162.—Historical Note on Root–Finding Machine. A machine which escaped the attention of J. S. Frame[1] is one proposed by Frank T. Freeland.[2] The machine is based upon a method for formation of linkages for the successive powers, published by Freeland in American Journal of Mathematics, v. 3, No. 2.

The machine provides n levers L_k, so connected by linkages that a displacement x of L_1 produces a displacement x^k of L_k, $k = 2$ to n. On each L_k is a pulley P_k with axis parallel to and distance $\frac{1}{2}c_k$ from the axis of L_k, where c_k is the coefficient of x^k. An inextensible cord is passed around the pulleys with the two portions of the cord on P_k parallel, perpendicular to L_k and on the same side of it, using fixed pulleys to change cord direction if necessary. To the end of the cord is fixed a pointer brought parallel to a scale marked off in units of x.

The pointer is set at c_0 with $x = 0$. As L_1 is displaced through values of x, the pointer indicates on the scale the corresponding values of the polynomial. A real root is indicated by zero and the real part of a pair of imaginary roots by a change of direction of the pointer.

Another machine is described for quadratic equations based on a linkage for x^2 suggested by Sylvester. A historical sketch is presented in an appendix, describing machines proposed by Clairaut in 1820 and de Roos in 1879 and mentioning the analytic engine of Charles Babbage. Finally, linkages are described, one for transporting a dimension parallel to itself and one for forming a product based on the differences of squares.

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[1] J. S. Frame, "Machines for solving algebraic equations," *MTAC*, v. 1, 1945, p. 337–353.

[2] Frank T. Freeland, "A machine for the solution of the equation of the n-th Degree," Engineers' Club of Philadelphia, *Proc.*, v. 2, Feb. 23, 1880.

QUERIES—REPLIES

50. Incomplete Hankel Function. (**Q. 43**, v. 8, p. 51).

Put

$$z = (x^2 - 1)^{\frac{1}{2}}, \quad s = y(t^2 - 1)^{\frac{1}{2}}, \quad u = yt = (s^2 + y^2)^{\frac{1}{2}}$$

to obtain

$$\begin{aligned} f(x, y) &= \int_x^\infty (t^2 - 1)^{-\frac{1}{2}} e^{-iyt} dt \\ &= \int_{yz}^\infty u^{-1} e^{-iu} ds \\ &= \int_0^\infty u^{-1} \cos u ds - i \int_0^\infty u^{-1} \sin u ds - \int_0^{yz} u^{-1} ds \\ &+ \int_0^{yz} u^{-1}(1 - \cos u) ds + i \int_0^{yz} u^{-1} \sin u ds. \end{aligned}$$

The first two integrals represent Bessel functions, and the third is an elementary integral. Thus

$$f(x, y) = \{\tfrac{1}{2}\pi Y_0(y) - \sinh^{-1}z + C(y, yz)\} - i\{\tfrac{1}{2}\pi J_0(y) - S(y, yz)\}.$$

Tables[1] of the integrals C and S have been reviewed in RMT **651** (*MTAC*, v. 3, 1948–49, p. 479–482).

A. E.

[1] HARVARD UNIVERSITY, COMPUTATION LABORATORY, *Annals*, v. 18, 19: *Tables of Generalized Sine- and Cosine-Integral Functions*, Parts I and II, 1949.

CORRIGENDUM

V. 4, p. 29, l. −13, for xx read 11.