[1] A. C. AITKEN, "The evaluation of the latent roots and latent vectors of a matrix," Roy. Soc. Edinburgh, *Proc.*, v. 57, 1936–1937, p. 269–304.
[2] E. BODEWIG, "Bericht über die Methoden zur numerischen Lösung algebraischer Eigenwertprobleme," Seminario matematico e fisico dell' Universita di Modena, *Atti*, v. 4, 1949–1950, v. 5, 1950–1951.
[3] R. VON MISES, "Praktische Methoden zur Gleichungsauflösung," *Zeit. angw. Math. Mech.*, v. 9, 1929, p. 62–77.

169.—ANALYTICAL APPROXIMATIONS. [See also NOTE **157.**] The following are approximations for the exponential integral and certain Bessel functions.

(56) $0 \le x \le 1 \quad |\epsilon|_{max} = 2 \times 10^{-7}$
$$-\,\mathrm{Ei}(-x) + \log_e x \doteq -\,.57721566 + .99999193\,x - .24991055\,x^2 + .05519968\,x^3 - .00976004\,x^4 + .00107857\,x^5$$

(57) $-3 \le x \le 3 \quad |\epsilon|_{max} = 10^{-7}$
$$J_0(x) \doteq 1 - 2.2499997\,(x/3)^2 + 1.2656208\,(x/3)^4 - .3163866\,(x/3)^6 + .0444479\,(x/3)^8 - .0039444\,(x/3)^{10} + .0002100\,(x/3)^{12}$$

(58) $0 \le x \le 3 \quad |\epsilon|_{max} = 2 \times 10^{-8}$
$$Y_0(x) - \frac{2}{\pi}\log_e\frac{x}{2}\,J_0(x) \doteq .36746691 + .60559366\,(x/3)^2 - .74350384\,(x/3)^4 + .25300117\,(x/3)^6 - .04261214\,(x/3)^8 + .00427916\,(x/3)^{10} - .00024846\,(x/3)^{12}$$

(59, 60) $3 \le x < \infty$
$$J_0(x) = x^{-\frac{1}{2}} f_0(3/x) \cos\{x - \varphi_0(3/x)\}$$
$$Y_0(x) = x^{-\frac{1}{2}} f_0(3/x) \sin\{x - \varphi_0(3/x)\}$$
$$|\epsilon|_{max} = 10^{-8}$$
$$f_0(3/x) \doteq .79788456 - .00000077\,(3/x) - .00552740\,(3/x)^2 - .00009512\,(3/x)^3 + .00137237\,(3/x)^4 - .00072805\,(3/x)^5 + .00014476\,(3/x)^6$$
$$|\epsilon|_{max} = 5 \times 10^{-8}$$
$$\varphi_0(3/x) \doteq .78539816 + .04166397\,(3/x) + .00003954\,(3/x)^2 - .00262573\,(3/x)^3 + .00054125\,(3/x)^4 + .00029333\,(3/x)^5 - .00013558\,(3/x)^6$$

(61) $0 \le x \le 3 \quad |\epsilon|_{max} = 5 \times 10^{-9}$
$$J_1(x)/x \doteq .5 - .56249985\,(x/3)^2 + .21093573\,(x/3)^4 - .03954289\,(x/3)^6 + .00443319\,(x/3)^8 - .00031761\,(x/3)^{10} + .00001109\,(x/3)^{12}$$

(62) $0 \le x \le 3 \quad |\epsilon|_{max} = 5 \times 10^{-8}$
$$\left\{Y_1(x) - \frac{2}{\pi}\log_e\frac{x}{2}\,J_1(x)\right\}x \doteq -\,.6366198 + .2212091\,(x/3)^2 + 2.1682709\,(x/3)^4 - 1.3164827\,(x/3)^6 + .3123951\,(x/3)^8 - .0400976\,(x/3)^{10} + .0027873\,(x/3)^{12}$$

(63, 64) $3 \le x < \infty$
$$J_1(x) = x^{-\frac{1}{2}} f_1(3/x) \sin\{x - \varphi_1(3/x)\}$$
$$Y_1(x) = -\,x^{-\frac{1}{2}} f_1(3/x) \cos\{x - \varphi_1(3/x)\}$$
$$|\epsilon|_{max} = 3 \times 10^{-8}$$

$$f_1(3/x) \doteq .79788456 + .00000156\,(3/x) + .01659667\,(3/x)^2 + .00017105\,(3/x)^3 - .00249511\,(3/x)^4 + .00113653\,(3/x)^5 - .00020033\,(3/x)^6$$

$$|\epsilon|_{max} = 8 \times 10^{-8}$$

$$\varphi_1(3/x) \doteq .78539816 - .12499612\,(3/x) - .00005650\,(3/x)^2 + .00637879\,(3/x)^3 - .00074348\,(3/x)^4 - .00079824\,(3/x)^5 + .00029166\,(3/x)^6$$

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170.—A SIEVE PROBLEM ON "PSEUDO-SQUARES." The following problem originated by KRAITCHIK,[1] and extended by LEHMER[2] by a special sieve, has recently been further extended by the SWAC. Let p be a prime. Let N_p be the least positive non-square integer of the form $8x + 1$ that is a quadratic residue of all primes $\leqslant p$. In this definition, zero is not counted as a quadratic residue so that N_p is not allowed to be divisible by any primes $\leqslant p$. Since squares are quadratic residues of any prime, the numbers N_p behave like squares and may be called pseudo-squares. This fact makes the problem of discovering pseudo-squares not only a sifting problem but also one of rejecting squares. Thus the problem is unsuitable for a high speed sieve alone since the output would be mostly squares, each of which would have to be tested by a more elaborate arithmetic unit. The problem is therefore one for an all-purpose computer programmed for sifting.[3]

Since for $p > 3$, N_p must be of the form $24x + 1$. One may proceed to exclude values of x using prime moduli between 5 and p inclusive. For every value of x not excluded the machine is programmed to extract the square root of $24x + 1$. If this is a perfect square, the machine returns to the sifting program for the next value of x. Fortunately the early part of the program, where the squares come thick and fast, had already been carried[2] as far as $N_{61} = 48473881$ in 1928 so that when programmed for the SWAC the routine spends most of its time sifting. Actually, for the record, the SWAC was instructed to print out every 64th square it produced. The complete table of N_p for $p < 83$ is as follows.

p	N_p	p	N_p	p	N_p
2	17	19	53881	47	9257329
3	73	23	87481	53	22000801
5	241	29	117049	59	48473881
7	1009	31	515761	61	48473881
11	2641	37	1083289	67	175244281
13	8089	41	3206641	71	427733329
17	18001	43	3818929	73	427733329
				79	898716289

All N_p above are primes except for

$$N_{11} = 19\cdot 139$$
$$N_{17} = 47\cdot 383$$
$$N_{29} = 67\cdot 1747$$
$$N_{41} = 643\cdot 4987$$