There are tables showing the variables in the design of synchronous machines and a table of operations and sample calculations for ascertaining performance of a $\frac{1}{2}$-hp. induction motor. The "digital approach" and its advantage over hand computations is discussed.

Discussion, pro and con, by ROBERT L. FILLMORE and C. G. VEINOTT is followed with a rebuttal by Saunders.

A. R. COCK

NBSCL

## NEWS

**Numerical Analysis Research, UCLA.**—Numerical Analysis Research, UCLA, is continuing much of the research program of the Institute for Numerical Analysis (INA) of the National Bureau of Standards; INA was disbanded on 30 June 1954 [*MTAC*, v. 8, p. 178].

The new organization, administered by the UCLA Mathematics Department, will receive support from the Office of Naval Research and the Office of Ordnance Research. Its primary mission includes pertinent fundamental research in mathematics and science and research in the application of computers to problems occurring in science and other applied fields. It will be aided in attaining these objectives by the use of the National Bureau of Standards Western Automatic Computer (SWAC) and other equipment which has been loaned to UCLA. In addition, the new organization has the use of INA's library. Most of the research staff of INA has joined the new organization.

The organization offers a training program in the efficient application of high-speed digital computers. A number of graduate assistantships offered in cooperation with various university departments are available each year. Seminars, and formal and informal courses in numerical analysis are conducted.

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## OTHER AIDS TO COMPUTATION

# A Useful Technique in Programming for Analog Computers

In programming for analog computers programmers have found certain types of linear differential equations difficult to handle even with the use of arbitrary function generators. In this paper a technique is presented which enables the analog computer to solve a heretofore difficult set of problems. The technique, while most useful in linear problems, is also useful in other problems. In many cases the technique will reduce the sensitivity of the solution to noise.

Let us consider the general ordinary differential equation

$$f(y,y',y'',\cdots,x) = 0 \tag{1}$$

with the parametric transformation

$$\phi(x,\dot{x},\ddot{x},\cdots,t) = 0. \tag{2}$$

One sets up feedback circuits to generate $x$ and functions of $x$ and uses these to generate $y$. In the case $\phi = x - t = 0$ one gets the usual setup. The special case $\phi = x \pm e^{at} = 0$ has several advantages in certain cases. Two examples following show its use:

Consider the equation

$$(3)\qquad (2x^{2/3} + x)y' + (3x^{-.1} + x^{-10})y = 2x^{.45}.$$

For $x > 1$, let $x = e^t$. Then one gets

$$(4)\qquad \dot{y} = [2e^{.45t} - y(3e^{-.1t} + e^{-10t})]/(1 + 2e^{-t/3})$$

which is easy to set up, generating the exponentials in individual integrator-pot-amplifier loops. Of course, one plots $y$ against $e^t$, not $t$.

An example for which this technique is useful in a non-linear equation is the following:

$$(5)\qquad Ay'' + By' + Cy^{1/2}e^{-D/y} = 0.$$

Let $v = dy/dx$ and let $y = e^t$. Then $dx/dt = e^t/v$ and thus (5) becomes

$$(6)\qquad \dot{v} = \alpha e^t + \beta v^{-1} \exp\left(\tfrac{3}{2}t - De^{-t}\right),$$

where

$$\alpha = -B/A, \quad \beta = -C/A,$$

which can be generated without the use of special function generators.

A parametric substitution as in the above examples can frequently eliminate or considerably reduce the need for special function generators. Other parameterizations can be used, but that which is best can only be determined by an investigation of the specific problem under consideration.

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## BIBLIOGRAPHY Z

**1152.** S. AMITSUR, U. OPPENHEIM & A. MANY, "An electrical computer for the solution of linear simultaneous equations," *Rev. Sci. Instruments*, v. 24, 1953, p. 112-116.

A variation of the Many-Meiboom network[1] is given which will solve a symmetrical system of linear equations. The given system of equations is analogous to the statement of Kirchhoff's laws for a circuit of pure reactances. Transformers permit the representation of negative nondiagonal quantities by condensers but diagonal negative quantities necessitate the use of coils. The problem of phase difference between the input and output alternating voltages is discussed in detail.

F. J. M.

[1] A. MANY & S. MEIBOOM, "An electrical network for determining the eigenvalues and eigenvectors of a real symmetric matrix," *Rev. Sci. Instruments*, v. 18, 1947, p. 831–836.

**1153.** V. C. ANDERSON & PHILIP RUDNICK, "A thermistor bridge correlator," *Rev. Sci. Instruments*, v. 24, 1953, p. 360–361.

Thermistors are used to obtain the squares of voltages for use in a quarter square multiplier.

F. J. M.

**1154.** N. F. BARBER, "Harmonic analysis," *Rev. Sci. Instruments*, v. 24, 1953, p. 329.

An improvement in the method of F. B. DANIELS[1] is suggested.

[1] F. B. DANIELS, "A planimetric method of harmonic analysis," *Rev. Sci. Instruments*, v. 23, 1952, p. 369–370; *MTAC*, v. 7, p. 120.

**1155.** M. L. BOAS, "Nomograms used in analysis of data in the reaction $Li^6(n,\alpha)H^3$," *Rev. Sci. Instruments*, v. 24, 1953, p. 356–359.

**1156.** S. L. BROWN & H. D. SCHWETMAN, "The application of the Laplace transformation and a mechanical harmonic synthesizer in the analysis of electric circuits," *Rev. Sci. Instruments*, v. 24, 1953, p. 375–379.

The harmonic synthesizer is a purely mechanical one, using gear ratios, scotch yokes and a chain adder. The harmonic synthesizer is used to plot the values of $P(z)$ on a circle $|z| = r$. Thus the real part of $P(z)$ is given by $\sum_{k=0}^{n} a_k r^k \cos k\varphi$ and the imaginary part by a similar expression. The synthesizer is used to plot the complex number $P(z)$ and the number of times this graph loops the origin is the number of roots of $P(z)$ within the circle. Two examples are given with detailed references to specific circuits.

F. J. M.

**1157.** G. L. BROWNELL, R. V. CAVICCHI & K. E. PERRY, "An electrical analog for analysis of compartmental biological systems," *Rev. Sci. Instruments*, v. 24, 1953, p. 704–710.

An analogy is set up between a biological system consisting of fluids in various compartments and an electrical circuit consisting of condensers connected by resistors. The condensers receive an initial charge at intervals of a tenth of a second and the transient response of the system is indicated on an oscilloscope. The system is used to analyze the results of tracer experiments. Models or simply diffusion constant values are hypothecated and tested by the analyzer and the results compared with experimental values obtained by the use of traces of radioactive elements.

F. J. M.

**1158.** A. B. CAMBEL & PAUL J. SCHNEIDER, "Membrane apparatus for analogic experiments," *Rev. Sci. Instruments*, v. 24, 1953, p. 513–514.

A soap film apparatus for solving the Laplacian equation is described.

**1159.** I. CEDERBAUM, "Note on high speed product integrator," *Rev. Sci. Instruments*, v. 24, 1953, p. 1072–1073.

The author obtains an exact expression for the error due to the variation of $y$ in the Macnee set up described in MACNEE (**1163**) in the case in which the kernel $K$ is trigonometric and $y$ varies either continuously or in small discrete steps.

F. J. M.

**1160.** C. B. Crumb, Jr., "Engineering uses of analog computing machines," *Mechanical Engineering*, v. 74, 1952, p. 635–639.

This is an introductory article which points out the nature of differential analyzers, both mechanical and electrical, various possible applications and a brief description of how such a computer is used.

F. J. M.

**1161.** C. A. Ludeke & R. T. Evans, "A coupling analog for nonlinear systems with more than one degree of freedom," *Jn. Appl. Physics*, v. 24, 1953, p. 119–122.

C. A. Ludeke & C. L. Morrison, "Analog computer elements for solving nonlinear differential equations," *Jn. Appl. Physics*, v. 24, 1953, p. 243–248.

The objective of the work of these papers is to develop a differential analyzer, whose elementary component contains a representation of a nonlinear differential equation of the second order, for instance

$$\frac{d^2\theta}{dt^2} + f(\theta) = 0.$$

In the second paper, which seems to precede the first logically, such an element is developed in the form of a coil suspended as a pendulum within another coil. The angle of rotation of the coil is the dependent variable and it can be used in turn to rotate a mask controlling the amount of light which falls on a photo tube. The output of the latter is amplified and the resulting current passes through the coil on the pendulum. This yields a possible representation of $f(\theta)$ as a restoring force dependent upon the dependent variable, $\theta$. A number of examples of periodic motions are quoted and good accuracy in the frequency is indicated. In the first paper, coupling between two such units by mounting two coils on each pendulum is described.

F. J. M.

**1162.** F. J. McDonal, "A Fourier analyzer," *Rev. Sci. Instruments*, v. 24, 1953, p. 272–276.

This instrument was developed to provide frequency analysis of transient signals, obtained on a multichannel recorder with band width of 5 to 200 cps, during seismic prospecting for oil. The record on variable area film is read by a photoelectric cell, and thus the function to be transformed is obtained as a voltage. A scotch yoke produces the sines and cosines in a form which yields a shaft rotation and hence a linear potentiometer can be used as a multiplier. Integration is by a tachometer type servo integrator and the output appears on a revolutions counter.

F. J. M.

**1163.** A. B. Macnee, "A high speed product integrator," *Rev. Sci. Instruments*, v. 24, 1953, p. 207–211.

The integrals

$$F(y) = \int_0^T f(t)K(y,t)dt \quad \text{or} \quad \int_0^y f(t)K(y,t)dt$$

are evaluated by means of high speed analog computer elements for various values of $y$. The variable $y$ is given a sequence of values and by a suitable switching procedure the values of $F(y)$ are evaluated at a rate of 60 per second and plotted with a reference value on a cathode ray tube. The functions $f(t)$ and $K(y,t)$ for $y$ fixed are obtained by function generators or by differential analyzer techniques. During the generation of $F(y)$, $y$ which should remain fixed does actually vary. The error due to this cause and the errors due to the multiplier and function generator are discussed.

F. J. M.

**1164.** J. R. Ragazzini & G. Reynolds, "The electronic complex plane scanner," *Rev. Sci. Instruments*, v. 24, 1953, p. 523–527.

A rational function $F(z)$ is supposed given in the form of a quotient of products of linear factors $z - \lambda_i$. It is required to obtain log $|F(jw)|$ and arg $F(jw)$ where $j = \sqrt{-1}$ and $w$ is real. Log $|F(jw)|$ is the sum of $\pm$ log $|(jw - \lambda_i)|$. The quantity $jw - \lambda_i$ is represented as a complex voltage and log $|(jw - \lambda_i)|$ is obtained by a suitable circuit. The argument of $F(jw)$ is obtained by differentiation using the Cauchy-Riemann equations.

F. J. M.

**1165.** J. Tadayon, "Measurement of the angle between two curves," *Rev. Sci. Instruments*, v. 24, 1953, p. 871–872.

An optical instrument for measuring the angle between two curves on a graph is based on the mirror principle for finding the normal to a curve.

F. J. M.

## NOTES

**168.**—A Practical Refutation of the Iteration Method for the Algebraic Eigenproblem. In the second part of my paper on algebraic eigenproblems[2] I have proved that the computation by means of the formation of the characteristic equation requires less computational work than the iteration method, and that this holds even when nothing but the first eigenvalue has to be calculated. This advantage grows with every accessory eigenvalue or vector. Further one has no trouble with deflation which requires a lot of multiplications. Also one can compute every eigenvalue and vector apart from the others, and do this to any desired accuracy, by the more powerful methods for algebraic equations. At last, there arise no difficulties from an unfavorable quotient of the two dominant eigenvalues.

The reason these facts are not yet universally acknowledged is that the iteration method seems to be simpler and more mechanical in its application. But one has to consider that iteration does not converge quickly enough in practice, unless the quotient of the two dominant eigenvalues is $\frac{1}{4}$ or less. This last will be rarely the case. For the eigenvalues must lie somewhere between two circles around the origin in the complex plane. The radius of