$$f_1(3/x) \doteq .79788456 + .00000156\,(3/x) + .01659667\,(3/x)^2 + .00017105\,(3/x)^3 - .00249511\,(3/x)^4 + .00113653\,(3/x)^5 - .00020033\,(3/x)^6$$

$$|\epsilon|_{\max} = 8 \times 10^{-8}$$

$$\varphi_1(3/x) \doteq .78539816 - .12499612\,(3/x) - .00005650\,(3/x)^2 + .00637879\,(3/x)^3 - .00074348\,(3/x)^4 - .00079824\,(3/x)^5 + .00029166\,(3/x)^6$$

E. E. ALLEN

Shell Development Co.
Houston, 25, Texas

170.—A SIEVE PROBLEM ON "PSEUDO-SQUARES." The following problem originated by KRAITCHIK,[1] and extended by LEHMER[2] by a special sieve, has recently been further extended by the SWAC. Let p be a prime. Let N_p be the least positive non-square integer of the form $8x + 1$ that is a quadratic residue of all primes $\leqslant p$. In this definition, zero is not counted as a quadratic residue so that N_p is not allowed to be divisible by any primes $\leqslant p$. Since squares are quadratic residues of any prime, the numbers N_p behave like squares and may be called pseudo-squares. This fact makes the problem of discovering pseudo-squares not only a sifting problem but also one of rejecting squares. Thus the problem is unsuitable for a high speed sieve alone since the output would be mostly squares, each of which would have to be tested by a more elaborate arithmetic unit. The problem is therefore one for an all-purpose computer programmed for sifting.[3]

Since for $p > 3$, N_p must be of the form $24x + 1$. One may proceed to exclude values of x using prime moduli between 5 and p inclusive. For every value of x not excluded the machine is programmed to extract the square root of $24x + 1$. If this is a perfect square, the machine returns to the sifting program for the next value of x. Fortunately the early part of the program, where the squares come thick and fast, had already been carried[2] as far as $N_{61} = 48473881$ in 1928 so that when programmed for the SWAC the routine spends most of its time sifting. Actually, for the record, the SWAC was instructed to print out every 64th square it produced. The complete table of N_p for $p < 83$ is as follows.

p	N_p	p	N_p	p	N_p
2	17	19	53881	47	9257329
3	73	23	87481	53	22000801
5	241	29	117049	59	48473881
7	1009	31	515761	61	48473881
11	2641	37	1083289	67	175244281
13	8089	41	3206641	71	427733329
17	18001	43	3818929	73	427733329
				79	898716289

All N_p above are primes except for

$$N_{11} = 19 \cdot 139$$
$$N_{17} = 47 \cdot 383$$
$$N_{29} = 67 \cdot 1747$$
$$N_{41} = 643 \cdot 4987$$

The interest in pseudo-squares is heightened by the fact that they may be used in tests for primality, as shown by MARSHALL HALL.[4] The operation of the SWAC and the reduction and checking of the output data was done by JOHN SELFRIDGE.

D. H. L.

[1] M. KRAITCHIK, *Recherches sur la Theorie de Nombres*, v. 1, Paris, 1924, p. 41–46.
[2] D. H. LEHMER, "The mechanical combination of linear forms," *Amer. Math. Monthly*, v. 35, 1928, p. 114–121.
[3] D. H. LEHMER, "The sieve problem for all-purpose computers," *MTAC*, v. 7, 1953, p. 6–14.
[4] M. HALL, "Quadratic residues in factorization," Amer. Math. Soc., *Bull.*, v. 39, 1933, p. 578–763.

171.—L. F. RICHARDSON (1881–1953). This English mathematician made several notable contributions to numerical analysis. A brief account of his life by P. A. SHEPPARD appears in *Nature* (v. 172, 1953, p. 1127–8).

His work on numerical analysis (apart from that appearing incidentally in his book[1]) was mainly contained in three long papers:

A: "The approximate arithmetical solution by finite differences of physical problems, involving differential equations with an application to the stresses in a masonry dam." Royal. Soc. *Phil. Trans.*, v. 210 A, 1910, p. 307–357.
B: "The deferred approach to the limit"—Part I, L. F. RICHARDSON, Part II, J. A. GAUNT, Royal Soc. *Phil. Trans.*, v. 226 A, 1926, p. 299–361.
C: "A purification method for computing the latent columns of numerical matrices and some integrals of differential equations," Royal Soc. *Phil. Trans.*, v. 242 A, 1950, p. 439–491.

There were minor contributions in

D: "Theory of the measurement of wind by shooting spheres upward," Royal Soc. *Phil. Trans.*, v. 223 A, 1923, p. 345–361.

An introduction to some of the material of A, B appeared in

E: "How to solve differential equations approximately by arithmetic," *Math. Gazette* v. 12, 1925, p. 415–421.

and one to some of the material of C in

F: "A method for computing principal axes," *British Jn. of Psychology*, v. 3, 1950, p. 16–20.

His work is highly individualistic and his language and symbolism picturesque. For instance he introduced the terms "marching" problem, for initial value problems of the form

$$y'' = ky, \quad y(0), \quad y'(0) \text{ given},$$

and "jury" problem for a problem of the form

$$y^{vi} - 3y^{iv} + 3y'' - y = \lambda y, \; y = y'' = y''' = 0 \text{ for } x = \pm 1.$$