The expressions of the partial derivatives up to the third order have been written out explicitly as shown in Tables V and VI.

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[1] H. E. SALZER, *Jn. Math. and Physics*, v. 26, 1948, p. 294–305.
[2] H. E. SALZER, Amer. Math. Soc., *Bull.*, v. 51, 1945, p. 279–280.
[3] *Nautical Almanac*, 1937, p. 802.

## TECHNICAL NOTES AND SHORT PAPERS

### The First Published Table of Logarithms to the Base Ten

The first two lines of the title-page of this Table, *LOGARITHMORVM/ Chillias Prima.*, are followed by 26 further lines. The full title is given in JAMES HENDERSON, *Biblioteca Tabularum Mathematicarum*, 1926, p. 30; a facsimile of the title-page is given in A. J. THOMPSON, *Logarithmetica Britannica*, part IX, 1924; or in his completed work, frontispiece to volume 2, 1954. There is nothing on the title-page to indicate the author, or the date and place of publication of the little 16-page pamphlet, which was, apparently, privately printed. On page 122 of JOHN WARD, *The Lives of the Professors of Gresham College*, London, 1740, there is the following quotation from a letter, dated 6 December 1617, written by Sir Henry Bourchier to Dr. Usher: "Our kind friend Mr. Briggs hath lately published a supplement to the most excellent table of logarithms, which I presume he sent to you." Thus in connection with other facts the author was revealed to be HENRY BRIGGS (1561–1631) and his table was printed in the latter part of 1617. John Napier (1550–1617) died in the previous April. Briggs had visited Napier in 1615, spent a month with him in 1616, and planned to show him this Table in the summer of 1617.

Copies of the Table are excessively rare; the only copies known to exist are two in the British Museum: (a) with press mark c.54e 10(1); (b) a copy in the Museum's Manuscript Room; and (c) a copy in the Savilian Library, Oxford. (a) and (c) are bound up with Edmund Gunter's *Canon Triangulorum*, 1623, so that the original may have been trimmed; its present size is 9.3 $\times$ 15.5 cm. A photostat copy of (a) is a recent acquisition of the Library of Brown University.

In this Table are given log $N$, $N = [1(1)1000; 14D]$, with the first four numbers of first differences, rounded for $N = 500(1)1000$. Characteristics are separated from the decimal parts by lines. The accuracy of this Table is very extraordinary. Every entry of the Table was compared with A. J. Thompson's log $N$, $N = [1(1)1000; 21D]$ and the only errors were 153 in the fourteenth decimal place: 150 unit errors, and 3 two-unit errors at $N = 154, 239, 863$.

In contrast to this, when we turn to Briggs' remarkable *Arithmetica Logarithmica*, London, 1624, giving log $N$ for $N = [1(1)20\,000, 90\,000(1)10\,000; 14D]$ with difference throughout, we find, for $N = 1(1)1000$, no less than 19 errors; 18 two-units in the fourteenth decimal place and one serious error of 6 units in the seventh decimal place. Of the 150 unit errors in the 1617 publication the same unit errors occur in 121 of the entries of the 1624 volume. In neither table did we make any checking of difference entries.

The idea of constructing a table in which the logarithm of unity was zero originated with Napier. Napier and Briggs never thought of logarithms as exponents of a base. An excellent exposition of their ideas is given in G. A. Gibson, "Napier's logarithms and the change to Briggs's logarithms," p. 111–137 of C. G. Knott, *Napier Tercentenary Memorial Volume*, London, 1915; see also, H. S. Carslaw, "The discovery of logarithms by Napier," *Mathematical Gazette*, v. 8, 1915, p. 76–84, 115–119. It was not till considerably later that our modern definition of a logarithm as an exponent was put forward by such mathematicians as David Gregory, 1684; Wm. Gardiner, 1742; Leonard Euler, 1748, 1770.

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## On the Numerical Integration of Functions Tabulated in Logarithmic Form*

In a number of physical problems which can be described by differential equations, it occurs that one or more of the variables show a range of several orders of magnitude. In such cases, it is convenient to tabulate the variables in logarithmic form. The purpose of this note is to show that the usual finite-difference formulae for numerical integration can be easily adapted to integrate a function when only its logarithm is given.

We shall limit our attention to the Newton-Gregory (backward-difference) formula, which is the most commonly used in the hand-integration of differential equations. Let $y'$ be the function to be integrated with respect to the independent variable $x$ between the limits $x_0 - h$ and $x_0$, where $h$ is the interval of tabulation. Writing $x = x_0 + hm$, we can approximate $\ln y'$ by the "Newton-backward" interpolating polynomial $f(m)$ as follows:

$$(1)\quad \ln y' = f(m) = f_0 + \binom{m}{1}\Delta'_{-\frac{1}{2}} + \binom{m+1}{2}\Delta''_{-1} + \cdots$$
$$= f_0 + \sum_{j=1}^{n}\binom{m+j-1}{j}\Delta^{(j)}_{-\frac{1}{2}j} + \text{truncation error.}$$
$$(j = 1, 2, 3, \cdots, n).$$

We now want to obtain an integration formula of the type

$$(2)\quad \ln\left(\frac{1}{h}\int_{x_0-h}^{x_0} y'dx\right) = \ln\int_{-1}^{0}\exp f(m)dm$$
$$= f_0 + \ln\int_{-1}^{0}\left\{1 + \sum_{j=1}^{n}\binom{m+j-1}{j}\Delta^{(j)}_{-\frac{1}{2}j} + \frac{1}{2!}\left[\sum_{j=1}^{n}\binom{m+j-1}{j}\Delta^{(j)}_{-\frac{1}{2}j}\right]^2 + \cdots\right\}dm$$
$$= f_0 + N'\Delta'_{-\frac{1}{2}} + N''\Delta''_{-1} + \cdots.$$