To make use of these expansions, for finding optimum polynomials for use with an automatic digital calculator, it is necessary to have numerical values for the functions $J_n(\pi/2)J_{n+1/2}(\pi/2)$. Since these do not appear to have been tabulated it was thought worth constructing the table given below. Values were obtained by means of the well-known series:

$$J_\nu(z) = \frac{(z/2)^\nu}{\Gamma(\nu+1)}\left\{1 - \frac{(z/2)^2}{1!(\nu+1)} + \frac{(z/2)^4}{2!(\nu+1)(\nu+2)} - \cdots\right\}$$

using the value $(z = \pi/2)$ taken to 20 decimal places. The resulting values were then rounded off to 11 decimal places and the resulting table checked by an application of the recursion formula:

$$J_\nu(z) = z/2\nu\{J_{\nu+1}(z) + J_{\nu-1}(z)\}.$$

In addition, the value of $J_{11.5}(\pi/2)$ was calculated directly from the recursion formula using the explicit value derived from the initial values:

$$J_{1/2}(\pi/2) = 2/\pi,$$

$$J_{3/2}(\pi/2) = 4/\pi^2,$$

and the table of values of π^{-n} to 25 decimal places computed by GLAISHER.[5]

n	$J_n(\pi/2)$	n	$J_n(\pi/2)$
0	0.47200 12157 7	.5	.63661 97723 7
1	0.56682 40889 1	1.5	.40528 47345 7
2	0.24970 16291 4	2.5	.13741 70540 3
3	0.06903 58882 9	3.5	.03212 73337 1
4	0.01399 60398 1	4.5	.00575 32170 8
5	0.00224 53571 2	5.5	.00083 61720 0
6	0.00029 83476 0	6.5	.00010 23428 0
7	0.00003 38506 4	7.5	.00001 08228 5
8	0.00000 33522 0	8.5	.00000 10077 8
9	0.00000 02945 7	9.5	.00000 00838 4
10	0.00000 00232 7	10.5	.00000 00063 0
11	0.00000 00016 7	11.5	.00000 00004 3
12	0.00000 00001 1	12.5	.00000 00000 3

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Continued Fraction Expansion of $2^{1/3}$

The Institute for Advanced Study computer is being used to compute extensive continued fraction expansions of certain real algebraic numbers. The interest lies in comparing statistics of such expansions with known distributions of these statistics over random numbers. For example, KHINTCHINE[1] has shown that over random numbers x uniformly distributed between 0 and 1, the sum $S_n(x)$ of the first n partial quotients of x is equivalent in the sense of Bernoulli to $Z_n = n \log n/\log 2$.

The first result is the computation of more than 2000 partial quotients of $2^{1/3}$. The table below shows $S_n(2^{1/3})$ for $n = 100(100)2000$, with Z_n given for comparison. It appears that $S_n(2^{1/3})$ oscillates considerably in relation to Z_n, being most of the time larger, up to a factor of about 2. We do not know whether this deviation is significant, since[1] oscillations of $S_n(x)$ of this type occur for almost all x. Expansions of additional numbers, as well as more detailed statistics, will follow.

The code depends on subroutines which do the necessary algebra on polynomials whose coefficients are p-tuples of computer words, for arbitrary and variable p. At present it handles cubic polynomials; a generalization to nth degree polynomials is planned. The methods and results will be reported later at greater length.

n	$n \log n/\log 2$	$S_n(2^{1/3})$
100	664.4	1384
200	1528.8	2283
300	2468.6	2834
400	3457.5	3471
500	4482.9	4191
600	5537.3	12636
700	6615.8	18190
800	7715.1	18777
900	8832.4	19139
1000	9965.8	19724
1100	11113.6	20322
1200	12274.6	21825
1300	13447.6	22873
1400	14631.7	23293
1500	15826.1	24271
1600	17030.2	25259
1700	18243.2	25819
1800	19464.8	26442
1900	20694.4	27063
2000	21931.6	41198

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The Values of $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ and their Logarithms Accurate to 28 Decimals

The values of $\Gamma(\frac{1}{3})$, $\Gamma(\frac{2}{3})$, $\log \Gamma(\frac{1}{3})$, $\log \Gamma(\frac{2}{3})$ were computed to 28 decimals using the series

$$\log \Gamma(2 + x) = C_1x + C_2x^2 - C_3x^3 + C_4x^4 - C_5x^5 + \cdots + (-1)^r C_r x^r + \cdots$$