

x	C_2/C_∞	C_2
10.0	.14302	.85262
10.5	.13006	.77535
11.0	.11873	.70779
11.5	.10897	.64965
12.0	.09998	.59604
12.5	.09220	.54965
13.0	.08528	.50840
13.5	.07910	.47157
14.0	.07357	.43857
14.5	.06859	.40890
15.0	.06411	.38218
15.5	.06003	.35789
16.0	.05634	.33589

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1. V. V. TARASOV, "Teoriia Teploemkosti Tsepnykh I Sloistykh Struktur" [The Theory of Heat Capacity of Chain and Linear Structures], *Zhurnal Fiz. Khimii*, XXIV, 1950, p. 111-128.

2. J. W. M. DUMOND & E. RICHARD COHEN, "Least-squares adjusted values of the atomic constants as of December, 1950," *Phys. Rev.*, v. 82, p. 555-556.

A Continued Fraction for e^x

Let A_n/B_n denote the convergents to the continued fraction

$$b_0 + \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n} + \cdots$$

It is well known [1] that the continued fraction

$$b_0 + \frac{a_1 b_2}{b_1 b_2 + a_2} - \frac{a_2 a_3 b_4}{(b_2 b_3 + a_3) b_4 + b_2 a_4} - \frac{a_4 a_5 b_6}{(b_4 b_5 + a_5) b_6 + b_4 a_6} - \frac{a_6 a_7 b_8}{(b_6 b_7 + a_7) b_8 + b_6 a_8} - \cdots$$

has convergents A_{2n}/B_{2n} .

From the continued fraction of Gauss, one can obtain [2]

$$e^x = 1 + \frac{x}{1} - \frac{x/1 \cdot 2}{1} + \frac{x/2 \cdot 3}{1} - \frac{x/2 \cdot 5}{1} + \frac{x/2 \cdot 5}{1} - \cdots$$

For this expansion, we have $b_0 = 1$, $a_1 = x$, $a_{2n} = -x[2(2n-1)]$, $a_{2n+1} = x/[2(2n+1)]$, and $b_n = 1$ ($n = 1, 2, 3, \dots$). Thus, we obtain

$$-a_{2n} a_{2n+1} b_{2n-2} b_{2n+2} = x^2/[4(4n^2-1)],$$

and

$$(b_{2n} b_{2n+1} + a_{2n+1}) b_{2n+2} + b_{2n} a_{2n+2} = 1, \quad (n = 2, 3, \dots);$$

and so

$$e^x = 1 + \frac{x}{1 - x/2} + \frac{x^2/(4 \cdot 3)}{1} + \frac{x^2/(4 \cdot 15)}{1} + \dots + \frac{x^2/\{4[4(\nu - 1)^2 - 1]\}}{1} + \dots$$

A convenient equivalent form is

$$e^x = 1 + \frac{x}{1 - x/2} + \frac{x^2/4}{3} + \frac{x^2/4}{5} + \dots + \frac{x^2/4}{2\nu - 1} + \dots$$

These expansions converge quite rapidly for all x . For example, if $-1 \leq x \leq 1$, we have $e^x - A_4/B_4 < .000084$, $e^x - A_5/B_5 < .000000033$, and $e^x - A_6/B_6 < .00000000081$.

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1. See, for example, OSKAR PERRON, *Die Lehre von den Kettenbrüchen*, 2nd ed., Teubner, Leipzig, 1929, p. 201.

2. PERRON, *op. cit.*, p. 312. Mr. Garwick also notes that this is credited to Thiele, see, for example, N. E. Nørlund, *Vorlesungen über Differenzenrechnung*, Springer, Berlin, 1924, Chapter 15, p. 415-455, especially p. 454.

3. The referee has noted that this work has been done independently by J. V. GARWICK of the Norwegian Defense Research Establishment.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

78[C, D, E].—FRIEDRICH LÖSCH, *Siebenstellige Tafeln der elementaren transzendenten Funktionen*. Springer, Berlin-Göttingen-Heidelberg, 1954, viii + 335 p. 27 cm. DM 49.80.

This volume is meant to supersede the 1926 table of K. HAYASHI [1], now out of print. The elementary transcendental functions are $\sin x$, $\cos x$, $\tan x$, $\sinh x$, $\cosh x$, $\tanh x$, $\ln x$, e^x , e^{-x} , $\arcsin x$, $\arctan x$, $\operatorname{arcsinh} x$, $\operatorname{arctanh} x$, together with φ , the angle corresponding to the arc x . In Table I (40 pages) these are given to 9D for $x = 0(.0001)0.1$, together with first differences (except for $\ln x$, and, of course, φ). In Table II (277 pages) these are given to 7D for $x = 0.1(.0005)3.15$, with (*double*) first differences for all functions (but φ) and for $x = 3(.01)10(.1)20$, without differences. For $x > 1$, $\operatorname{arccoth} x$ is tabulated in place of $\operatorname{arctanh} x$ and $\operatorname{arccosh} x$ in place of $\operatorname{arcsin} x$. There are supplementary tables of $\tan x$ to 7D for $x = 1.5680(.0001)1.5730$, $\operatorname{arctanh} x$ for $x = 0.9980(.0001)1$, $\operatorname{arccoth} x$ for $x = 1(.0001)1.0020$.

In Table III, there are given $\sin x$, $\cos x$, $\ln x$, $\operatorname{arcsinh} x$, $\operatorname{arccosh} x$ to 7D, $e^{\pm x}$ to 7S, for $x = 0(1)100$. Table IV gives $\frac{1}{2}n\pi$ to 12D for $n = 0(1)100$. Table V gives $\sin \frac{1}{2}\pi x$, $\cos \frac{1}{2}\pi x$ to 7D for $x = 0(.001)0.5$. Table VI gives $\exp(\pm \frac{1}{2}\pi x)$, $\sinh \frac{1}{2}\pi x$, $\cosh \frac{1}{2}\pi x$ to 7D for $x = 0(.01)2$. Table VII gives $\exp(\pm \pi x/180)$, $\sinh(\pi x/180)$, $\cosh(\pi x/180)$ to 7D for $x = 0(1)180$. Table VIII gives φ in degrees