Then application of the recurrence relation (6) shows that

$$f(x) = b_0 p_0(x) + b_1\{p_1(x) + \alpha_0 p_0(x)\}.$$

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[1] C. W. CLENSHAW, "Polynomial approximations to elementary functions," *MTAC*, v. 8, 1954, p. 143–147.

[2] NBS Applied Mathematics Series 9, *Tables of Chebyshev Polynomials* $S_n(x)$ *and* $C_n(x)$. U. S. Govt. Printing Office, Washington, 1952.

Conjectures Concerning the Mersenne Numbers

Conjectures concerning the Mersenne numbers are appropriate since they were inaugurated with one. A conjecture [1] that seems likely to be false, but unlikely to be proved false, is that all numbers p_n are prime ($n = 1, 2, 3, \cdots$), where, for example, p_4 is

$$2^{2^{2^{2^2-1}-1}-1}-1$$

Recursively, $p_1 = 3$, $p_{n+1} = 2^{p_n} - 1$. The first four are 3, 7, 127 and $2^{127} - 1$, all known to be prime. Any factor of p_5 is congruent to 1 modulo p_4, so p_5 certainly has no factor less than 2^{127}. Similarly

$$2^{2^{2281}-1} - 1$$

is not divisible by any known prime, if $2^{2281} - 1$ is still the largest known prime [2]. One can try to argue about the probability that a number of the form $2^p - 1$ is prime, when p is known to be prime. The probability that a whole number x is prime is about $1/\log x$, and is close to

$$\tag{1} \tfrac{1}{2}e^{\gamma}(1 - \tfrac{1}{2})(1 - \tfrac{1}{3})(1 - \tfrac{1}{5}) \cdots \left(1 - \frac{1}{q}\right)$$

where $q \doteqdot \sqrt{x}$, so the factors $(1 - \frac{1}{2})$, $(1 - \frac{1}{3})$, etc., can be regarded as probabilities that are not far from independent. But if $x = 2^p - 1$, only every pth factor of (1) should be taken, and the probability apparently ought to be about the pth root of $1/p \cdot \log 2$, which is approximately 1 when p is large. But this argument is also invalid, as we may see from the statistics of Mersenne primes [2]. We may see from these statistics (assuming them to contain no gaps), that, if m_n denotes the nth Mersenne prime ($m_1 = 3$), then

$$2.18 \log\log m_n < n < 2.72 \log\log m_n \quad (3 \leqslant n \leqslant 17)$$

while

$$2.31 \log\log m_{17} = 17.$$

It is reasonable to suppose that the number of Mersenne primes less than x, when x is large, is about 2.3 log log x. This conjecture may be shown to be equivalent to the assertion that the probability of $2^p - 1$ being prime, when p is known to be prime and is large, is about $1.6(\log p)/p$, and is perhaps asymptotically $(\log_2 p)/p$. If so, the probability that p_5 is prime is negligible, and we should be able to say with confidence that our original conjecture was the exact opposite of the truth.

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[1] E. Catalan, *Nonu. Corresp. Math.*, v. 2, 1876, p. 96; cf. L. E. Dickson, *History of the theory of numbers*, v. 1, 1934, p. 22, ref. 116.
[2] D. H. Lehmer, *MTAC*, v. 7, 1953, p. 72.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

55[A, F].—Horace S. Uhler, "*Hamartiexéresis* as applied to tables involving logarithms," Nat. Acad. Sci., *Proc.*, v. 40, 1954, p. 728–731 [1].

Hamartiexéresis appears to be a technical term in theology, meaning the absolute removal of sin.

This paper contains in tabular form, the exponents of the prime factors $(2, 3, \cdots, 997)$ in the product $(1!)(2!)\cdots(1000!)$.

This table was used to check the first thousand entries in the table of F. J. Duarte [2]. Two errors were found:

log 99!: the seventh quartet *should read* 8029 *instead of* 8929.

log 266!: the eighth quartet *should read* 1897 *instead of* 1987.

Later calculations indicate no (non-cancelling) errors in the range from $n = 1001$ to $n = 1200$.

J. T.

[1] See also Nat. Acad. Sci., *Proc.*, v. 41, 1955, p. 183, for errata.
[2] F. J. Duarte, Nouvelles tables de log $n!$ à 33 décimales, depuis $n = 1$ jusqu'à $n = 3000$. Geneva and Paris, 1927.

56[C, D, E, K, L, S].—Cecil Hastings, Jr., Jeanne T. Hayward, & James P. Wong, Jr. *Approximations for Digital Computers.* Princeton University Press, Princeton, N. J., 1955, viii + 201 p., 25 cm. Price \$4.00.

This book contains rational approximations of the following functions with approximate precision as indicated (there are several approximations to each function and the approximate precision of each is shown):

$\log_{10} x$, $10^{-\frac{1}{2}} \le x \le 10^{\frac{1}{2}}$, 3D, 5D, 6D, 7D; $\varphi(x) = (1 - e^{-x})/x$, $0 \le x < \infty$, 3D, 4D, 5D; $\arctan x$, $-1 \le x \le 1$, 3D, 4D, 5D, 6D, 7D, 8D; $\sin \frac{1}{2}\pi x$, $-1 \le x \le 1$, 4S, 6S, 8S; 10^x, $0 \le x \le 1$, 4S, 6S, 7S, 9S; $W(x) = e^{-x}/(1 + e^{-x})^2$, $-\infty < x < \infty$, 3D, 4D, 5D; $E^1(x) = e^{-x^2/2}/\sqrt{2\pi}$, $-\infty < x < \infty$, 3D, 3D, 4D; $K(n) = (n - 2n^2 - 2n^3)\ln(1 + 2/n) + (2n + 18n^2 + 16n^3 + 4n^4)(2 + m)^{-2}$, $0 \le n < \infty$, 3D; $\Gamma(1 + x)$, $0 \le x \le 1$, 5D, 5D, 6D, 7D; $\Psi(x) = (\pi/2 - \arcsin x)(1 - x)^{-\frac{1}{2}}$, $0 \le x \le 1$, 4D, 5D, 6D, 7D, 8D; $\log_2 x$, $2^{-\frac{1}{2}} \le x \le 2^{\frac{1}{2}}$,