In these formulas, $\epsilon = \dfrac{\text{calculated } F - \text{tabular } F}{\text{tabular } F}$, where $F = I_0$, I_1, K_0, or K_1. [ϵ given in the approximations to J_0, Y_0, J_1, and Y_1 previously mentioned was the absolute error in the functions represented by polynomials.]

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1. BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, Mathematical Tables, v. X, *Bessel Functions*, Cambridge Univ. Press, 1952.
2. E. E. ALLEN, "Analytical approximations," *MTAC*, v. 8, 1954, p. 240–241.

Radix Tables for sin x and cos x, $x = a \cdot 10^k$ degrees, $a = 1(1)9$, $k = -3(1)1$

These tables permit calculation of sin x or cos x for any argument x expressed to 3D by means of summation formulas listed in Table I and requiring at most 64 multiplications and 15 additions or subtractions.

No. of decimals in x	Maximum number of multiplications	Maximum number of additions or subtractions
3	64	15
2	24	7
1	8	3
0	2	1

The table was computed on the SWAC machine using double precision routines. (Each SWAC number has a sign plus 36 binary digits.) Values of $1/n!$ for $n = 2(1)20$ were taken from Peters and Stein [1].

For $x = .001(.001).01(.01).1(.1)1(1)10(10)50°$, expressed in radians, double precision, the routine computed sin x and cos x by

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots - \frac{x^{19}}{19!}$$

and

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{x^{20}}{20!}.$$

For each x, as a term $\dfrac{x^m}{m!}$ was computed, it was accumulated into the sin x or the cos x cell (alternating) with the proper sign attached. The limit of 20 was chosen for n in the formulas above, since there would be no contribution from succeeding

terms in the power series for the functions. The values for $\sin x$ and $\cos x$ where $x = 60(10)80°$ were lifted from the $\cos x$ and $\sin x$ for $x = 10(10)30°$.

The values were converted by SWAC to decimal form, tabulated on an International Business Machines Corporation model 402 accounting machine, and rounded to 20D by hand. Accuracy to 0.6×10^{-20} is believed assured.

The tables were computed in connection with the review of AMS 40 in this issue, p. 169.

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1. J. PETERS & J. STEIN, Appendix to *Zehnstellige Logarithmentafel. Band I. Zehnstellige Logarithmen der Zahlen von 1 bis 100,000 nebst einem Anhang mathematischer Tafeln*, Berlin, Reichsamt für Landesaufnahme, 1922. A reprint is being issued by F. Ungar, New York.

TABLE I. *Summation Formulas for Sine and Cosine*

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\begin{aligned}\sin(\alpha+\beta+\gamma) = {} & \sin\alpha\cos\beta\cos\gamma - \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\beta\sin\gamma \\ & + \cos\alpha\sin\beta\cos\gamma\end{aligned}$$

$$\begin{aligned}\cos(\alpha+\beta+\gamma) = {} & \cos\alpha\cos\beta\cos\gamma - \cos\alpha\sin\beta\sin\gamma - \sin\alpha\sin\beta\cos\gamma \\ & - \sin\alpha\cos\beta\sin\gamma\end{aligned}$$

$$\begin{aligned}\sin(\alpha+\beta+\gamma+\delta) = {} & \sin\alpha\cos\beta\cos\gamma\cos\delta - \sin\alpha\sin\beta\sin\gamma\cos\delta \\ & - \sin\alpha\sin\beta\cos\gamma\sin\delta - \sin\alpha\cos\beta\sin\gamma\sin\delta \\ & + \cos\alpha\sin\beta\cos\gamma\cos\delta + \cos\alpha\cos\beta\sin\gamma\cos\delta \\ & + \cos\alpha\cos\beta\cos\gamma\sin\delta - \cos\alpha\sin\beta\sin\gamma\sin\delta\end{aligned}$$

$$\begin{aligned}\cos(\alpha+\beta+\gamma+\delta) = {} & \cos\alpha\cos\beta\cos\gamma\cos\delta - \cos\alpha\sin\beta\sin\gamma\cos\delta \\ & - \cos\alpha\sin\beta\cos\gamma\sin\delta - \cos\alpha\cos\beta\sin\gamma\sin\delta \\ & + \sin\alpha\sin\beta\sin\gamma\sin\delta - \sin\alpha\sin\beta\cos\gamma\cos\delta \\ & - \sin\alpha\cos\beta\sin\gamma\cos\delta - \sin\alpha\cos\beta\cos\gamma\sin\delta\end{aligned}$$

$$\begin{aligned}\sin(\alpha+\beta+\gamma+\delta+\epsilon) = {} & \sin\alpha\sin\beta\sin\gamma\sin\delta\sin\epsilon - \sin\alpha\cos\beta\cos\gamma\sin\delta\sin\epsilon \\ & - \sin\alpha\cos\beta\sin\gamma\cos\delta\sin\epsilon - \sin\alpha\sin\beta\cos\gamma\cos\delta\sin\epsilon \\ & - \sin\alpha\cos\beta\sin\gamma\sin\delta\cos\epsilon - \sin\alpha\sin\beta\cos\gamma\sin\delta\cos\epsilon \\ & - \sin\alpha\sin\beta\sin\gamma\cos\delta\cos\epsilon + \sin\alpha\cos\beta\cos\gamma\cos\delta\cos\epsilon \\ & - \cos\alpha\cos\beta\sin\gamma\sin\delta\sin\epsilon - \cos\alpha\sin\beta\cos\gamma\sin\delta\sin\epsilon \\ & - \cos\alpha\sin\beta\sin\gamma\cos\delta\sin\epsilon + \cos\alpha\cos\beta\cos\gamma\cos\delta\sin\epsilon \\ & - \cos\alpha\sin\beta\sin\gamma\sin\delta\cos\epsilon + \cos\alpha\cos\beta\cos\gamma\sin\delta\cos\epsilon \\ & + \cos\alpha\cos\beta\sin\gamma\cos\delta\cos\epsilon + \cos\alpha\sin\beta\cos\gamma\cos\delta\cos\epsilon\end{aligned}$$

$$\begin{aligned}\cos(\alpha+\beta+\gamma+\delta+\epsilon) = {} & \cos\alpha\cos\beta\cos\gamma\cos\delta\cos\epsilon - \cos\alpha\sin\beta\sin\gamma\cos\delta\cos\epsilon \\ & - \cos\alpha\sin\beta\cos\gamma\sin\delta\cos\epsilon - \cos\alpha\cos\beta\sin\gamma\sin\delta\cos\epsilon \\ & - \cos\alpha\sin\beta\cos\gamma\cos\delta\sin\epsilon - \cos\alpha\cos\beta\sin\gamma\cos\delta\sin\epsilon \\ & - \cos\alpha\cos\beta\cos\gamma\sin\delta\sin\epsilon + \cos\alpha\sin\beta\sin\gamma\sin\delta\sin\epsilon \\ & - \sin\alpha\sin\beta\cos\gamma\cos\delta\cos\epsilon - \sin\alpha\cos\beta\sin\gamma\cos\delta\cos\epsilon \\ & - \sin\alpha\cos\beta\cos\gamma\sin\delta\cos\epsilon + \sin\alpha\sin\beta\sin\gamma\sin\delta\cos\epsilon \\ & - \sin\alpha\cos\beta\cos\gamma\cos\delta\sin\epsilon + \sin\alpha\sin\beta\sin\gamma\cos\delta\sin\epsilon \\ & + \sin\alpha\sin\beta\cos\gamma\sin\delta\sin\epsilon + \sin\alpha\cos\beta\sin\gamma\sin\delta\sin\epsilon\end{aligned}$$

TABLE II. *Radix Tables for Sine x and Cosine x in Degrees*

x (deg.)	sin x	cos x
.001	.00001 74532 92519 05720	.99999 99998 47691 29011
.002	.00003 49065 85032 79782	.99999 99993 90765 16049
.003	.00005 23598 77535 90529	.99999 99986 29221 61127
.004	.00006 98131 70023 06303	.99999 99975 63060 64270
.005	.00008 72664 62488 95446	.99999 99961 92282 25508
.006	.00010 47197 54928 26301	.99999 99945 16886 44885
.007	.00012 21730 47335 67209	.99999 99925 36873 22451
.008	.00013 96263 39705 86514	.99999 99902 52242 58266
.009	.00015 70796 32033 52557	.99999 99876 62994 52401
.01	.00017 45329 24313 33680	.99999 99847 69129 04933
.02	.00034 90658 43310 09671	.99999 99390 76516 66127
.03	.00052 35987 51673 70300	.99999 98629 22164 22770
.04	.00069 81316 44087 57925	.99999 97563 06074 06842
.05	.00087 26645 15235 14954	.99999 96192 28249 43114
.06	.00104 71973 59799 83861	.99999 94516 88694 49148
.07	.00122 17301 72465 07198	.99999 92536 87414 35299
.08	.00139 62629 47914 27617	.99999 90252 24415 04715
.09	.00157 07956 80830 87881	.99999 87662 99703 53332
.1	.00174 53283 65898 30884	.99999 84769 13287 69880
.2	.00349 06514 15223 73227	.99999 39076 57790 38148
.3	.00523 59638 31419 58009	.99998 62922 47426 79269
.4	.00698 12602 97961 55247	.99997 56307 05394 79311
.5	.00872 65354 98373 93496	.99996 19230 64171 28874
.6	.01047 17841 16245 79346	.99994 51693 65512 13198
.7	.01221 70008 35247 16888	.99992 53696 60451 99446
.8	.01396 21803 39145 27162	.99990 25240 09304 21155
.9	.01570 73173 11820 67575	.99987 66324 81660 59864
1	.01745 24064 37283 51282	.99984 76951 56391 23916
2	.03489 94967 02500 97165	.99939 08270 19095 73001
3	.05233 59562 42943 83272	.99862 95347 54573 87378
4	.06975 64737 44125 30078	.99756 40502 59824 24761
5	.08715 57427 47658 17356	.99619 46980 91745 53229
6	.10452 84632 67653 47140	.99452 18953 68273 33692
7	.12186 93434 05147 48111	.99254 61516 41322 03498
8	.13917 31009 60065 44411	.99026 80687 41570 31508
9	.15643 44650 40230 86901	.98768 83405 95137 72619
10	.17364 81776 66930 34885	.98480 77530 12208 05937
20	.34202 01433 25668 73304	.93969 26207 85908 38405
30	.50000 00000 00000 00000	.86602 54037 84438 64676
40	.64278 76096 86539 32632	.76604 44431 18978 03520
50	.76604 44431 18978 03520	.64278 76096 86539 32632
60	.86602 54037 84438 64676	.50000 00000 00000 00000
70	.93969 26207 85908 38405	.34202 01433 25668 73304
80	.98480 77530 12208 05937	.17364 81776 66930 34885
90	1.00000 00000 00000 00000	.00000 00000 00000 00000