interest lies in comparing statistics of such expansions with known distributions of these statistics over random numbers. For example, KHINTCHINE[1] has shown that over random numbers $x$ uniformly distributed between 0 and 1, the sum $S_n(x)$ of the first $n$ partial quotients of $x$ is equivalent in the sense of Bernoulli to $Z_n = n \log n/\log 2$.

The first result is the computation of more than 2000 partial quotients of $2^{\frac{1}{3}}$. The table below shows $S_n(2^{\frac{1}{3}})$ for $n = 100(100)2000$, with $Z_n$ given for comparison. It appears that $S_n(2^{\frac{1}{3}})$ oscillates considerably in relation to $Z_n$, being most of the time larger, up to a factor of about 2. We do not know whether this deviation is significant, since[1] oscillations of $S_n(x)$ of this type occur for almost all $x$. Expansions of additional numbers, as well as more detailed statistics, will follow.

The code depends on subroutines which do the necessary algebra on polynomials whose coefficients are $p$-tuples of computer words, for arbitrary and variable $p$. At present it handles cubic polynomials; a generalization to $n$th degree polynomials is planned. The methods and results will be reported later at greater length.

| $n$ | $n \log n/\log 2$ | $S_n(2^{\frac{1}{3}})$ |
|---|---|---|
| 100 | 664.4 | 1384 |
| 200 | 1528.8 | 2283 |
| 300 | 2468.6 | 2834 |
| 400 | 3457.5 | 3471 |
| 500 | 4482.9 | 4191 |
| 600 | 5537.3 | 12636 |
| 700 | 6615.8 | 18190 |
| 800 | 7715.1 | 18777 |
| 900 | 8832.4 | 19139 |
| 1000 | 9965.8 | 19724 |
| 1100 | 11113.6 | 20322 |
| 1200 | 12274.6 | 21825 |
| 1300 | 13447.6 | 22873 |
| 1400 | 14631.7 | 23293 |
| 1500 | 15826.1 | 24271 |
| 1600 | 17030.2 | 25259 |
| 1700 | 18243.2 | 25819 |
| 1800 | 19464.8 | 26442 |
| 1900 | 20694.4 | 27063 |
| 2000 | 21931.6 | 41198 |

JOHN VON NEUMANN
BRYANT TUCKERMAN

Institute for Advanced Study
Princeton, New Jersey

[1] A. KHINTCHINE, "Metrische Kettenbruchprobleme," *Compositio Mathematica*, v. 1, 1935, p. 361–382.

## The Values of $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ and their Logarithms Accurate to 28 Decimals

The values of $\Gamma(\frac{1}{3})$, $\Gamma(\frac{2}{3})$, $\log \Gamma(\frac{1}{3})$, $\log \Gamma(\frac{2}{3})$ were computed to 28 decimals using the series

$$\log \Gamma(2 + x) = C_1 x + C_2 x^2 - C_3 x^3 + C_4 x^4 - C_5 x^5 + \cdots + (-1)^r C_r x^r + \cdots$$

where $C_1 = 1 - \gamma$, $\gamma = \lim \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \log n\right)$: Euler's constant

$$C_r = \frac{1}{r}\left(\frac{1}{2^r} + \frac{1}{3^r} + \frac{1}{4^r} + \cdots\right) \quad (r = 2, 3, \cdots).$$

The values of $S_r = \frac{1}{1^r} + \frac{1}{2^r} + \frac{1}{3^r} + \cdots$ and $\gamma$ were taken from Stieltjes' table.[1] Part of the calculation was done with the assistance of Mr. E. V. Hankam on an IBM (602-A type) calculating punch. Uhler's radix table was used for getting the antilog of $\log \Gamma$. The values $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ were required for calculating the power series coefficients of Bessel functions of order $\frac{1}{3}$ and of functions related to them.

The values were checked by the identity

$$\sqrt{3}\Gamma(\tfrac{1}{3})\Gamma(\tfrac{2}{3}) = 2\pi$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Gamma(\frac{1}{3})$ = | 2.67893 | 85347 | 07747 | 63365 | 56929 | 410 |
| $\Gamma(\frac{2}{3})$ = | 1.35411 | 79394 | 26400 | 41694 | 52880 | 282 |
| $\log \Gamma(\frac{1}{3})$ = | .98542 | 06469 | 27767 | 06918 | 71740 | 370 |
| $\log \Gamma(\frac{2}{3})$ = | .30315 | 02751 | 47523 | 56867 | 58628 | 174 |

B. Zondek

Watson Scientific Computing Laboratory
New York, N. Y.

[1] H. T. Davis, *Tables of Higher Mathematical Functions.* v. II, The Principia Press, 1935, p. 244.

## Modification of a Method for Calculating Inverse Trigonometric Functions

The 605 programming that I gave recently[1] fails for arguments near $2^{-\frac{1}{2}}$. The reason for this failure is that the double angle formulations used multiply round-off errors until they are intolerably large. These formulations were originally introduced to assure that $\cos 2\theta$ depend on both $\sin \theta$ and $\cos \theta$. Upon closer examination it was found that it is only necessary that $\cos 2\theta$ depend on $\sin \theta$, hence we may use

$$\cos 2\theta = 1 - 2 \sin^2 \theta.$$

The use of the above formula and

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

avoids the errors mentioned and is just as easily programmed for the 605.

R. L. La Fara

U. S. Naval Ordnance Plant
Indianapolis 18, Indiana

[1] Richard L. La Fara, *MTAC*, v. 8, 1954, p. 132–139.