* v. 1, Paris, 1922, ix + 229 p., v. 2, Paris, 1926.
[2] M. Kraitchik, *Recherches sur la Théorie des Nombres.* v. 1, Paris, 1924.

1044[F].—A. Natucci, "Osservazioni sul problema di Fermat," Un. Mat. Ital., *Boll.*, s. 3, v. 6, 1951, p. 245–248

This paper contains a short table of

$$F_n(x, u, v) = (x + u + v)^n - \{x^n + (x + u)^n\}$$

for $v = 1$, $u = 1, 2$ and $n = 3, 4, 5$. The variable x ranges over integers until F becomes negative. Two or three negative values are given and x does not exceed 13.

D. H. L.

1045[F].—Giuseppe Palamà, "Numeri primi e composti contenuti nella forma $1848x^2 + y^2$ dell' intervallo 11000000 − 11100000," Unione Mat. Ital., *Boll.*, s. 3, v. 7, p. 168–171, 1952.

The author uses a recent list of primes of the eleventh million[1] to find all the primes of the form $1848x^2 + y^2$ between the above limits together with values x and y. There are 202 primes. This is similar to a list above 10 million given by Cunningham & Cullen.[2] The author also lists 121 composite numbers of this form.

D. H. L.

[1] J. P. Kulik, L. Poletti & R. J. Porter, *Liste des nombres premiers du onzième million.* Amsterdam, 1951 [*MTAC*, v. 6, p. 81, errata p. 163].

[2] J. C. Cunningham & J. Cullen, "On idoneal primes," Br. Asso. Adv. Sci., *Report*, 1901, p. 552.

1046[H].—P. T. Smolîakov & A. N. Hovanskiĭ, "K resheniu algebraicheskikh uravneniĭ 3–ĭ stepeni" [On the solution of algebraic equations of the third degree], Kazan Filial. Akad. Nauk. SSSR., *Izvestiîa* Fiz-Mat. Tekhn. Nauk, v. 1, 1948, p. 85–92.

The authors take as a reduced form the cubic

$$y^3 + Py + 1 = 0.$$

On p. 87 is a 2D table giving one real root y corresponding to 271 given values of P between -10 and 10.

D. H. L.

1047[K].—G. E. Albert & R. B. Johnson, "On the estimation of central intervals which contain assigned proportions of a normal univariate population," *Annals Math. Stat.*, v. 22, 1951, p. 596–599.

Let Y be a random variable with the cumulative probability function $F(y) = (\sigma^2 2\pi)^{-\frac{1}{2}} \int_{-\infty}^{y} \exp[-(u - m)^2/2\sigma^2]du$. The statistic considered is

$$A(\bar{Y}, s; \lambda) = F(\bar{Y} + \lambda s) - F(\bar{Y} - \lambda s)$$

where $\bar{Y} = \sum_{i=1}^{N} Y_i/N$, $s = \left\{\sum_{i=1}^{N} (Y_i - \bar{Y})^2/(N-1)\right\}^{\frac{1}{2}}$.

Wilks has shown[1] that if for given p, $0 < p < 1$, one sets

$$\lambda_{p,N} = t_p[(N+1)/N]^{\frac{1}{2}},$$

where t_p is Student's t, then the expectation of this statistic is

$$E[A(\bar{Y}, s; \lambda_{p,N})] = 1 - p.$$

The problem treated in the present paper is: for given p, α, d_1, d_2, such that $0 < p < 1$, $0 < \alpha < 1$, $0 < d_1 \leqq 1 - p$; $0 < d_2 \leqq p$, find the smallest sample size N for which one has

$$\text{Prob. } \{1 - p - d_1 \leqq A(\bar{Y}, s; \lambda_{p,N}) \leqq 1 - p + d_2\} \geqq \alpha.$$

A table of such smallest sample values is given for $p = .01$, $\alpha = .95, .99$, and for $p = .05, .25, .50$, $\alpha = .80, .95, .99$, in each case for $(d_1, d_2) = (.075, .05)$, $(.05, .05)$, $(.025, .025)$, $(.035, .015)$, $(.05, .01)$, $(.025, .01)$, $(.02, .01)$, $(.01, .01)$.

Z. W. BIRNBAUM

Univ. of Washington
Seattle, Wash.

[1] S. S. WILKS, "On the determination of sample sizes for setting tolerance limits," *Annals Math. Stat.*, v. 12, 1941, p. 91–96.

1048[K].—D. H. BHATE, "A note on the significance levels of the distribution of the mean of a rectangular population," Calcutta Stat. Assn., *Bulletin*, v. 3, 1951, p. 172–173.

For means of samples of n drawn from a continuous rectangular universe on the interval (0, 1) the author gives the 5%, 2.5% and .5% points to 4D for $n = 1(1)16$. He remarks that for $n \geqq 16$ the corresponding percentage points for the normal distribution function approximation are correct to 4D.

C. C. C.

1049[K].—J. M. CAMERON, "Tables for constructing and for computing the operating characteristics of single-sampling plans," *Industrial Quality Control*, v. 9, 1952, no. 1, p. 37–39.

Let the probability of acceptance of a lot be denoted by $P(A)$ and the lot fraction defective by p. For a single-sampling plan with sample size n and allowable number of defectives c, Table 1 provides 13 paired values of p and $P(A)$ which can be used to plot the respective operating characteristic (OC) curves. The entries in Table 1 are values of np which are given to 3D for $c = 0(1)49$ and for $P(A) = .995, .990, .975, .950, .900, .750, .500, .250, .100, .050, .025, .010, .005$.

Table 2 is designed for constructing single-sampling plans whose OC curve passes through two points $(p_1, 1 - \alpha)$ and (p_2, β) where p_1 is the fraction defective for which the risk of rejection is to be α, and where p_2 is the fraction defective for which the risk of acceptance is to be β. The entries in Table 2 are values, given to 3D, of the ratio p_2/p_1 which are associated with three sets of paired values of α and β; namely, $(\alpha = .05, \beta = .10)$, $(\alpha = .05, \beta = .05)$, and $(\alpha = .05, \beta = .01)$. Corresponding to the selected value of p_2/p_1 are values of np_1 and c. The sample size is determined by dividing np_1 by p_1 and the acceptance number is read directly from the table.

Since the tables were computed from the Poisson series, they are exact only when the probability distribution of the number of defectives in a sample of n follows the Poisson law. However, for most practical cases, the tables will give satisfactory approximations when the distribution of defectives is binomial.

G. W. McElrath

University of Minnesota
Minneapolis, Minnesota

1050[K].—J. H. Chung & D. B. DeLury. *Confidence Limits for the Hypergeometric Distribution.* Univ. of Toronto Press, Toronto 1950, 72 leaves, 22.5 × 30.9 cm. Price $2.25.

The charts in this book for the hypergeometric distribution are similar to the ones for the binomial distribution given by Clopper & Pearson.[1] Charts are given for population sizes 500, 2500, and 10,000, for sampling rates 5%, 10% (10%) 90%, and for confidence coefficients 90%, 95%, and 99%. The methods of constructing the charts are given explicitly, as well as means for performing interpolation and extrapolation to cases not included.

I. R. Savage

National Bureau of Standards
Statistical Engineering Laboratory
Washington, D. C.

[1] C. J. Clopper & E. S. Pearson, "The use of confidence and fiducial limits applied to the case of the binomial," *Biometrika*, v. 26, 1934, p. 404–413.

1051[K].—A. Hald & S. A. Sinkbaek, "A table of percentage points of the χ^2-distribution," *Skandinavisk Aktuarietidskrift*, v. 33, 1950, p. 168–175.

The present table gives values of χ_0^2 to 3D for which $P(\chi^2 \geqq \chi_0^2)$ in which χ^2 has f degrees of freedom for $P = .0005, .001, .005, .01, .025, .05, .1(.1).9, .95, .975, .99, .995, .999, .9995$ and $f = 1(1)100$.

C. C. C.

1052[K].—H. Levene, "On the power function of tests of randomness based on runs up and down," *Annals Math. Stat.*, v. 23, 1952, p. 34–56.

In the set of observed values, $x_1, x_2, \cdots, x_n$ of a continuous statistical variable X, let B^* be the sequence of signs ($+$ or $-$) of the differences $(x_{i+1} - x_i)$ for $i = 1, \cdots, n - 1$. A sequence of successive $+$ $(-)$ signs not immediately preceded or followed by a $+$ $(-)$ sign is called a run up (down). Let s be the number of runs up and let $E'(s) = \lim_{n\to\infty} \frac{1}{n} E(s)$, $\sigma'^2(s) = \lim_{n\to\infty} \frac{1}{n} \sigma^2(s)$, where $E(s)$ is the expected value of s and $\sigma^2(s)$ is the variance of s. Also let k be the total number of $+$ signs in B^*, with $E'(k)$ and $\sigma'^2(k)$ defined analogously to $E'(s)$ and $\sigma'^2(s)$. s and k are members of a class of u-run statistics which may be used for tests of randomness of the sample values, other members of which are also discussed by the author.

One of the alternatives to randomness, precisely defined in this paper, is that x_i obeys a normal distribution law with mean $i\vartheta$ and unit variance, $i = 1, \cdots, n$. For this case $E'(s)$, $1 - E'(k)$, $\sigma'^2(k)$, and $\sigma'(k)$ are given to 6D for the first three and to 3D for the fourth for $\vartheta/\sqrt{2} = 0(.1)3(.2)4$ in Table 1. An interesting graph of $E'(s)$ for the linear trend considered compares the results for normal and rectangular universes.

L. A. Aroian

Hughes Aircraft Company
Culver City, California

1053[K].—F. H. C. Marriott, "Tests of significance in canonical analysis," *Biometrika*, v. 39, 1952, p. 58–64.

Given two sets of variates $x_1, \cdots, x_p$ and $y_1, \cdots, y_q$ ($p \leqq q$) and $n + 1$ observations on each set, the canonical correlations are defined as the p roots ($l_1^2 \geqq l_2^2 \geqq \cdots \geqq l_p^2$) of the determinantal equation $|Q - l^2T| = 0$, where T is the dispersion matrix of the y's; W is the dispersion matrix of the y's with the x's eliminated; $Q = T - W$ is the dispersion matrix due to regression.

The exact null distribution of the largest root l_1^2 is given by Roy[1] in a recursive form involving multiple integrals. The author gives the exact distribution of l_1^2 for $p = 2$ and $p = 3$, $q = 4$. A significance test which is exact for practical purposes is given for $p = 3$ and $p = 4$, $q = 5$. 5% and 1% significance levels to 2D (3 or 4S) for $\frac{1}{2}n\, l_1^2$ (n large) are given for $p = 2$, $q = 2(1)12, 21$; $p = 3$, $q = 3(1)12, 21$; $p = 4$, $q = 5$.

An approximate test,

$$\chi^2_{[D]} = -\{n - \tfrac{1}{2}(p + q + 1)\} \log (1 - l_1^2)$$

where $D = p + q - 1 + \frac{1}{2}\{(p - 1)(q - 1)\}^{\frac{1}{2}}$, based on Wilks' criterion[2] is proposed. 5% and 1% significance levels to 2D (3 or 4S) for $\frac{1}{2}n\, l_1^2$ (n large) are given for $p = 2$, $q = 2, 6, 12, 21$; $p = 3$, $q = 3, 6, 12, 21$; $p = 4$, $q = 5$ which compare favorably with the exact values. The derivation of the test is omitted, with reference to the author's unpublished thesis.[3] The 5% points of l_1^2 for the exact and approximate tests for $p = 2$, $q = 5$ are given to 2 or 3D for $n = 10, 20, 50, 100, \infty$, which indicate good results for n down to 20.

I. Olkin

Michigan State College
East Lansing, Michigan

[1] S. N. Roy, "The individual sampling distribution of the maximum, the minimum, and any intermediate of the p-statistics on the null hypothesis," *Sankhyā*, v. 7, 1945, p. 133–158.

[2] S. S. Wilks, "Certain generalizations in the analysis of variance," *Biometrika*, v. 24, 1932, p. 471–494.

[3] F. H. C. Marriott, *The Analysis and Interpretation of Multiple Measurements*. 1951, University of Aberdeen.

1054[K].—Sigeiti Moriguti, "A lower bound for a probability moment of any absolutely continuous distribution with finite variance," *Annals Math. Stat.*, v. 23, 1952, p. 286–289.

The n-th probability moment of a population with probability density function $f(x)$ is defined as

$$\Omega_n = \int_{-\infty}^{\infty} [f(x)]^n\, dx.$$

The author considers populations with zero mean and finite variance σ^2. A table is computed for the lower bound of the reduced probability moment $\Omega_n\sigma^{n-1}$ for $n = 2(1)10$ to 5D and a population which attains the lower bound is exhibited. The problem arose in the distribution of sample ranges.

J. R. Vatnsdal

State College of Washington
Pullman, Washington

1055[K].—SEIJI NABEYA, "Absolute moments in 2-dimensional normal distribution," Inst. Stat. Math., *Annals*, v. 3, 1951, p. 2–6.

Let x and y have the joint probability density function

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_1^2}-\frac{2\rho xy}{\sigma_1\sigma_2}+\frac{y^2}{\sigma_2^2}\right)\right].$$

This paper presents expressions for $E(|x^m y^n|)$ which are functions of σ_1, σ_2, and ρ. The cases considered are those for which simultaneously $m \geqq n$, $m \geqq 1$, $n \geqq 0$, $m + n \leqq 12$.

J. E. WALSH

U. S. Naval Ordnance Test Station
China Lake, California

1056[K].—K. R. NAIR, "Some three-replicate partially balanced designs," Calcutta Stat. Assn., *Bulletin*, v. 4, 1951, p. 39–42.

Four types of two-associate and four types of three-associate designs, using three replicates of each treatment, are discussed. A list of 48 useful designs is presented for which v (number of treatments) $\geqq 10$ and k (number of treatments per block) $\leqq 10$. The number of associate classes (classes of treatment comparisons) and the efficiency factor are given for each design.

R. L. ANDERSON

North Carolina State College
Raleigh, N. C.

1057[K].—M. H. QUENOUILLE, "The variate-difference method in theory and practice," International Stat. Inst., *Review*, v. 19, 1951, p. 121–129.

The author discusses the usual use of the variate difference method to estimate the random variance in a time series and to test for the existence of trend after the i-th difference. He develops a new test of the existence of trend and a formula to estimate the random variance based on differences of the variances of the successive variate differences. If we let V_i be the variance of the i-th set of variate differences (as defined by the author), then the variance of $\Delta^m V_i$ is $\alpha_{mi}\sigma^4/n$, where n is the number of observations, σ^2 is the true random variance, and values of α_{mi} are given by the author for $i = 1(1)10$ and $m = 0(1)5$, $m + i \leqq 10$, to 4D for $m = 0, 1$ and to 6D for $m \geqq 2$.

The author often interchanges his notation (e.g., m is an order of differencing and also indicates both a terminal and an intermediate variance, V_m). He does not indicate how to decide on the best of many estimates to use for the random variance. However, his methods may be quite useful, if a systematic procedure is developed for using them.

Some discussion is made of the difficulties involved when the residuals are serially correlated. The author discusses two empirical examples not involving serial correlation and one practical example with serially correlated residuals. More discussion is needed of how the empirical data were generated.

R. L. ANDERSON

North Carolina State College
Raleigh, N. C.

1058[K].—P. V. Sukhatme, V. D. Thawani, V. G. Pendharkar, & N. P. Natu, "Revised tables for the d-test of significance," Indian Soc. of Agricultural Stat., *Jn.*, v. 3, 1951, p. 9–23.

The d-test under discussion is the Behrens-Fisher test for the difference of two sample means drawn from normal universes with possibly unequal variances. Specifically,

$$d = (\bar{x}_2 - \bar{x}_1)/(s_2^2 + s_1^2)^{\frac{1}{2}} = t_2 \cos \vartheta - t_1 \sin \vartheta$$

in which $\bar{x}_1$ and $\bar{x}_2$ are the sample means, s_1^2 and s_2^2 are unbiased estimates of the population variances, $\tan \vartheta = s_1/s_2$ and t_1 and t_2 are Student-Fisher t's with n_1 and n_2 degrees of freedom. Sukhatme's original tables of 5% and 1% values of d appeared in the 1943 edition of Fisher & Yates' tables.[1] Meanwhile Fisher developed an asymptotic method of calculating these percentage points[2] and detected a small positive bias in Sukhatme's values. The latter investigated further and found that his errors were due to his use of linear interpolation in Student's original tables of the t-distribution.[3] His corrected 5% values were used in the 1948 edition of the Fisher & Yates' tables but his 1% values were corrected by Fisher for a positive bias shown in comparison with those of Fisher. In the present paper the authors report the results of a further investigation. They find that the positive bias reported by Fisher did not exceed unity in the third decimal place and that the use of quadratic interpolation is sufficient to give accuracy to 3D for the arguments used. Their investigation of Fisher's method casts doubt on its adequacy at all points. Their finally corrected values appear as their Tables III and VII. These are for the same arguments as those in the Fisher & Yates tables and are also to 3D. (Values are apparently given to 4D when the final figure is 5 but no indication is given whether such a 5 is 5^+ or 5^-.) The 5% values agree with the 1948 Fisher & Yates values but there are three discrepancies in the 1% values as follows:

n_1	n_2	ϑ	Sukhatme	Fisher & Yates
24	24	15°(75°)	2.784	2.785
6	6	30°(60°)	3.558	3.557
8	8	30°(60°)	3.241	3.239

C. C. C.

[1] R. A. Fisher & F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research.* London. 2nd ed. 1943, 3rd ed. 1948. Table V_1.
[2] R. A. Fisher, "The asymptotic approach to Behrens's integral, with further tables for the d-test of significance," *Ann. Eugenics*, v. 11, 1941, p. 141–172.
[3] Student, "New tables for testing the significance of observations," *Metron*, v. 5, 1925, p. 105–120.

1059[L].—D. Caligo, "Complementi analitici e numerici allo studio delle aste vibranti," Accad. naz. dei Lincei, Rome, *cl. sci. fis. mat. nat.*, *Rendiconti*, s. 7, v. 12, 1952, p. 76–83, 277–285.

Let β_ρ, $\rho = 0, 1, \cdots$ be the roots of $\cosh x \cos x = 1$, $\rho = 0$ and $\rho = 1$ being the double root. Table I, p. 82, gives values of $\beta_\rho, (-1)^\rho[\beta_\rho - (\rho - \frac{1}{2})\pi]$, and $\sinh \beta_\rho/[\cosh \beta_\rho + (-1)^\rho]$ for $\rho = 2(1)5$.

Table V, p. 282, contains similar information for the roots of

$$\cosh x \cos x = -1.$$

Six other tables give values of certain integrals occurring in the physical problem.

A. E.

1060[L].—M. A. Dengler, M. Goland & Y. L. Luke, *Tables of the functions* $\int_0^y (e^{iu} - 1) \sqrt{u^2 + b^2}\, du/u$. Midwest Research Institute, *Report* August 1952, 19 p.

This report tabulates

$$A(y, b) = \int_0^y u^{-1}(1 - \cos u)\, (u^2 + b^2)^{\frac{1}{2}}\, du$$

$$B(y, b) = \int_0^y u^{-1} \sin u\, (u^2 + b^2)^{\frac{1}{2}}\, du$$

for

$$b = 0(.2)1.8, \quad y = 0(.05)2(.1)2.5$$

and

$$b = 2(.2)4, \quad y = 0(.05).5(.1)2.$$

The introduction describes the computation. 9D were carried, and the results have been rounded to 7D for presentation in these tables. The entries should be correct to within one unit in the seventh place. Five-point Lagrangean interpolation in the y direction gives results correct to about two units in the sixth place. If $b > 1$, similar interpolation in the b-direction is reliable to about one unit in the fifth place. If $b < 1$, interpolation in the b direction is poor.

A. E.

1061[L].—C. Ferrari, "The turbulent boundary layer in a compressible fluid with positive pressure gradient," *Jn. Aero. Sci.*, v. 18, 1951, p. 460–477.

There is a table (p. 476) of $G_n(x) = 2^{2n} n!\, i^{2n} \operatorname{erfc} x$ for $x = 0(.05)1(.1)2.2$, $n = 2(1)5$. The values are to 4D but "the fourth place is not accurate." This table extends one by Hartree.[1]

D. H. L.

[1] D. R. Hartree, "Tables of the error function," Manchester Lit. and Phil. Soc. *Memoirs and Proceedings*, v. 80, 1935, p. 85. [Reprinted in Carslaw & Jaeger, *Conduction of Heat in Solids*, p. 373, Appendix III.]

1062[L].—H. J. Godwin, "A method for the evaluation of

$$\int_0^\infty x^m \left(\sqrt{(2/\pi)} \int_x^\infty \exp\left(-\tfrac{1}{2}t^2\right) dt \right)^n dx,"$$

Quart. Jn. Mech. Appl. Math., v. 5, 1952, p. 109–115.

Put

$$(2/\pi)^{\frac{1}{2}} \int_x^\infty \exp\left(-\tfrac{1}{2}t^2\right) dt = g(x),$$

$$\int_0^\infty x^m \{g(x)\}^n \, dx = I_{mn},$$

$$P(m, 0, x) = x^m,$$

$$P(m, r, x) = rxP(m, r-1, x) + P'(m, r-1, x),$$

$$\gamma_{m,v} = P(m, m+2v, 0)\, (\tfrac{1}{2}\pi)^{\frac{1}{2}(m+2v+1)}/(m+2v)\,!$$

The author computes $\gamma_{m,v}$ and I_{mn} by expansion in inverse factorial series. He gives 9–10D tables of I_{mn} for $m = 0, 1, 2$, $n = 1(1)20$, and 9–15D tables of γ_{mv} for $m = 0$, $v = 0(1)18$; $m = 1, 2$, $v = 0(1)10$.

A. E.

1063[L].—V. R. THIRUVENKATACHAR & B. S. RAMAKRISHNA, "A case of combined radial and axial heat flow in composite cylinders," *Quart. Appl. Math.*, v. 10, 1952, p. 255–262.

On p. 261 there is a 2D table for the first four roots of the equation

$$10x \frac{J_1(x)}{J_0(x)} = \frac{J_1(3y/2)\,Y_1(y) - J_1(y)\,Y_1(3y/2)}{J_1(3y/2)\,Y_0(y) - J_0(y)\,Y_1(3y/2)}\, y$$

where

$$y = [10x^2 + .09\pi^2 n^2]^{\frac{1}{2}}, \quad n = 1, 3, 5.$$

A. E.

1064[V].—ROBERT W. SMITH, JR., HELEN E. EDWARDS & STUART R. BRINKLEY, JR., *Tables of Velocity of Steady Laminar Flow in Channels of Rectangular Cross Section.* Report of Investigations 4885, U. S. Dept. of the Interior, Bureau of Mines, July 1952, 41 p.

Table 2 contains the solution of the boundary value problem: $\omega_{xx} + \omega_{\bar{y}\bar{y}} = -2$ in the rectangle $R(|x| \leq 1, |\bar{y}| \leq 1/\sigma)$, $\omega = 0$ on the boundary of R. For each $\sigma = .1(.1)1$, $\omega(x, y)$ is presented as a function of $x = 0(.1).5(.05).7(.02)1$ and $y = \sigma\bar{y}$. In the first 20 columns $y = 0(.1).5(.05).7(.02).9$; in the remaining columns $y = .905(.005)1$ for $\sigma = .1$ and $.2$, $y = .91(.01)1$ for $\sigma = .3(.1).7$, and $y = .92(.02)1$ for $\sigma = .8(.1)1$.

For the same range of σ Table 1 has $\omega_0 \equiv \omega(0, 0)$, $\bar{\omega} \equiv \int_0^1 \int_0^1 \omega(x, y) \times dxdy$, $k' \equiv \sigma\omega_0/8$, $k \equiv \sigma\bar{\omega}/8$; Table 3 has $(-\partial\omega/\partial x)_{x=1}$ for the above pairs of values of σ and y; and Table 4 has $(-\partial\omega/\partial y)_{y=1}$ for the above values of x.

In hydrodynamics ω is the reduced velocity of the steady laminar flow of a viscous fluid through a channel with the rectangular cross section R.

In the problem of twisting an isotropic prismatic bar with cross section R and shear modulus μ by a couple M whose moment is directed along the axis of the bar, ω is the stress function. Moreover, if α is the angular twist per unit length of the bar, $M = 8\mu\alpha\bar{\omega}/\sigma$ and the stress components are $\tau_{zx} = \sigma\mu\alpha\partial\omega/\partial y$ and $\tau_{zy} = -\mu\alpha\partial\omega/\partial x$.[1]

The tabulated functions were computed to 10D from Fourier series and rounded off to 6D.

The very last factor in equations (17) should be $[g(0, y, \sigma)]^2$. The second of equations (12) is $k' = \cdots$. The denominators of the factor before $\sum$ in equations (5), (7), (9), and (14) are π^3, π^3, π^5, and π^2.

R. R. REYNOLDS

NBSINA

[1] I. S. SOKOLNIKOFF, *Mathematical Theory of Elasticity*. New York, McGraw-Hill, 1946. See under "Torsion" in index for references to electrical, hydrodynamic, and membrane analogies. The right side of equation (38.15), p. 149, of the above reference is the series for $\omega + (x^2 + \bar{y}^2)/2$ provided y, a, and b are replaced by $\bar{y}$, 2, and $2/\sigma$.

MATHEMATICAL TABLES—ERRATA

In this issue references have been made to RMT **1043**, **1058**, **1061**, and **1064**.

218.—N. W. MCLACHLAN & P. HUMBERT, *Formulaire pour le Calcul Symbolique*. Mémorial des Sciences Mathématiques, fasc. 100, 2nd ed., Paris 1950.

Errata in the first edition have been noted in MTE **205** [*MTAC*, v. 6, p. 100-101]. Beside these errata the second edition contains also the following:

p. 4, formula 5; *for* $t^2/4$ *read* $t^2/2$
p. 4, formula 9; *for* E *read* F
p. 6, formula 8; *for* t^α *read* $t^{-\alpha}$
p. 11, formula 2; *for* $\frac{p}{}\,_a$ *read* $\frac{p}{a}$
p. 12, formula 7; *for* $-\phi'(-\log p)$ *read* $\phi(\log p) - f(0)$
p. 12, formula 9; *for* 2^n *read* $2^{n/2}$ and *for* $x/2\sqrt{t}$ *read* $x/\sqrt{2t}$
p. 16, formula 9; *add* $R(\nu) > 1$
p. 21, formulas 6, 7; *add* $0 < t < 1$
p. 21, formula 11; *add* $t > a$
p. 22, formula 10; *for* $\mathrm{ch}^{2n}t$ *read* $\mathrm{sh}^{2n+1}t$ (see *Supplément*, p. 10)
p. 22, formula 11; *for* $\mathrm{sh}^{2n+1}t$ *read* $\mathrm{sh}\sqrt{t}$
p. 22, last formula; *delete* the brackets [ ]
p. 23, formula 11; *for* $p \log (p/\sqrt{p^2 - 1})$ *read* $p \log (\sqrt{p^2 - 1}/p)$
p. 23, formula 12; *for* $p \log \left(\frac{\sqrt{p^2 - 1}}{p}\right)$ *read* $p^2 \log \left(\frac{\sqrt{p^2 - 1}}{p}\right)$
p. 25, formula 6; the l.h.s. is equal to $\nu^{-1}\,\mathrm{sh}(\nu \arg \mathrm{ch}\, t)$
p. 27, formula 6; *for* $1/(p + 1)$ *read* $p/(p - 1)$
p. 27, last formula but one; first term, *for* numerator on r.h.s., *read* a
p. 28, formula 6; *for* $R(\nu) > -\frac{1}{2}$ *read* $R(\nu) > -1$
p. 28, third last formula; *for* 2^ν *read* 2^μ and *for* $R(\mu + \nu) > 0$ *read* $R(\mu + \nu) > -1$
p. 28, last formula; *for* $J_\nu(t)$ *read* $J_{2\nu}(t)$, *for* $(2\nu q - p)$ *read* $(2\nu q + p)$ and *for* $R(\nu) > 1$ *read* $R(\nu) > \frac{1}{2}$
p. 29, formula 5; *add* $R(\nu) > -2$
p. 29, formula 9; multiply the r.h.s. by $n!$ (see *Supplément* p. 11)