interest lies in comparing statistics of such expansions with known distributions of these statistics over random numbers. For example, KHINTCHINE[1] has shown that over random numbers x uniformly distributed between 0 and 1, the sum $S_n(x)$ of the first n partial quotients of x is equivalent in the sense of Bernoulli to $Z_n = n \log n / \log 2$.

The first result is the computation of more than 2000 partial quotients of $2^{\frac{1}{3}}$. The table below shows $S_n(2^{\frac{1}{3}})$ for $n = 100(100)2000$, with Z_n given for comparison. It appears that $S_n(2^{\frac{1}{3}})$ oscillates considerably in relation to Z_n, being most of the time larger, up to a factor of about 2. We do not know whether this deviation is significant, since[1] oscillations of $S_n(x)$ of this type occur for almost all x. Expansions of additional numbers, as well as more detailed statistics, will follow.

The code depends on subroutines which do the necessary algebra on polynomials whose coefficients are p-tuples of computer words, for arbitrary and variable p. At present it handles cubic polynomials; a generalization to nth degree polynomials is planned. The methods and results will be reported later at greater length.

n	$n \log n / \log 2$	$S_n(2^{\frac{1}{3}})$
100	664.4	1384
200	1528.8	2283
300	2468.6	2834
400	3457.5	3471
500	4482.9	4191
600	5537.3	12636
700	6615.8	18190
800	7715.1	18777
900	8832.4	19139
1000	9965.8	19724
1100	11113.6	20322
1200	12274.6	21825
1300	13447.6	22873
1400	14631.7	23293
1500	15826.1	24271
1600	17030.2	25259
1700	18243.2	25819
1800	19464.8	26442
1900	20694.4	27063
2000	21931.6	41198

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[1] A. KHINTCHINE, "Metrische Kettenbruchprobleme," *Compositio Mathematica*, v. 1, 1935, p. 361–382.

The Values of $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ and their Logarithms Accurate to 28 Decimals

The values of $\Gamma(\frac{1}{3})$, $\Gamma(\frac{2}{3})$, $\log \Gamma(\frac{1}{3})$, $\log \Gamma(\frac{2}{3})$ were computed to 28 decimals using the series

$$\log \Gamma(2 + x) = C_1 x + C_2 x^2 - C_3 x^3 + C_4 x^4 - C_5 x^5 + \cdots + (-1)^r C_r x^r + \cdots$$

where $C_1 = 1 - \gamma$, $\gamma = \lim \left(1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{n} - \log n \right)$: Euler's constant

$$C_r = \frac{1}{r} \left(\frac{1}{2^r} + \frac{1}{3^r} + \frac{1}{4^r} + \cdots \right) \quad (r = 2, 3, \cdots).$$

The values of $S_r = \dfrac{1}{1^r} + \dfrac{1}{2^r} + \dfrac{1}{3^r} + \cdots$ and γ were taken from Stieltjes' table.[1] Part of the calculation was done with the assistance of Mr. E. V. HANKAM on an IBM (602-A type) calculating punch. Uhler's radix table was used for getting the antilog of log Γ. The values $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ were required for calculating the power series coefficients of Bessel functions of order $\frac{1}{3}$ and of functions related to them.

The values were checked by the identity

$$\sqrt{3}\, \Gamma(\tfrac{1}{3}) \Gamma(\tfrac{2}{3}) = 2\pi$$

$$\begin{aligned}
\Gamma(\tfrac{1}{3}) &= 2.67893\ 85347\ 07747\ 63365\ 56929\ 410 \\
\Gamma(\tfrac{2}{3}) &= 1.35411\ 79394\ 26400\ 41694\ 52880\ 282 \\
\log \Gamma(\tfrac{1}{3}) &= .98542\ 06469\ 27767\ 06918\ 71740\ 370 \\
\log \Gamma(\tfrac{2}{3}) &= .30315\ 02751\ 47523\ 56867\ 58628\ 174
\end{aligned}$$

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[1] H. T. DAVIS, *Tables of Higher Mathematical Functions.* v. II, The Principia Press, 1935, p. 244.

Modification of a Method for Calculating Inverse Trigonometric Functions

The 605 programming that I gave recently[1] fails for arguments near $2^{-\frac{1}{2}}$. The reason for this failure is that the double angle formulations used multiply round-off errors until they are intolerably large. These formulations were originally introduced to assure that $\cos 2\theta$ depend on both $\sin \theta$ and $\cos \theta$. Upon closer examination it was found that it is only necessary that $\cos 2\theta$ depend on $\sin \theta$, hence we may use

$$\cos 2\theta = 1 - 2 \sin^2 \theta.$$

The use of the above formula and

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

avoids the errors mentioned and is just as easily programmed for the 605.

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[1] RICHARD L. LA FARA, *MTAC*, v. 8, 1954, p. 132–139.