means, for example, that if we use a method in which we find the characteristic equation we will almost certainly obtain the latter exactly, which will not be true if the coefficients were numbers with several digits.

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TECHNICAL NOTES AND SHORT PAPERS

The Specific Heat Function for a Two-Dimensional Continuum

Numerical Values of

$$\frac{C_2}{C_\infty} = \frac{6}{x^2}\int_0^x \frac{\xi^2 d\xi}{e^\xi - 1} - \frac{2x}{e^x - 1}.$$

This function which appears in the theory of low-temperature specific heats of two-dimensional (layer) structures [1] was computed as follows:

(1) For $0 \leqslant x \leqslant 2.0$, the formula

$$\frac{C_2}{C_\infty} = 1 - \frac{x^2}{24} + \frac{x^4}{720} - \frac{x^6}{24{,}192} + \frac{x^8}{864{,}000} - \frac{x^{10}}{31{,}933{,}440} + \cdots$$

was used. The maximum error (using seven terms) is no greater than 0.5×10^{-5}.

(2) For $2.0 \leqslant x \leqslant 16.0$, the formula

$$\frac{C_2}{C_\infty} = \frac{14.424684}{x^2} - 6x\sum_{n=1}^{\infty} e^{-nx}\left(\frac{1}{nx} + \frac{2}{(nx)^2} + \frac{2}{(nx)^3}\right) - \frac{2x}{e^x - 1}$$

was used. The maximum error was approximately 2×10^{-6}.

The value $C_\infty = 3R = 5.9616$ cal mol^{-1} deg^{-1} was used throughout [2].

x	C_2/C_∞	C_2
0.0	1.00000	5.9616
0.1	0.99958	5.95911
0.2	.99833	5.95167
0.3	.99625	5.93925
0.4	.99333	5.92186
0.5	.98959	5.89955

x	C_2/C_∞	C_2
0.6	.98502	5.87227
0.7	.97961	5.84006
0.8	.97340	5.80300
0.9	.96632	5.76081
1.0	.95843	5.71378
1.1	.95155	5.67274
1.2	.94276	5.62037
1.3	.93336	5.56432
1.4	.92337	5.50478
1.5	.91284	5.44198
1.6	.90179	5.37610
1.7	.89026	5.30737
1.8	.87829	5.23601
1.9	.86592	5.16224
2.0	.85318	5.08629
2.1	.84024	5.00919
2.2	.82677	4.92890
2.3	.81327	4.84841
2.4	.79936	4.76548
2.5	.78526	4.68138
2.6	.77111	4.59707
2.7	.75689	4.51225
2.8	.74243	4.42607
2.9	.72802	4.34016
3.0	.71353	4.25379
3.1	.69901	4.16719
3.2	.68454	4.08097
3.3	.67014	3.99511
3.4	.65582	3.90974
3.5	.64159	3.82488
3.6	.62746	3.74068
3.7	.61347	3.65725
3.8	.59961	3.57461
3.9	.58594	3.49313
4.0	.57240	3.41244
4.1	.55910	3.33315
4.2	.54597	3.25483
4.3	.53306	3.17787
4.4	.52037	3.10225
4.5	.50791	3.02793
4.6	.49568	2.95502
4.7	.48368	2.88352
4.8	.47193	2.81343
4.9	.46041	2.74480
5.0	.44915	2.67766

x	C_2/C_∞	C_2
5.1	.43814	2.61202
5.2	.42738	2.54789
5.3	.41687	2.48521
5.4	.40661	2.42403
5.5	.39659	2.36434
5.6	.38683	2.30611
5.7	.37731	2.24937
5.8	.36803	2.19404
5.9	.35900	2.14019
6.0	.35019	2.08771
6.1	.34163	2.03664
6.2	.33328	1.98691
6.3	.32517	1.93853
6.4	.31728	1.89152
6.5	.30961	1.84577
6.6	.30215	1.80129
6.7	.29490	1.75805
6.8	.28785	1.71604
6.9	.28100	1.67519
7.0	.27434	1.63554
7.1	.26788	1.59699
7.2	.26160	1.55954
7.3	.25550	1.52318
7.4	.24959	1.48797
7.5	.24382	1.45353
7.6	.23823	1.42021
7.7	.23280	1.38786
7.8	.22753	1.35643
7.9	.22241	1.32591
8.0	.21744	1.29628
8.1	.21258	1.26733
8.2	.20792	1.23954
8.3	.20337	1.21239
8.4	.19894	1.18602
8.5	.19465	1.16041
8.6	.19048	1.13554
8.7	.18642	1.11137
8.8	.18248	1.08789
8.9	.17866	1.06508
9.0	.17494	1.04291
9.1	.17132	1.02136
9.2	.16781	1.00043
9.3	.16440	.98009
9.4	.16108	.96030

x	C_2/C_∞	C_2
10.0	.14302	.85262
10.5	.13006	.77535
11.0	.11873	.70779
11.5	.10897	.64965
12.0	.09998	.59604
12.5	.09220	.54965
13.0	.08528	.50840
13.5	.07910	.47157
14.0	.07357	.43857
14.5	.06859	.40890
15.0	.06411	.38218
15.5	.06003	.35789
16.0	.05634	.33589

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A Continued Fraction for e^x

Let A_n/B_n denote the convergents to the continued fraction

$$b_0 + \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n} + \cdots.$$

It is well known [1] that the continued fraction

$$b_0 + \frac{a_1 b_2}{b_1 b_2 + a_2} - \frac{a_2 a_3 b_4}{(b_2 b_3 + a_3) b_4 + b_2 a_4} - \frac{a_4 a_5 b_2 b_6}{(b_4 b_5 + a_5) b_6 + b_4 a_6} - \frac{a_6 a_7 b_4 b_8}{(b_6 b_7 + a_7) b_8 + b_6 a_8} - \cdots$$

has convergents A_{2n}/B_{2n}.

From the continued fraction of Gauss, one can obtain [2]

$$e^x = 1 + \frac{x}{1} - \frac{x/1\cdot2}{1} + \frac{x/2\cdot3}{1} - \frac{x/2\cdot5}{1} + \frac{x/2\cdot5}{1} - \cdots.$$

For this expansion, we have $b_0 = 1$, $a_1 = x$, $a_{2n} = - x[2(2n - 1)]$, $a_{2n+1} = x/[2(2n + 1)]$, and $b_n = 1$ $(n = 1, 2, 3, \cdots)$. Thus, we obtain

$$- a_{2n} a_{2n+1} b_{2n-2} b_{2n+2} = x^2/[4(4n^2 - 1)],$$

and

$$(b_{2n} b_{2n+1} + a_{2n+1}) b_{2n+2} + b_{2n} a_{2n+2} = 1, \quad (n = 2, 3, \cdots);$$