Choosing one of these classes, say (301, 149, 697), we can duplicate it by compounding (301, 149, 697) with (697, −149, 301).

This is done most simply by a method devised by the writer:

Let the classes be $(a, b, \cdots)$ and $(a', b', \cdots)$ where a and a' are prime to each other, and take p and q such that either ap and $a'q$, or $a'p$ and aq differ by unity. This can nearly always be done mentally, but when a and a' are not small, the values of p and q are more quickly found as the constituents of the penultimate convergent of the continued fraction representing a/a' or a'/a. The required compound class is then given by $(aa', a'bp + ab'q, \cdots)$ or by $(aa', a'bq + ab'p, \cdots)$, care being taken that the signs of p and q are so chosen that the smaller of the two products, *e.g.*, aq and $a'p$, say, shall be negative. Applying this to the case in hand, we get:

$$(697.301, 697.149(-19) + 301(-149)44, \cdots),$$

or

$$(209797, -1973207 -1973356, \cdots),$$
$$(209797, -3946563, \cdots),$$
$$(209797, +39580, 7468),$$
$$(7468, 2240, 697),$$
$$(697, 149, 301),$$
$$(301, -149, 697),$$

which shows that (301, 149, 697) is a critical class, and each of the twelve other classes when similarly tested is found to be a critical class.

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1. G. B. Mathews, *Theory of Numbers*, Part I, 1892.
2. A. Scholz & O. Taussky, "Die Hauptideale der kubischen Klassenkörper imaginär-quadratischer Zahlkörper: ihre rechnerische Bestimmung und ihr Einfluss auf den Klassenkörperturm," *Jahrbuch über die Fortschritte der Mathematik*, 60, 1934, p. 126, *J. reine angew. Math.* 171, 1934, p. 19–41.
3. T. Pepin, *Atti. Acad. Pont. Nuovi Lincei*, 33, 1881, p. 354–391.
4. C. F. Gauss, *Disquisitiones Arithmeticae*, Art. 306, VIII, 1801; in *Werke* I, 1863, p. 371; German transl. by H. Maser, 1889, p. 653–654.
5. The 11th member of the second series, *i.e.* −297675, has exponent 27, with 40 critical classes.

TECHNICAL NOTES AND SHORT PAPERS

Selected References on Use of High-Speed Computers for Scientific Computation

The author is often asked to recommend reading to orient mathematicians in the impact of high-speed computers on numerical analysis. The following list was prepared in answer to one such request, but does not pretend to be definitive. The author is indebted to C. B. Tompkins for several suggestions.

For a list of books not necessarily influenced by high-speed computers, but highly pertinent to their use, see G. E. Forsythe, "A numerical analyst's fifteen-foot shelf," *MTAC*, v. 7, 1953, p. 221–228.

I. BOOKS

A. D. BOOTH & K. H. V. BOOTH, *Automatic Digital Calculators*, Butterworths Scientific Publications, London, 1953, 231 p.

B. V. BOWDEN, *Faster than Thought. A Symposium on Digital Computing Machines*, Isaac Pitman, London, 1953, 416 p.

ENGINEERING RESEARCH ASSOCIATES, INC., *High-Speed Computing Devices*, McGraw-Hill, New York, 1950, 451 p.

D. R. HARTREE, *Numerical Analysis*, Clarendon Press, Oxford, 1952, 287 p.

CECIL HASTINGS, JR., J. T. HAYWARD, & J. P. WONG, JR., *Approximations for Digital Computers*, University Press, Princeton, 1955, 201 p.

A. S. HOUSEHOLDER, *Principles of Numerical Analysis*, McGraw-Hill, New York, 1953 (extensive bibliography), 274 p.

NATIONAL BUREAU OF STANDARDS, *Monte Carlo Method*, Applied Mathematics Series 12, U. S. Gov. Printing Office, 1951, 42 p.

H. RUTISHAUSER, *Automatische Rechenplanfertigung bei programmgesteuerten Rechenmaschinen*, Birkhäuser, Basel, 1952, 45 p.

M. V. WILKES, D. J. WHEELER, & S. GILL, *The Preparation of Programs for an Electronic Digital Computer*, Addison-Wesley, Cambridge, Mass., 1951, 170 p.

II. JOURNALS

Computers and Automation (New York)
Journal of the Association for Computing Machinery
Journal of Research of the U. S. National Bureau of Standards
Journal of the Society for Industrial and Applied Mathematics
Mathematical Reviews (Numerical and Graphical Methods section)
Mathematical Tables and Other Aids to Computation
Naval Research Logistics Quarterly
Proceedings of the Association for Computing Machinery (terminated)
Proceedings of the Cambridge Philosophical Society
Quarterly of Applied Mathematics
Quarterly Journal of Mechanics and Applied Mathematics
Vychislitel'naia͡ Matematika i Vychislitel'naia͡ Tekhnika (Moscow)
Zeitschrift für angewandte Mathematik und Mechanik
Zeitschrift für angewandte Mathematik und Physik

III. SOME ARTICLES NOT IN ABOVE JOURNALS

AMERICAN MATHEMATICAL SOCIETY, "Proceedings of symposia in applied mathematics," vol. 6, to appear.

V. BARGMANN, D. MONTGOMERY, & J. VON NEUMANN, "Solution of linear systems of high order," Institute for Advanced Study, Princeton, 1946.

WALLACE GIVENS, "Numerical computation of the characteristic values of a real symmetric matrix," ORNL 1574, Oak Ridge National Laboratory, 1954, 107 p.

H. H. GOLDSTINE & J. VON NEUMANN, "Planning and coding of problems for an electronic computing instrument," issued in three parts as part II, vols. 1–3, Institute for Advanced Study, Princeton, 1947–1948.

HARVARD UNIVERSITY, "Proceedings of a symposium on large-scale digital calculating machinery," Cambridge, Mass., 1948.

HARVARD UNIVERSITY, "Proceedings of a second symposium on large-scale digital calculating machinery," Cambridge, Mass., 1951.

M. A. HYMAN, "On the numerical solution of partial differential equations," thesis, Delft, 1953, 108 p.

N. METROPOLIS & S. ULAM, "The Monte Carlo method," Am. Stat. Assn., *Jn.*, v. 44, 1949, p. 335–341.

J. VON NEUMANN & H. H. GOLDSTINE, "Numerical inverting of matrices of high order," Am. Math. Soc., *Bull.*, v. 53, 1947, p. 1021–1099, and Am. Math. Soc., *Proc.*, v. 2, 1951, p. 188–202.

F. W. J. OLVER, "The evaluation of zeros of high-degree polynomials," R. Soc. London, *Phil. Trans.*, v. 244, 1952, p. 385–415.

E. STIEFEL, "Relaxationsmethoden bester Strategie zur Lösung linearer Gleichungssysteme," *Commentarii Mathematici Helvetici*, v. 29, 1955, p. 157–179.

JOHN TODD, "Motivation for working in numerical analysis," *Comm. on Pure and Applied Math.*, v. 8, 1955, p. 97–116.

DAVID YOUNG, "Iterative methods for solving partial difference equations of elliptic type," Am. Math. Soc., *Trans.*, v. 76, 1954, p. 92–111.

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Modified Quotients of Cylinder Functions

The name in the title of this note is applied to the function, $\mathfrak{C}_\nu(z)$, defined by the following equation,

$$\mathfrak{C}_\nu(z) = \frac{zC_{\nu-1}(z)}{C_\nu(z)} \tag{1}$$

where $C_\nu(z)$ is a cylinder function [1] which satisfies the pair of recurrence formulae,

$$\frac{2\nu}{z} C_\nu(z) = C_{\nu-1}(z) + C_{\nu+1}(z) \tag{2}$$

$$2C_\nu'(z) = C_{\nu-1}(z) - C_{\nu+1}(z)$$

$\mathfrak{C}_\nu(z)$ has not repeated zeros and poles with possible exception of the origin and satisfies the following RICCATI's equation,

$$\frac{dy}{dz} + \frac{1}{z}(y^2 - 2\nu y) + z = 0. \tag{3}$$

Its derivative,

$$\frac{\partial \mathfrak{C}_\nu(z)}{\partial z} = \frac{z}{C_\nu^2(z)} \{C_{\nu-1}(z)C_{\nu+1}(z) - C_\nu^2(z)\} \tag{4}$$