where only fourth differences are retained and it is clear that there is stability as h tends to zero. Now from equation (8), $F(-\infty) < 0$, and $F(-1) = -2 + 551h/45$. There is therefore a root of modulus greater than unity when h exceeds 90/551, and the method is stable only for sufficiently small tabular interval. Moreover if higher order differences are retained, the maximum value of h for which the method is stable is decreased.

Similar arguments show that Moulton's method based on the formula

$$(9) \quad (y_0 - y_{-1})/h = y_0' - 1/2\nabla y_0' - 1/12\nabla^2 y_0' - 1/24\nabla^3 y_0' - 19/720\nabla^4 y_0' - \cdots$$

is also unstable for large values of the tabular interval when differences higher than the first are retained. The upper limit on h for stability for a given number of differences is very much higher than in Adams' method.

It has been remarked by RUTISHAUSER[2] that the error equation, corresponding to a non-linear differential equation of the form

$$y^{(n)} = f(x, y, y^{(1)}, \cdots, y^{(n-1)}),$$

is linear. The above arguments with certain modifications can therefore be applied to the stability associated with equations of this form.

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[1] J. TODD, "Solution of differential equations by recurrence relations," *MTAC*, v. 4, 1950, p. 39–44.
[2] H. RUTISHAUSER, "Uber die Instabilität von Methoden zur Integration gewöhnlicher Differentialgleichungen," *Zeit. angew. Math. Phys.*, v. 3, 1952, p. 65–74.

148.—TWO NON-ELEMENTARY DEFINITE INTEGRALS. The two integrals in question are

$$F(x) = \int_0^x t^t dt, \qquad G(x) = \int_0^x t^{-t} dt$$

and are of interest because of the peculiar branching properties of the integrands and because they lead to series with unusually rapid convergence. Integrals of these types have been encountered in some recent studies of transients in networks. They can be evaluated numerically as follows.

As usual, we interpret t^t as

$$e^{t \log t} = \sum_{n=0}^{\infty} (t \log t)^n / n!$$

The integral of the general term of this series

$$I_n = \frac{1}{n!} \int_0^x (t \log t)^n dt$$

can be expressed in terms of the complete and incomplete Gamma function by means of the transformation

$$u = -(n + 1) \log t.$$

In fact

$$(-1)^n(n+1)^{n+1}I_n = 1 - \frac{\Gamma_y(n+1)}{\Gamma(n+1)}$$

where

$$\Gamma_y(n+1) = \int_0^y e^{-u}u^n du$$

and

$$y = -(n+1)\log x.$$

However, since available tables[1] of the incomplete Gamma function are not convenient for this application, it was decided to use a direct evaluation of the series.

The integral for I_n can be expressed as a sum of $n+1$ terms as follows:[2]

$$I_n = x^{n+1}\sum_{r=0}^{n}\frac{(-1)^r(\log x)^{n-r}}{(n+1)^{r+1}(n-r)!}.$$

Summing over n and collecting the coefficient of each power of $\log x$ we obtain the double series,

$$F(x) = \sum_{n=0}^{\infty} I_n = \sum_{r=0}^{\infty}(\log x)^r/r!\sum_{s=1}^{\infty}(-1)^{s+1}x^{s+r}(s+r)^{-s}.$$

Similarly

$$G(x) = \sum_{n=0}^{\infty}(-1)^n I_n = \sum_{r=0}^{\infty}(-1)^r(\log x)^r/r!\sum_{s=1}^{\infty}x^{s+r}(s+r)^{-s}.$$

When $x = 1$ we have the unusually rapidly converging series

$$F(1) = 1 - 2^{-2} + 3^{-3} - 4^{-4} + \cdots,$$
$$G(1) = 1 + 2^{-2} + 3^{-3} + 4^{-4} + \cdots.$$

The following values of $F(x)$ and $G(x)$ were computed to 10D by the above double series and checked by numerical integration using tables of fractional powers.[3] Values have been rounded off to 8D. Most of the calculation was made by Mrs. Joan M. Clay.

x	$F(x)$	$G(x)$
0	0.0000 0000	0.0000 0000
.1	0.0870 9546	0.1152 6443
.2	0.1625 6731	0.2478 5617
.3	0.2334 0100	0.3890 5018
.4	0.3027 3442	0.5332 8125
.5	0.3726 1486	0.6763 8768
.6	0.4446 5226	0.8152 2316
.7	0.5202 8866	0.9474 7031
.8	0.6009 4326	1.0715 0820
.9	0.6881 0902	1.1862 9918
1.0	0.7834 3051	1.2912 8600

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[1] KARL PEARSON, *Tables of the Incomplete* Γ-*Function*. London, H. M. Stationery Office. (Reissue 1934; London, Biometrika Office, University College.)
[2] W. GRÖBNER & N. HOFREITER, *Integraltafel*. Erster Teil, Unbestimmte Integrale. Vienna, 1949, p. 111, formula 2a.
[3] NBSMTP, *Tables of Fractional Powers*. New York, Columbia University Press, 1946.

149.—The CANON DOCTRINAE TRIANGVLORVM (1551) of RHETICUS (1514–1576). Some facts with reference to this excessively rare publication have been given in material about PITISCUS and RHETICUS in *MTAC*, v. 3, p. 394, 396, 553–554, 559–560. It is here noted that the only copies known to have been preserved were in the Bibliothèque Nationale and British Museum. DEMORGAN had a copy in 1845 when he published[1] a description of the work, but this was doubtless in his Library at the University of London, destroyed during the recent World War.

In Catalogue 19, 1952, of the London bookseller E. WEIL, a copy was offered for 27 £ 10 s. Mr. WILLIAM D. MORGAN, of 1764 St. Anthony Ave., St. Paul 4, Minnesota, was so fortunate as to secure this item for adding to his already valuable collection (see *MTAC*, v. 3, p. 562–563). Since Mr. Morgan graciously loaned this precious work to me that a microfilm copy might be made for the Brown University Library, I take the opportunity to add a little to the information already published in *MTAC*. The complete title is as follows: *Canon Doctrinae Triangvlorvm. Nvnc primvm a Georgio Ioachimo Rhetico, in lvcem editvs, cvm privilegio imperiali, Ne qvis haec intra decennivm, quacunque; forma ac compositione, edere, neve sibi vendicere aut operibvs suis inserere ausit. Lipsiae ex officina Wolphgangi Gvnteri. Anno M.D. LI.* In the title page is an obelisk with a man drawing a diagram on the base.

The back of the title page is blank; then follows a page of Latin verses. On the back of this page is the first of 14 pages of 7D tables of the six trigonometric functions, at interval 10′, arranged for the first time in semiquadrantal display. The degrees are in black, and the minutes and differences are in red. This is the first table in which all trigonometric functions are brought together. Rheticus was the first to define trigonometric functions by means of a right-angled triangle without any reference to a circle.

Immediately following the tables are 6 pages of dialogue between Philomathes, a supposed friend of Rheticus, and Hospes, his pupil. The pupil asks what the intention of the book is, and is answered at length. He suggests that, perhaps, the intention may be to complete the system of Copernicus, by publishing tables from it resembling those then in use. But he is answered that Rheticus would rather that Copernicus himself had not done so much in this line, as he thereby diminished the geometrical practice of the learner, and so on.

An undated 1580 reprint of the *Canon* is in the British Museum.

The copy of the *Canon* before me has evidently had its pages trimmed; but the present size of its pages is 15.8 × 22.5 cm.

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[1] DEMORGAN, "On the almost total disappearance of the earliest trigonometrical canon," RAS, *Mo. Not.*, v. 6, 1845, p. 221–228; reprinted with an addition in *Phil. Mag.*, s. 3, v. 26, 1845, p. 517–526.