Some Comments on a NORC Computation of π

Among the problems suggested as demonstration routines for the NORC [1] was the calculation of π to a large number of digits. In 1949, π was calculated to 2035D on the ENIAC [2]. The present computation on the NORC, which was carried out by the authors at the Watson Scientific Computing Laboratory, produced 3089D. This limit was chosen since the entire calculation could be contained in the NORC's high speed memory (2000 locations). The program was designed to produce any number of digits up to this limit. More extensive programming involving use of the magnetic tapes would have increased the number of digits by several orders of magnitude.

The formula used was the same as that employed for the ENIAC computation:

$$\pi/4 = 4 \text{ arc tan } 1/5 - \text{arc tan } 1/239$$

in conjunction with the GREGORY series

$$\text{arc tan } x = \sum_{n=0}^{\infty} (-1)^n (2n+1)^{-1} x^{2n+1}.$$

The form of the expression used for the NORC computation was:

$$\frac{\pi}{4} = \sum_{n=0}^{\infty} (-1)^n \left[\frac{100(0.2)^{2n+3} - (1/239)^{2n+1}}{2n+1} \right].$$

All intermediate factors were truncated without rounding to 3093D. For $n = 0$, the value of $100(0.2)^{2n+3}$ was set at 0.8; the value of this factor at each succeeding n was computed by multiplying the value at $n - 1$ by .04, with truncation of the product to 3093D. For $n = 0$, the value of $(1/239)^{2n+1}$ was computed with truncation of the quotient to 3093D; the value of this factor at each succeeding n was computed by dividing the value at $n - 1$ by 239^2, with truncation of the quotient to 3093D. For each n, the tens complement of this second factor was taken and added to the first factor; the sum was divided by $2n + 1$, with truncation to 3093D. This result or its tens complement, according to the value of n, was added to $\sum_{0}^{n-1}$. This process was continued until the first 3093D of the second factor were zeros; computations involving this factor were then bypassed. The process terminated when the first 3093D of the first factor were zeros. A count was kept of the number of terms required for the calculation; at termination, n equaled 2214.

A second run, with the same program, of the calculation was made about two months after the first run. A comparison of the two results showed agreement. The first 2035D of the ENIAC and NORC results agree. Since the programming for the computation on the NORC is completely general for any number of digits, the correctness of the digits up to 3089D is strongly indicated. Further assurance of the correctness of the result is afforded by the NORC's inherent checking system [3]. Note that the only expected error results from truncation of the intermediate calculations; this error is a function of n and could affect the last

NORC Computation of π

3.14159	26535	89793	23846	26433	83279	50288	41971	69399	37510
58209	74944	59230	78164	06286	20899	86280	34825	34211	70679
82148	08651	32823	06647	09384	46095	50582	23172	53594	08128
48111	74502	84102	70193	85211	05559	64462	29489	54930	38196
44288	10975	66593	34461	28475	64823	37867	83165	27120	19091
45648	56692	34603	48610	45432	66482	13393	60726	02491	41273
72458	70066	06315	58817	48815	20920	96282	92540	91715	36436
78925	90360	01133	05305	48820	46652	13841	46951	94151	16094
33057	27036	57595	91953	09218	61173	81932	61179	31051	18548
07446	23799	62749	56735	18857	52724	89122	79381	83011	94912
98336	73362	44065	66430	86021	39494	63952	24737	19070	21798
60943	70277	05392	17176	29317	67523	84674	81846	76694	05132
00056	81271	45263	56082	77857	71342	75778	96091	73637	17872
14684	40901	22495	34301	46549	58537	10507	92279	68925	89235
42019	95611	21290	21960	86403	44181	59813	62977	47713	09960
51870	72113	49999	99837	29780	49951	05973	17328	16096	31859
50244	59455	34690	83026	42522	30825	33446	85035	26193	11881
71010	00313	78387	52886	58753	32083	81420	61717	76691	47303
59825	34904	28755	46873	11595	62863	88235	37875	93751	95778
18577	80532	17122	68066	13001	92787	66111	95909	21642	01989
38095	25720	10654	85863	27886	59361	53381	82796	82303	01952
03530	18529	68995	77362	25994	13891	24972	17752	83479	13151
55748	57242	45415	06959	50829	53311	68617	27855	88907	50983
81754	63746	49393	19255	06040	09277	01671	13900	98488	24012
85836	16035	63707	66010	47101	81942	95559	61989	46767	83744
94482	55379	77472	68471	04047	53464	62080	46684	25906	94912
93313	67702	89891	52104	75216	20569	66024	05803	81501	93511
25338	24300	35587	64024	74964	73263	91419	92726	04269	92279
67823	54781	63600	93417	21641	21992	45863	15030	28618	29745
55706	74983	85054	94588	58692	69956	90927	21079	75093	02955
32116	53449	87202	75596	02364	80665	49911	98818	34797	75356
63698	07426	54252	78625	51818	41757	46728	90977	77279	38000
81647	06001	61452	49192	17321	72147	72350	14144	19735	68548
16136	11573	52552	13347	57418	49468	43852	33239	07394	14333
45477	62416	86251	89835	69485	56209	92192	22184	27255	02542
56887	67179	04946	01653	46680	49886	27232	79178	60857	84383
82796	79766	81454	10095	38837	86360	95068	00642	25125	20511
73929	84896	08412	84886	26945	60424	19652	85022	21066	11863
06744	27862	20391	94945	04712	37137	86960	95636	43719	17287
46776	46575	73962	41389	08658	32645	99581	33904	78027	59009
94657	64078	95126	94683	98352	59570	98258	22620	52248	94077
26719	47826	84826	01476	99090	26401	36394	43745	53050	68203
49625	24517	49399	65143	14298	09190	65925	09372	21696	46151
57098	58387	41059	78859	59772	97549	89301	61753	92846	81382
68683	86894	27741	55991	85592	52459	53959	43104	99725	24680
84598	72736	44695	84865	38367	36222	62609	91246	08051	24388
43904	51244	13654	97627	80797	71569	14359	97700	12961	60894
41694	86855	58484	06353	42207	22258	28488	64815	84560	28506
01684	27394	52267	46767	88952	52138	52254	99546	66727	82398
64565	96116	35488	62305	77456	49803	55936	34568	17432	41125
15076	06947	94510	96596	09402	52288	79710	89314	56691	36867
22874	89405	60101	50330	86179	28680	92087	47609	17824	93858
90097	14909	67598	52613	65549	78189	31297	84821	68299	89487
22658	80485	75640	14270	47755	51323	79641	45152	37462	34364
54285	84447	95265	86782	10511	41354	73573	95231	13427	16610
21359	69536	23144	29524	84937	18711	01457	65403	59027	99344
03742	00731	05785	39062	19838	74478	08478	48968	33214	45713
86875	19435	06430	21845	31910	48481	00537	06146	80674	91927
81911	97939	95206	14196	63428	75444	06437	45123	71819	21799
98391	01591	95618	14675	14269	12397	48940	90718	64942	31961
56794	52080	95146	55022	52316	03881	93014	20937	62137	85595
66389	37787	08303	90697	92077	34672	21825	6259		

four digits of the final result. The result has been tabulated to 3089D; the final digit is unrounded.

Running time for the 3093D was approximately thirteen minutes. The programming takes account of the number of zeros generated to the right of the decimal point in each factor, so that the number of operations required for each term in the series decreases. This leads to the following statement—if the time to compute π to m digits is t units, then the time to produce km digits is roughly k^2t units; this holds true as long as the calculation is contained in high-speed storage.

The following table gives a count of each of the digits in π.

(1)	(2) 1–3090	(3) 1–2036	(4) 2037–3090	(5) (4)/(3)
0	269	184	85	.46
1	315	213	102	.47
2	314	210	104	.50
3	276	191	85	.45
4	322	198	124	.63
5	326	211	115	.54
6	311	204	107	.52
7	297	200	97	.49
8	318	207	111	.54
9	342	218	124	.57
Σ	3090	2036	1054	.52

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1. The IBM-Naval Ordnance Research Calculator, now located at Naval Proving Ground, Dahlgren, Virginia.
2. George W. Reitwiesner, "An ENIAC determination of π and e to more than 2000 decimal places," *MTAC*, v. 4, 1950, p. 11–15.
3. For a description of the NORC checking system, see W. J. Eckert & R. B. Jones, *Faster, Faster*, McGraw-Hill Book Company, New York, 1955, p. 98–104.

Orthogonal Polynomials Arising in the Numerical Evaluation of Inverse Laplace Transforms

Abstract. In finding $f(t)$, the inverse Laplace transform of $F(p)$, where (1) $f(t) = (1/2\pi j)\int_{c-j\infty}^{c+j\infty} e^{pt}F(p)dp$, the function $F(p)$ may be either known only numerically or too complicated for evaluating $f(t)$ by Cauchy's theorem. When $F(p)$ behaves like a polynomial without a constant term, in the variable $1/p$, along $(c - j\infty, c + j\infty)$, one may find $f(t)$ numerically using new quadrature formulas (analogous to those employing the zeros of the Laguerre polynomials in the direct Laplace transform). Suitable choice of p_i yields an n-point quadrature