

Then application of the recurrence relation (6) shows that

$$f(x) = b_0 p_0(x) + b_1 \{p_1(x) + \alpha_0 p_0(x)\}.$$

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<sup>1</sup> C. W. CLENSHAW, "Polynomial approximations to elementary functions," *MTAC*, v. 8, 1954, p. 143-147.

<sup>2</sup> NBS Applied Mathematics Series 9, *Tables of Chebyshev Polynomials  $S_n(x)$  and  $C_n(x)$* . U. S. Govt. Printing Office, Washington, 1952.

### Conjectures Concerning the Mersenne Numbers

Conjectures concerning the Mersenne numbers are appropriate since they were inaugurated with one. A conjecture [1] that seems likely to be false, but unlikely to be proved false, is that all numbers  $p_n$  are prime ( $n = 1, 2, 3, \dots$ ), where, for example,  $p_4$  is

$$\begin{array}{r} 2^2 - 1 \\ 2^2 - 1 \\ 2^2 - 1 \\ 2^2 - 1 \end{array}$$

Recursively,  $p_1 = 3$ ,  $p_{n+1} = 2^{p_n} - 1$ . The first four are 3, 7, 127 and  $2^{127} - 1$ , all known to be prime. Any factor of  $p_5$  is congruent to 1 modulo  $p_4$ , so  $p_5$  certainly has no factor less than  $2^{127}$ . Similarly

$$2^{2^{281}} - 1$$

is not divisible by any known prime, if  $2^{2281} - 1$  is still the largest known prime [2]. One can try to argue about the probability that a number of the form  $2^p - 1$  is prime, when  $p$  is known to be prime. The probability that a whole number  $x$  is prime is about  $1/\log x$ , and is close to

$$(1) \quad \frac{1}{2} e^{\gamma} \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{q}\right)$$

where  $q \doteq \sqrt{x}$ , so the factors  $(1 - \frac{1}{2})$ ,  $(1 - \frac{1}{3})$ , etc., can be regarded as probabilities that are not far from independent. But if  $x = 2^p - 1$ , only every  $p$ th factor of (1) should be taken, and the probability apparently ought to be about the  $p$ th root of  $1/p \cdot \log 2$ , which is approximately 1 when  $p$  is large. But this argument is also invalid, as we may see from the statistics of Mersenne primes [2]. We may see from these statistics (assuming them to contain no gaps), that, if  $m_n$  denotes the  $n$ th Mersenne prime ( $m_1 = 3$ ), then

$$2.18 \log \log m_n < n < 2.72 \log \log m_n \quad (3 \leq n \leq 17)$$

while

$$2.31 \log \log m_{17} = 17.$$

It is reasonable to suppose that the number of Mersenne primes less than  $x$ , when  $x$  is large, is about  $2.3 \log \log x$ . This conjecture may be shown to be equivalent to the assertion that the probability of  $2^p - 1$  being prime, when  $p$  is known to be prime and is large, is about  $1.6(\log p)/p$ , and is perhaps asymptotically  $(\log_2 p)/p$ . If so, the probability that  $p_5$  is prime is negligible, and we should be able to say with confidence that our original conjecture was the exact opposite of the truth.

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<sup>1</sup> E. CATALAN, *Nonu. Corresp. Math.*, v. 2, 1876, p. 96; cf. L. E. DICKSON, *History of the theory of numbers*, v. 1, 1934, p. 22, ref. 116.

<sup>2</sup> D. H. LEHMER, *MTAC*, v. 7, 1953, p. 72.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

55[A, F].—HORACE S. UHLER, “*Hamartixéresis* as applied to tables involving logarithms,” *Nat. Acad. Sci., Proc.*, v. 40, 1954, p. 728–731 [1].

*Hamartixéresis* appears to be a technical term in theology, meaning the absolute removal of sin.

This paper contains in tabular form, the exponents of the prime factors (2, 3, . . . , 997) in the product  $(1!)(2!) \cdots (1000!)$ .

This table was used to check the first thousand entries in the table of F. J. DUARTE [2]. Two errors were found:

log 99!: the seventh quartet *should read 8029 instead of 8929*.

log 266!: the eighth quartet *should read 1897 instead of 1987*.

Later calculations indicate no (non-cancelling) errors in the range from  $n = 1001$  to  $n = 1200$ .

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<sup>1</sup> See also *Nat. Acad. Sci., Proc.*, v. 41, 1955, p. 183, for errata.

<sup>2</sup> F. J. DUARTE, *Nouvelles tables de log n! à 33 décimales, depuis n = 1 jusqu'à n = 3000*. Geneva and Paris, 1927.

56[C, D, E, K, L, S].—CECIL HASTINGS, JR., JEANNE T. HAYWARD, & JAMES P. WONG, JR. *Approximations for Digital Computers*. Princeton University Press, Princeton, N. J., 1955, viii + 201 p., 25 cm. Price \$4.00.

This book contains rational approximations of the following functions with approximate precision as indicated (there are several approximations to each function and the approximate precision of each is shown):

$\log_{10} x$ ,  $10^{-1} \leq x \leq 10^1$ , 3D, 5D, 6D, 7D;  $\varphi(x) = (1 - e^{-x})/x$ ,  $0 \leq x < \infty$ , 3D, 4D, 5D;  $\arctan x$ ,  $-1 \leq x \leq 1$ , 3D, 4D, 5D, 6D, 7D, 8D;  $\sin \frac{1}{2}\pi x$ ,  $-1 \leq x \leq 1$ , 4S, 6S, 8S;  $10^x$ ,  $0 \leq x \leq 1$ , 4S, 6S, 7S, 9S;  $W(x) = e^{-x}/(1 + e^{-x})^2$ ,  $-\infty < x < \infty$ , 3D, 4D, 5D;  $E^1(x) = e^{-x^{1/2}}/\sqrt{2\pi}$ ,  $-\infty < x < \infty$ , 3D, 3D, 4D;  $K(n) = (n - 2n^2 - 2n^3) \ln(1 + 2/n) + (2n + 18n^2 + 16n^3 + 4n^4)(2 + m)^{-2}$ ,  $0 \leq n < \infty$ , 3D;  $\Gamma(1 + x)$ ,  $0 \leq x \leq 1$ , 5D, 5D, 6D, 7D;  $\Psi(x) = (\pi/2 - \arcsin x)(1 - x)^{-1/2}$ ,  $0 \leq x \leq 1$ , 4D, 5D, 6D, 7D, 8D;  $\log_2 x$ ,  $2^{-1} \leq x \leq 2^1$ ,