tinuing. The operation of the SWAC has been handled, for the most part, by John L. Selfridge.

Fourteen new factors of Fermat numbers have been found, namely

$$21\cdot 2^{41}+1\,|\,F_{39},\quad 29\cdot 2^{57}+1\,|\,F_{55},\quad 9\cdot 2^{67}+1\,|\,F_{63},$$
$$7\cdot 2^{120}+1\,|\,F_{117},\quad 5\cdot 2^{127}+1\,|\,F_{125},\quad 17\cdot 2^{147}+1\,|\,F_{144},$$
$$1575\cdot 2^{157}+1\,|\,F_{150},\quad 3\cdot 2^{209}+1\,|\,F_{207},\quad 15\cdot 2^{229}+1\,|\,F_{226},$$
$$29\cdot 2^{231}+1\,|\,F_{228},\quad 21\cdot 2^{276}+1\,|\,F_{268},\quad 7\cdot 2^{290}+1\,|\,F_{284},$$
$$7\cdot 2^{320}+1\,|\,F_{316},\quad 27\cdot 2^{455}+1\,|\,F_{452}.$$

Also, the number $N = k\cdot 2^n + 1$ was found to be prime in just the following cases with $k = 3, 5, 7$ and $20 < n < 1024$.

$k = 3$: $n = 30, 36, 41, 66, 189, 201, 209, 276, 353, 408, 438, 534.$

$k = 5$: $n = 25, 39, 55, 75, 85, 127.$

$k = 7$: $n = 26, 50, 52, 92, 120, 174, 180, 190, 290, 320, 390, 432, 616, 830.$

After the project is completed, it is planned to publish a list of all the primes found.

RAPHAEL M. ROBINSON

University of California
Berkeley, Calif.

The work reported was sponsored by the Office of Naval Research.

1. L. E. DICKSON, *History of the Theory of Numbers*, Carnegie Inst. of Washington, v. 1, 1919, p. 92.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[A].—H. S. UHLER, "Exact values of 996!, and 1000!, with skeleton tables of antecedent constants," *Scripta Mathematica*, v. 21, 1955, p. 261–268.

This paper gives the values of 996! and 1000! without the terminal zeros of which there are 246 and 249 respectively. There is also given the exact value of (996! + 1)/997, together with a frequency census of the digits 0–9 in each of these three large numbers. The last mentioned number is a "Wilson quotient" and the fact that it is an integer constitutes a check on the work. Certain partial products of 25 consecutive integers beyond 750 are also given.

The work was done by desk calculator as far as 750! and then finished on the UNIVAC. The reviewer has compared the value of 1000! with the result obtained directly on the SWAC by Kenneth Ralston. The agreement was perfect.

D. H. LEHMER

University of California
Berkeley, California

2[A].—S. C. NICHOLSON & J. JEENEL, "Some comments on a NORC computation of π," *MTAC*, v. 9, 1955, p. 162–164.

This gives π, 3089D. It also includes frequency counts of the digits in π.

C. B. T.

3[B, C, D].—E. H. NEVILLE, editor, *Rectangular-Polar Conversion Tables*. Designed and compiled by E. H. Neville, Roy. Soc. Math. Tables, v. 2, Camb. Univ. Press, New York, 1956, xxxii + 109 p., 28 cm. Price $5.50.

The stated purpose of this set of double-entry conversion tables is to provide values of r and θ corresponding to pivotal values of x and y. To this end, for each positive integer M not exceeding 105 there appears a table of r and θ in degrees to 13D for $x = M$ and $y = 1(1)M$, followed by a companion table of ln r and θ in radians to 15D for $y = M$ and $x = M(1)105$.

E. H. Neville has written an extensive introduction, which includes an account of the evolution of these tables, together with a full description of the elaborate checking procedures employed, which insure the attainment of the exemplary tabular accuracy exhibited earlier in the publications of the British Association Mathematical Tables Committee, on which Professor Neville served as chairman for sixteen years.

As explained in the Introduction, the calculation of the polar angle θ was made to depend basically on the Haros property of Farey series and an auxiliary table of the denominators of the Farey series of order 105 is included to expedite interpolation. A scholarly historical account of Farey series appears in R. C. Archibald's review [*MTAC*, RMT **881**, v. 5, 1951, p. 135–139] of the first volume of the present series of tables.

Both direct and inverse interpolation in all tabulated quantities, to the full tabular accuracy, are illustrated by detailed numerical examples. (It should be remarked that a desk calculator is almost a necessity in such interpolation.)

Additional supplementary tables give ln x, $x = 1(1)160$, 15D, and 15D values of the first 20 multiples of ln 10. The volume is concluded with page indexes for r in the range 70.0 to 109.9 and ln r in the interval 4.400 to 4.709 designed to aid in the location of close tabular approximations, prior to interpolation by series. A small table of "adjusting factors" and their natural logarithms to 15D is included to bring general values of r and ln r within the prescribed range.

The versatility of the conversion tables is illustrated by their application to the calculation of the principal value of $(e + i\gamma)^{\pi}$ to about eleven significant figures in both rectangular and polar form.

The application of the tables to the conversion of oblique Cartesian coordinates to polar coordinates is also illustrated.

An extensive bibliography, compiled by J. C. P. Miller, has been included. This lists—with appropriate comments—the most important earlier tables in this field and also enumerates the works used in the preparation of the present compilation.

The high standard of typographical excellence characteristic of the earlier publications of both this committee and their predecessors has been maintained. The present set of tables, in providing reliable key values for future tabulations, as well as very accurate working data, constitutes a valuable addition to the growing literature of such numerical information.

JOHN W. WRENCH, JR.

4505 Strathmore Avenue
Garrett Park, Maryland

4[C].—HANS J. MAEHLY, *Rational Approximation*, Cecil Hastings' "Approximation Newsletter," October 25, 1956, Note-16.

Rational approximations for

$$\log_2(x),\ \tfrac{1}{2} \leq x \leq 1,\ 10\text{D};\quad 2^x,\ -\tfrac{1}{2} \leq x \leq \tfrac{1}{2},\ 10\text{D},$$

and

$$\arctan x,\ -1 \leq x \leq 1,\ 9\text{D}.$$

C. B. T.

5[D].—R. B. HORGAN, "Radix tables for sin x and cos x," *MTAC*, v. 10, 1956, p. 164–166.

Sin x and cos x for $x = a \cdot 10^k$ degrees, $a = 1(1)9$, $k = -3(1)1$, 20D. Also included are summation formulas for sin and cos of sums of 2, 3, 4, and 5 angles.

C. B. T.

6[I].—G. A. CHISNALL, "A modified Chebyshev-Everett interpolation formula," *MTAC*, v. 10, 1956, p. 66–73.

Coefficients for obtaining appropriately modified differences $N^j (j = 2(2)10)$ in terms of ordinary central differences $\delta^j (j = 2(2)10)$ are given to 12D; so, too, are the corresponding multipliers $\beta_i{}^j$ in the interpolation formula of the title, which is of the form

$$\begin{aligned} f(m) &= f(0) + m[f(1) - f(0)] + \sum \{\beta_0{}^j C_{2j+1}\{(2 - 2_m)\theta_j\} + \beta_1{}^j C_{2j+1}(2m\theta_j)\} \\ &= f(0) + m[f(1) - f(0)] \\ &\qquad + \sum k_j \{N_0{}^{(2j)} C_{2j+1}\{(2 - 2m)\theta_j\} + N_1{}^{(2j)} C_{2j+1}(2m\theta_j)\}, \end{aligned}$$

where

$$k_j = 2^{-2j-1} \sec^{2j+1} \theta_j / (2_{j+1})!,\ \theta_j = \cos(\pi/(4j+2)) \text{ and } \check{C}_j(2x) = 2\cos(j \arccos x).$$

J. T.

7[I, X].—P. C. HAMMER, O. J. MARLOWE, & A. H. STROUD, "Numerical integration over simplexes and cones," *MTAC*, v. 10, 1956, p. 130–137.

Let $q_m(x) = q_m{}^n(x)$, $m = 1, 2, \cdots$, denote the polynomials orthogonal with respect to x^n in the interval $(0, 1)$. These are given for $m = 1(1)5$, $n = 1, 2$ and for $m = 1(1)4$, $n = 3$. The zeros of the $q_m{}^n$ are given to 18S and so are the corresponding weights b_j in the approximate quadrature formula

$$\int_0^1 x^n g(x)dx \doteqdot \sum b_j g(x_j).$$

Combination of two of these formulas gives a quadrature formula of the form

$$\begin{aligned} \int_\Delta f d\sigma &= \sum \sum b_j a_k f(x_j, x_j y_k) \\ &= \sum \sum w_{jk} f(x_j, x_j y_k) \end{aligned}$$

which is exact for a polynomial of degree 7, when Δ is a triangle (normalized to (0, 0), (1, 1), (1, −1)). The sixteen weights and abscissae are given to 18S.

J. T.

8[I, X].—K. S. Kunz, "High accuracy quadrature formulas from divided differences with repeated arguments," *MTAC*, v. 10, 1956, p. 87–90.

Let $S_r = \sum_{i=1}^{r} i^{-1}$ for $r > 0$. There is a table of the rational numbers, $2[S_p - S_{n-p}]\binom{n}{p}^2$ and the integers $\binom{n}{p}^2$ for $p = 0(1)6$, $n = 1(1)p$. The values of the integers $(2n + 1)!/(n!)$ are given for $n = 1(1)6$. There are various minor typographical faults in the paper.

J. T.

9[J, X].—Herbert E. Salzer, "Formulas for the partial summation of series," *MTAC*, v. 10, 1956, p. 149–156.

The table lists coefficients $A_m(n)$ in the "partial" summation formula $S_n \doteq \sum_{m=4}^{10} A_m(n)S_m$, $m = 4(1)10$, $n = 11(1)50(5)100(10)200(50)500(100)1000$. Exact as rational fractions. See also Review **10** below.

C. B. T.

10[J].—R. B. Horgan, "Coefficients for the partial summation of series," *MTAC*, v. 10, 1956, p. 156–162.

This table lists $A_m(n)$ (see Review **9**, this issue) for $m = 4(1)10$ and $n = 11(1)50(5)100(10)200(50)500(100)1000$, 15D.

C. B. T.

11[K].—Harvard University, Computation Laboratory, *Annals*, v. 35, *Tables of the Cumulative Binomial Probability Distribution*, Cambridge, Mass., Harvard University Press, 1955, lxi + 503 p., 27.3 cm. Price **$8.00.**

These tables give cumulative binomial probabilities

$$E(n, r, p) = \sum_{x=r}^{n} \binom{n}{x} p^x(1 - p)^{n-x}, \quad 0 \leqq r \leqq n, \quad \text{to 5D}$$

for $p = .01(.01).5$, 1/16, 1/12, 1/8, 1/6, 3/16, 5/16, 1/3, 3/8, 5/12, 7/16, and $n = 1(1)50(2)100(10)200(20)500(50)1000$. Of course cumulative binomial probabilities for $p = .51(.01)1$ are immediately available from the tables since $E(n, r, p) = 1 - E(n, n - r + 1, 1 - p)$.

The present tables should be compared with the Army Ordnance "Tables of the Cumulative Binomial Probabilities" [1] which give cumulative binomial probabilities for $p = .01(.01).5$ and $n = 1(1)150$ to 7D. The Harvard tables therefore extend previously available tables of binomial probabilities to some useful higher values of n, while leaving some important gaps in sample sizes $n \leq 150$ and recording fewer decimal places as compared to the Ordnance tables. Furthermore, interpolation in the tables for missing values of n is not necessarily a pleasant pastime. The additional probabilities for specific fractional values of p such as $p = 1/12, 1/6$, etc., are a good feature of the new Harvard tables and will serve a use, perhaps to dice throwers, certainly for game probabilities and indeed for computations on many more or less natural phenomena.

In using the Harvard tables, one finds that they are not consecutive for the sample size n; rather they are divided into six parts on the chance of success in a single trial, p, as follows: (1) Table I, $.01 \leqq p \leqq 1/12$, (2) Table II, $.09 \leqq p \leqq 1/6$, (3) Table III, $.17 \leqq p \leqq 1/4$, (4) Table IV, $.26 \leqq p \leqq 1/3$, (5) Table V, $.34 \leqq p \leqq 5/12$, and (6) Table IV, $.42 \leqq p \leqq 1/2$. For fixed n, the tables are further subdivided into subtables. A user of the tables will nevertheless become accustomed to the required manipulations rapidly.

Much is to be said for the Introduction which consists of: (1) the mathematical characteristics of the cumulative binomial probability distribution, (2) methods of preparation of the tables, (3) methods of interpolation for the tables, and (4) the very excellent section IV on applications by Frederick Mosteller.

It is considered somewhat unfortunate by the reviewer that here is a large new table which does not encompass readily or include in one volume or set all binomial probabilities tabulated to date. One will still want to use these tables along with other existing tabulations.

F. E. Grubbs

Ballistic Research Laboratories
Aberdeen Proving Grounds, Md.

1. L. E. Simon & F. E. Grubbs, *Tables of the Cumulative Binomial Probabilities*, U. S. Gov. Printing Office, Washington, D. C., 1954.

12[K].—E. C. Fieller, T. Lewis, & E. S. Pearson, *Correlated Random Normal Deviates*, Tracts for Computers, No. XXVI, Cambridge Univ. Press, New York, 1955, xv + 60 p., 23.2 cm. Price $2.00.

"These tables give in effect 3000 random pairs from each of nine bivariate normal distributions with respective correlation coefficients .1, .2, .3, $\cdots$, .9. They have been compiled from Wold's table of random normal deviates [1].

"The data are set out in the form: ith line: $x_{0i}, x_{1i}, x_{2i}, \cdots, x_{9i}$ ($i = 1, 2, \cdots, 3000$), where the ten values are random members of normal populations with means 0 and variances 1, and with correlation coefficients $\rho_t = t/10$ between x_0 and x_t ($t = 1, 2, \cdots, 9$)." All values are given to 2D.

In all, 25,000 lines were computed in which the x_{0i} were the 25,000 random deviates of Wold's table and values of x_{ti} were obtained as suggested by Wold (p. xii) by evaluating, $x_{ti} = \rho_t x_{0i} + (1 - \rho_t^2)^{\frac{1}{2}} z_{ti}$ in which the z_{ti} are a new random arrangement of the x_{0i}. The 3000 lines printed were selected according to the x_{0i} by making a random choice of 6 of the 20 pages of 500 which constitute the 3rd and 4th of the 5 blocks of Wold's tables. These blocks passed all of Wold's tests for randomness.

At the foot of each column of 50 entries there are given for the x_{ti} in that column, $\sum x_{ti}$ to 2D, $\sum x^2_{ti}$ to 4D, $\sum x_{0i}x_{ti}$ to 4D, and the sample correlation coefficients r_{0t} to 4D. In addition in table 1 for the 3000 values of each x_t, the ranges (2D), means (4D), standard deviations (4D), and third and fourth standard cumulants (4D) are listed.

The authors describe the application of four tests of randomness which were used on the complete set, on six sets of 500, and on sixty sets of 50 into which the table was divided. Performance on all of these tests seemed quite satisfactory.

C. C. C.

1. HERMAN WOLD, *Random Normal Deviates*, Tracts for Computers, No. XXV, Cambridge Univ. Press, New York, 1948.

13[K].—E. J. HANNAN, "An exact test for correlation between time series," *Biometrika*, v. 42, 1955, p. 316–326.

This article is concerned with tests for correlation between two time series x_t and y_t with serially correlated normal residuals. The estimates compared are: (1) the partial correlation between x_{2t} and y_{2t} when the effects of $(y_{2t-1} + y_{2t+1})$, x_{2t-1} and x_{2t+1} have been removed; (2) the ordinary correlation coefficient r between the two series, and (3) the partial correlation between x_t and y_t given x_{t-1} and y_{t-1}. The asymptotic efficiencies of these statistics are compared under the conditions: (a) the residual process from the regression of y_t and x_t is independent of the x_t process and comes from a Gaussian Markov process; (b) the two series are Markovian and are correlated through correlated errors; (c) same as (b) but with second order autoregression. The paper shows statistic (3) to lead to the asymptotically most efficient test for conditions (a), (b), and (c), except for some cases under (c) where the first partial correlation of the x_t process is high and positive. The criterion used for comparison requires the evaluation of the quantities:

$$\frac{(1 + p_1p_2)(1 - p_2^2)}{(1 - p_1p_2)(1 - p_1^2)}$$

and

$$\frac{1}{2}\,\frac{(1 - p_1^2p_2^2)(1 + p_1p_2)^2}{(1 - p_1^4)(1 - p_2^4)}$$

which are tabulated to 2D for $p_1, p_2 = -.8(.2).8$ and

$$\frac{(1 + p_1p_2 - b)^2[1 + p_1p_2 - b(p_1^2 + p_1p_2)][1 - p_1p_2 - b(p_1p_2 - p_1^2)]}{2(1 - p_1^4)(1 - p_2^2)(1 - b)[1 + p_2^2 - b(1 - p_2^2)]}$$

which is tabulated to 2D for $p_1 = p_2 = .4$ and all combinations of $p_1, p_2 = .6, .8$ for $b = -.6(.2).6$.

W. J. DIXON

University of California
Los Angeles, California

14[K].—G. E. NOETHER, "Use of the range instead of the standard deviation," Amer. Stat. Assn., *Jn.*, v. 50, 1955, p. 1040–1055.

This expository article summarizes methods which use the range instead of the standard deviation: (1) to obtain confidence intervals for the mean μ of a normal population, the difference of two means, and (2) to estimate the standard deviation σ of a normal population. Table 1 gives for $N = 2(1)100$ the appropriate subsample size, the necessary factors to 3S to obtain an unbiased estimate of σ and to 2D to find 90% and 98% upper and lower confidence limits. This table is based on the optimum procedure given by Grubbs and Weaver [1]. It is pointed out that the loss in efficiency due to the use of equal subsamples is slight compared to the gain in computational ease. For use in obtaining confidence intervals for the mean and difference of means, Tables I and II of Jackson and Ross [2], which were derived from earlier tables by Lord, are recommended.

C. A. Bennett

Hanford Atomic Products Operation
General Electric Company
Richland, Washington

1. F. E. Grubbs & C. L. Weaver, "The best unbiased estimate of population standard deviation based on group ranges," Amer. Stat. Assn. *Jn.*, v. 42, 1947, p. 224–241.
2. J. E. Jackson & E. L. Ross, "Extended tables for use with the 'G' test for means," Amer. Stat. Assn., *Jn.*, v. 50, 1955, p. 416–433.

15[K].—D. P. Banerjee, "A note on the distribution of the ratio of sample standard deviations in random samples of any size from a bivariate correlated normal population," Indian Soc. Agricultural Stat. *Jn.*, v. 6, 1954, p. 93–100.

For samples of N from a normal bivariate universe, the author has tabulated to 3S the upper 80%, 90%, 95%, and 99% points of the distribution of the ratio of the two sample standard deviations for $N = 3(1)30$ and $\rho = 0(.1).9$, where ρ is the universe coefficient of correlation. For $\rho = 0$ the values given in a high proportion of cases are one less in the third significant figure than the 3S square root of the corresponding variance ratio, F, given in the standard tables.

C. C. C.

16[K].—D. A. S. Fraser & Irwin Guttman, "Tolerance regions," *Annals Math. Stat.*, v. 27, 1956, p. 162–179.

Tables in this article give factors for obtaining certain tolerance regions for univariate and multivariate normal distributions.

The tolerance intervals (regions) are termed similar β-expectation if the average probability content of the interval (region) is β. The appropriate factors are given to 4S for five cases: univariate normal with unknown mean and variance, univariate normal with known variance, bivariate, trivariate, and quadri-variate normal with unknown means, and variance-covariance matrices for $\beta = .75, .9, .95, .975, .99, .995$ for sample size $n = 2(1)31, 41, 61, 121, \infty$ for the univariate cases and n increased by one less than the number of variates for the multivariate cases.

W. J. Dixon

University of California
Los Angeles, California

17[K].—J. Arthur Greenwood & David Durand, "The distribution of length and components of the sum of n random unit vectors," *Annals Math. Stat.*, v. 26, 1955, p. 233–246.

The n random unit vectors of the title of the paper are in Euclidean 2-space and are drawn independently from a distribution with probability density for the angle ξ, measured from some fixed direction, $g(\xi) = [\exp\{k \cos(\xi - \alpha)\}]/2\pi I_0(k)$. Mises [1] introduced this distribution of points on a circle as the distribution (under certain requirements with respect to continuity of derivatives) for which the maximum likelihood estimate of the center of gravity of the distribution is the center of gravity of the sample points. Gumbel, Greenwood, and Durand [2] in 1953 gave tables of the distribution function of $g(\xi)$ for various values of k and a table to facilitate calculation of the maximum likelihood estimate of k. The present paper gives functional forms for the distribution of $V = \sum_{\nu=1}^{n} \cos \xi_\nu$ for $k = 0$ and for $\alpha = 0$, the joint distribution of V and $W = \sum_{\nu=1}^{n} \sin \xi_\nu$ for $\alpha = 0$, and the distribution of $R = \sqrt{V^2 + W^2}$.

The tables in the present paper deal with the distribution of R when $k = 0$, i.e., with samples from the uniform angular distribution, $g(\xi) = 1/2\pi$. Table 1 gives the probability that $R \leq r$, $P(r, n) = r \int_0^\infty [J_0(x)]^n J_1(rx)dx$ to 5D for $n = 6(1)24$, $r = .5(.5)12.(1)n$. $P(n, n) = 1$ and the value 1.00000 is entered only for the lowest value of r for which it is appropriate. Table 2 gives 95th and 99th percentiles of the distribution of R and of various functions of r. The solutions of $P(r, n) = .95$ and $P(r, n) = .99$ are given to 3D. Corresponding values of r/n are given to 4D, r^2 to 2D, $z = r^2/n$ to 4D. The two limiting values of z for $n \to \infty$ are given. The authors describe the computational procedures used and give reasons for believing the tables accurate to the number of places given, except that $P(r, n)$ for $n = 6$ may be in error by 1 in the fifth place and z in error by 2 in the fourth place. Table 3 compares $P(r, n)$ with the approximation $1 - \exp(-r^2/n)$ and with two other approximations. Table 4 compares percentage points with various approximations to them. An example of a test of significance of R is given with n outside the range of the tables and, therefore, an approximation to a percentage point of $P(r, n)$ is calculated.

K. J. Arnold

Michigan State University
East Lansing, Mich.

1. R. v. Mises, "Über die 'Ganzzahligkeit' der Atomgewichte und verwandte Fragen," *Physikalische Zeitschrift*, v. 19, 1918, p. 490–500.
2. E. J. Gumbel, J. A. Greenwood, & D. Durand, "The circular normal distribution: theory and tables," Am. Stat. Assn. *Jn.*, v. 48, 1953, p. 131–152. [*MTAC*, v. 8, RMT 1151, 1954, p. 19.]

18[K].—Peter Ihm, "Ein Kriterium für zwei Typen zweidimensionaler Normalverteilungen," *Mitteilungsblatt für Math. Stat.*, v. 7, 1955, p. 46–52.

A significance test is constructed for the composite hypothesis H_0: $\sigma_1 = \sigma_2$; $\rho = 0$ for a bivariate normal distribution p and a sample size N. The alternative H_1 is $\sigma_1 \neq \sigma_2$ or $\rho \neq 0$ or both. Let $L = p_0/p_1$ be the likelihood ratio. The test function used is $Z = L^{2/N}$. In order to test H_0 against H_1, the author calculates the probability $P(Z \leqq Z_0 | H_0) = \alpha$. If $Z \leqq Z_0$, then H_0 is rejected and H_1 is accepted. If $Z > Z_0$, then H_1^* is rejected and H_0^* accepted, where H_1^* and H_0^* stand respectively for $\rho^2 \gtrless \rho^{*2}$. A table gives Z_0 to 3D as a function of $N = 3(1)30$, 40, 60, 120 for $\alpha = .01$ and $\alpha = .05$.

E. J. Gumbel

Columbia University
New York, N. Y.

19[K].—F. M. LORD, "Nomograph for computing multiple correlation coefficients," Amer. Stat. Assn., *Jn.*, v. 50, 1955, p. 1073–1078.

The author presents a nomograph for obtaining the multiple correlation coefficient $R_{1.23} = \{(r^2_{12} + r^2_{13} - 2r_{12}r_{13}r_{23})/(1 - r^2_{23})\}^{\frac{1}{2}}$, where r_{12}, r_{13}, and r_{23} are the ordinary coefficients of correlation.

L. A. AROIAN

Hughes Aircraft Co.
Culver City, Calif.

20[K].—MOTOSABURA MASUYAMA, "Tables of two-sided 5% and 1% control limits for individual observations of the r-th order," *Sankhyā*, v. 15, 1955, p. 291–294.

The 95% and 99% confidence limits here found for the rth order statistic in a sample of n from any continuous distribution are obtained by finding 2.5% and 97.5% (or 0.5% and 99.5%) points of the corresponding incomplete β distribution, using a well known relation. These limits are tabulated to 5S for $n = 3(1)6, 8, 10$ and $r = 1(1)n$. Most of the values listed could be copied directly from the Thompson tables [1]; it is not stated how the ones not found there were obtained. In addition in the case where the distribution sampled is $N(0, 1)$, the probability integral variates tabled are converted to standardized normal deviates to 4S apparently by linear interpolation in the Kelley tables [2], which, with the Thompson tables, are listed in the author's bibliography.

C. C. C.

1. C. M. THOMPSON, "Tables of percentage points of the incomplete beta-function and of the χ^2 distribution," *Biometrika*, v. 32, 1941, p. 151–181. Also Table 16 in *Biometrika Tables for Statisticians*, vol. 1, Cambridge, 1954. [*MTAC*, v. 1, 1943, RMT 99, p. 76–77.]
2. *The Kelley Statistical Tables*, Cambridge, Mass., 1948. [*MTAC*, v. 1, 1944, RMT **130**, p. 151–152.]

21[K].—M. R. SAMPFORD, "The truncated negative binomial distribution," *Biometrika*, v. 42, 1955, p. 58–69.

This paper is concerned with estimating the parameters p and k of the negative binomial distribution, $P(r) = \binom{k+r-1}{r}p^r(1 + p)^{-(k+r)}$, $(r = 0, 1, \cdots; p, k > 0)$, when truncation has resulted in elimination of the class corresponding to $r = 0$. Moment estimating equations obtained by obtaining the population mean and variance to corresponding sample values and maximum likelihood estimating equations are also derived.

Explicit solutions in terms of elementary functions are not possible for either the moment or the maximum likelihood estimating equation, and iterative procedures must be employed. The author gives illustrative examples to demonstrate that the moment estimating equations are simpler to solve than the maximum likelihood equations. A table of efficiencies of the moment method is given to 3S for $k = 0.5$, $1(1)5$ with means of 0.5, 1.0, 2.0, 5.0. To facilitate the iterative solution of both moment and maximum likelihood equations, the function $\varphi(x) = -x \log_e x/(1 - x)$, which occurs in both, is tabulated to 4D for $x = 0(.01)0.99$.

A. C. COHEN, JR.

University of Georgia
Athens, Georgia

22[K].—J. L. HODGES, JR., "On the non-central beta-distribution," *Annals Math. Stat.*, v. 26, 1955, p. 648–653.

Nicholson [1] has derived a closed expression for B, the noncentral beta-distribution in case b is an integer:

$$B(x; a, b, \lambda) = 1 - e^{-\lambda x}\{I_{1-x}(a, b) + (1 - x)^a \sum_{j=1}^{b-1} [x(1 - x)\lambda]^j (P_{j/j!})\}$$

where

$$P_j = \sum_{k=0}^{b-j-1} \left[(-1)^k \binom{b - j - 1}{k} \frac{(a + b - 1)(a + b - 2)\cdots(a + j)}{(b - j - 1)!(a + j + k)} \right] (1 - x)^k,$$

and $I_x(a, b)$ is the beta-distribution. The author proves that $P_j = \sum_{t=0}^{b-j-1} \binom{A+t}{t} x^t$, $A = a + j - 1$. A table of $\binom{A+t}{t}$ to 7S is provided for $A = .5(1)19.5$, $t = 1(1)18$. The author compares these direct methods for computing B with Tang's recursion formula [2].

L. A. AROIAN

Hughes Aircraft Co.
Culver City, Calif.

1. W. L. NICHOLSON, "A computing formula for the power of the analysis of variance," *Annals Math. Stat.*, v. 25, 1954, p. 607–610.
2. P. C. TANG, "The power function of the analysis of variance test with tables and illustrations of their use," *Stat. Res. Memoirs*, v. 2, 1938, p. 126–149.

23[K].—L. H. MILLER, "Table of percentage points of Kolmogorov statistics," Amer. Stat. Assn., *Jn.*, v. 51, 1956, p. 111–121.

Let $S_n(X)$ be the observed cumulative distribution of a random sample of n observations from a population having a continuous cumulative distribution $F(X)$. Let $D_n = \max \{S_n(X) - F(X)\}$ and $D_n^* = \max|S_n(X) - F(X)|$. Table 1 gives for $\alpha = .005, .01, .025, .05, .10$ and $n = 1(1)100$ values of ϵ to 5D such that $\alpha = \text{Prob.} (D_n \geq \epsilon)$. For $\alpha < .1$, $P = \text{Prob.} \{D_n^* \leq \epsilon\}$ is close to $1 - 2\alpha$ and the table also gives in its heading $P = .99, .98, .95, .9$.

The author believes that the greatest error in Table 1 (considering ϵ as a function of n and α) does not exceed one unit in the fifth decimal place. For $n \leq 20$ an exact formula was used, and Table 3 gives a number of checks of the asymptotic formula for $n \geq 20$. Table 2 gives an illustration of part of the computing technique.

FRANK MASSEY

University of California
Los Angeles, Calif.

24[K].—C. W. TOPP & F. C. LEONE, "A family of J-shaped frequency functions," Amer. Stat. Assn., *Jn.*, v. 50, 1955, p. 209–219.

The authors consider a family of cumulative frequency functions defined by

$$(1) \quad \begin{cases} F(x) = \dfrac{a}{b^{2r}} (2bx - x^2)^t + (1 - a)\dfrac{x}{b}, & 0 \leq x \leq b < \infty, \\ F(x) = 0, & x < 0, \\ F(x) = 1, & x > b, \end{cases}$$

where $0 < r < 1$, and $0 < a \leq 1$. The paper takes its title from the fact that the corresponding family of frequency functions obtained by differentiation of (1) are J-shaped.

The parameters a and r are expressed as functions of α_3 and δ where α_k is the k-th standard moment and $\delta = (2\alpha_4 - 3\alpha_3^2 - 6)(\alpha_4 + 3)^{-1}$. To facilitate graduation of observed data by means of (1) in the special case where $b = 1$, α_3^2 and δ are tabulated to 3D for the arguments a and r, with $r = .01, .02, .05, .08, .09, .10(.05).95$ and $a = .05(.05)1.00$. The tabled information is also presented graphically in the form of an (α_3^2, δ) chart.

A. C. COHEN, JR.

University of Georgia
Athens, Georgia

25[K].—SIGEITI MORIGUTI, "Confidence limits for a variance component," Union of Japanese Scientists and Engineers, *Reports of Statistical Application Research*, v. 3, 1954, p. 29–41.

If, as in the case of an analysis of variance with random effects, one has a mean square V_1 with f_1 degrees of freedom, whose expected value is $\sigma^2 + nv$, such that $f_1V_1/\sigma^2 + nv$ obeys a χ^2 distribution, and also has an independent mean square V with f degrees of freedom, whose expected value is σ^2 such that fV/σ^2 obeys a χ^2 distribution, it is of interest to determine confidence intervals for the variance component v, n being a known constant and σ^2 a nuisance parameter. The author derives his approximation formulas for $100(1 - \alpha)\%$ confidence limits in which for each limit two parameters enter linearly. These parameters are tabulated for $\alpha = .1$ to 4S or 3D for $f = 6(2)12, 15, 20, 30, 60$ and $f_1 = 1(1)6(2)12, 15, 20, 30, 60, \infty$. Comparisons are made with previously obtained approximations which favor the present one.

C. C. C.

26[K].—SHOJI URA, "A table of the power function of the analysis of variance tests," Union of Japanese Scientists and Engineers, *Reports of Statistical Application Research*, v. 3, 1954, p. 23–28.

The author extends the inverse tables of E. Lehmer [1] for probabilities of errors of the second kind in the variance ratio test ordinarily used in the analysis of variance. He develops a formula for the necessary power function which he credits to J. Yamauti (apparently hitherto unpublished) and employs it to tabulate values of the quantity $\psi = [(f_1 + 1)/f_1]^{\frac{1}{2}}\varphi$, where φ is the quantity tabled by Lehmer and introduced by P. C. Tang [2], for which the significance level is .05 and the probability of an error of the second kind is .1. Values are given to 2D for the degrees of freedom, $f_1 = 1(1)10, 12, 15, 20, 24, 30, 60, 120, \infty$ and $f_2 = 2(2)20, 24, 30, 40, 60, 120, \infty$.

C. C. C.

1. EMMA LEHMER, "Inverse tables of probabilities of errors of the second kind," *Annals Math. Stat.*, v. 15, 1944, p. 388–398.
2. P. C. TANG, "The power function of the analysis of variance tests with tables and illustrations of their use," *Stat. Res. Memoirs*, v. 2, 1938, p. 126–194 + tables.

27[K].—MICHIO TAKASHIMA, "Tables for testing randomness by means of lengths of runs," *Bull. Math. Stat.*, v. 6, 1955, p. 17–23.

In testing for randomness an ordered arrangement of $(m + n)$ objects of two kinds (say m A's and n B's), one may use as test criterion the length of the longest run, or of the longest A-run. The author has tabulated the critical run-lengths for tests based on these criteria.

Let $Q(t)$ be the probability that there appears at least one run (of A's or B's) of length t or longer. Let $Q_1(t)$ be the probability that there appears at least one A-run of length t or longer. Let $t_\alpha(t_\alpha)$ be the smallest integer such that $Q(t) \leq \alpha$; $(Q_1(t) \leq \alpha)$.

The tables give t_α and (t_α) for $\alpha = .01, .05$ and for $m, n = 1(1)25$. Calculations were based on an investigation by Mood [1].

Misprints in the introduction:

$$\text{line 9, } \textit{read } m = \sum_{i=1}^{m} ir_i,\ n = \sum_{j=1}^{n} js_j;\ \text{line 10, } \textit{read } u = r + s.$$

JOAN R. ROSENBLATT

Statistical Engineering Laboratory
National Bureau of Standards
Washington, D. C.

1. A. M. MOOD, "The distribution theory of runs," *Annals Math. Stat.*, v. 11, 1940, p. 367–392.

28[K].—J. F. SCOTT & V. J. SMALL, "A numerical investigation of least squares regression involving trend-reduced Markoff series," Roy. Stat. Soc., *Jn.*, s. B., v. 17, 1955, p. 105–114.

Tables 1 and 2 give for a number of values of two parameters, ρ_e and ρ_x (representing serial correlations of independent and residual variables), factors which facilitate the computation in the formula for the asymptotic variance of an estimate of a regression slope in a trend-reduced Markoff time series. Two smoothing formulas, each extending over $2k + 1$ terms, were considered: (i) a moving average of 3 separated terms, and (ii) an equally weighted moving average. Tables 1 and 2 give results to 2D for $k = 1, 2, 3, 5, 10, \infty$ and ρ_e, $\rho_x = 0(.1).9$ which allows one to make an optimum choice of k if the values of other parameters are known.

Tables 5 and 6 give, for the same values of k and the two parameters noted above, correction factors to 2D for converting a classical estimate of this same regression slope into an estimate which is adjusted for autocorrelations in the series.

Tables 3 and 4 give, for the above smoothing formulas, the first serial correlation to 2D of the reduced series for the same values of k in terms of the Markoff parameter $\rho = 0(.1).9$. Used inversely, these tables may be used to obtain estimates of ρ from the observed serial correlations.

FRANK MASSEY

University of California
Los Angeles, Calif.

29[K].—E. S. Page, "A test for a change in a parameter occurring at an unknown point," *Biometrika*, v. 42, 1955, p. 523–527.

The statistic $m = \max_{0 \le i \le n} \{S_r - \min_{0 \le i < r} S_i\}$, where $S_r = \sum_{j=1}^{r} (x_j - \theta)$, $S_0 = 0$, is suggested to test the hypothesis that the observations x_i, $i = 1, \cdots, k$ (in order of observation) have mean value θ and the observations x_i, $i = k + 1, \cdots, n$ have mean value $\theta' > \theta$. For the special case $y_i = \text{sgn}(x_i - \theta)$ and x_i symmetrically distributed, 5% and 1% points are derived for values of n ranging from 21 to 185 (Table 1). The power of this test is compared to that of the usual sign test (values to 3D) for $k = 0$ and $p = \Pr[x_i - \theta > 0 | E(x_i) = \theta' > \theta] = 0.5(.05).8$, and for $p = .75$ and $k = 0(10)50$ (Tables 2 and 3).

C. A. Bennett

Hanford Atomic Products Operation
General Electric Company
Richland, Washington

30[K].—R. L. Storer & W. R. Davison, "Simplified procedures for sampling inspection by variables," *Industrial Quality Control*, v. 12, no. 1, 1955, p. 15–18.

The authors present master tables for sampling inspection by inspection based on the average range ($\bar{R}$) and sample standard deviation (V). The procedure based on the average range is: (1) accept the lot if $\bar{X} + k\bar{R} \le U$ when there is a one-sided upper specification limit (U) given; (2) accept the lot if $\bar{X} - k\bar{R} \ge L$ when there is a one-sided lower specification limit (L) given; and (3) accept the lot if $\bar{X} + k\bar{R} \le U$ and $\bar{X} - k\bar{R} \ge L$ and $\bar{R} \le$ maximum allowable average range ($MA\bar{R}$) where $MA\bar{R} = F(U - L)$ when there is a two-sided specification limit given. The authors give values of k to 2D and F to 3D for sampling plans indexed according to acceptable quality levels of .065, .1, .15, .25, .4, .65, 1, 1.5, 2.5, 4, and 6.5 (in percent) and sample sizes of 5, 10, 15, 20, 25, 30, 35, 40, 50, 70, 130, and 200.

The procedure based on the sample standard deviation is the same as the above with $\bar{R}$ replaced by V. The authors give values of k to 2D and F to 3D for the same parameters as given above.

The authors state that the plans based on the average range and sample standard deviation give essentially equal protection for a given sample size in the sense of having the same operating characteristic curves. However, no operating characteristic curves are presented in the article but appear in the stated references.

G. J. Lieberman

Stanford University
Stanford, Calif.

31[K].—G. J. Lieberman & Herbert Solomon, "Multi-level continuous sampling plans," *Annals Math. Stat.*, v. 26, 1955, p. 686–704.

We quote from page 687 of the article:

"0) At the outset inspect 100 percent consecutively as produced and continue such inspection until i units in succession are found clear of defects.

"1) When i units in succession are found clear of defects, discontinue 100 percent inspection and inspect only a fraction f of the units (i.e., one out of every

$1/f$ where $1/f$ is an integer). If the next i inspected units are non-defective, proceed to the next level; if a defective occurs, revert immediately to 100 percent inspection.

"2) When at rate f, i inspected units are found clear of defects, discontinue sampling at rate f and proceed to sampling at rate f^2. If the next i inspected units are non-defective, proceed to the next level; if a defective occurs, revert immediately to sampling at rate f.

"3) When at rate f^2, i inspected units are found clear of defects, discontinue sampling at rate of f^2 and proceed to sampling at rate f^3. If the next i inspected units are non-defective, proceed to the next level; if a defective occurs, revert immediately to sampling at rate f^2.

"$k - 1$) When at rate f^{k-2}, i inspected units are found clear of defects, discontinue sampling at rate f^{k-2} and proceed to sampling at rate f^{k-1}. If the next i inspected units are non-defective, proceed to the next level; if a defective occurs, revert immediately to sampling at rate f^{k-2}.

"k) When at rate of f^{k-1}, i inspected units are found clear of defects, discontinue sampling at rate f^{k-1} and proceed to sampling at rate f^k. If a defective occurs, revert immediately to sampling at rate f^{k-1}, otherwise, continue sampling at rate f^k.

"Whenever sampling is in operation, one item should be selected at random from each segment of $1/f^j$ ($j = 0, 1, 2, \cdots, k$) production items. During both sampling inspection and 100 percent inspection all defective items found should either be corrected or replaced with good items."

For $k = 1$, the Lieberman-Solomon plan reduces to Dodge's 1943 plan [1].

Charts show the relationship of f to i for the average outgoing quality limit ($AOQL$) in percents .01, .02, .03, .05, .1, .2, .3, .5, 1(1)6, 8, 10, for $k = 1, 2, \infty$.

Harold Freeman

Massachusetts Institute of Technology
Cambridge, Mass.

1. H. F. Dodge, "Skip-lot sampling plan," *Industrial Quality Control*, v. XI, no. 5, 1955, p. 3–5. [*MTAC*, v. 10, 1956, RMT **19**, p. 47.]

32[K].—H. F. Dodge, "Chain sampling inspection plan," *Industrial Quality Control*, v. 11, No. 4, 1955, p. 10–13.

A chain sampling plan introduces a somewhat different consideration in sampling inspection. The plan overcomes—to a degree—the shortcomings of a sampling plan involving a single small sample with an acceptance number, $c = 0$.

The procedure of the plan is as follows: (a) For each lot, select a sample of n units and test each unit for conformance to a specified requirement; (b) The acceptance number of defects is $c = 0$; except that $c = 1$ if no defects are found in the immediately preceding "chain" of i samples of size n. ($i = 1, 2, 3, \cdots$.) That is, a lot is accepted if no defects are found in its sample of n units. A lot is rejected if two or more defects are found in its sample. But if one defect is found, the lot can still be accepted if the last defect was far enough back in the history of the product, as determined by the choice of i.

The characteristic curves for four sets of chain sampling plans designated by ChSP-1 are given. The sample sizes are $n = 4, 5, 6$ and 10 with values of $i = 1(1)5$.

A formula is presented to be used for drawing up curves for arbitrarily chosen values of n and i.

Because of the cumulative aspect of chain sampling, it is obvious that there exist conditions under which the plan is best applied. The author clearly states these conditions which have been assumed when generating the theory fundamental to chain sampling.

G. W. McElrath

University of Minnesota
Minneapolis, Minn.

33[K, L].—NBS Applied Mathematics Series, No. 41, *Tables of the Error Function and its Derivative* $H(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\alpha^2} d\alpha$ *and* $H'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$, U. S. Gov. Printing Office, Washington, D. C., 1954, xi + 302 p., 26 cm. Price $3.25.

Here are listed $H'(x)$ and $H(x)$, $x = 0(.0001)1(.001)5.6$(various), 15D and $H'(x)$ and $1 - H(x)$, $x = 4(.01)10$, 8S, where $H(x)$ is the error function,

$$H(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\alpha^2} d\alpha.$$

This is a reissue of "Tables of probability functions," v. 1, 1941, Mathematical Table 8, prepared by the Mathematical Tables Projects of the Federal Works Agency, Works Projects Administration (see RMT **91**, *MTAC*, v. 1, 1943, p. 48–51). The only change has been the correction of two minor misprints, and one major misprint; the correct value at $x = 1.742$ is $2\pi^{-\frac{1}{2}}e^{-x^2} = .05427\ 01046\ 62097$.

The bibliography of tables in this volume was not supposed to be complete, but it is regrettable that this new edition included no reference to [1] (see RMT **1034**, *MTAC*, v. 6, 1952, p. 232). Incidentally in [1] the phrase 'error function' is used to denote a quantity which is one-half less than the normal probability integral—a notation not in conformance with the NBS notation or with Fletcher, Miller, and Rosenhead's, *An Index of Mathematical Tables*.

Topography of the new edition remains adequate.

C. B. T.

1. Harvard University, Computation Laboratory, *Annals*, v. 23, *Tables of the Error Function and its First Twenty Derivatives*, Cambridge, Mass., Harvard University Press, 1952.

34[L].—S. Johnston, *Tables of Sievert's Integral*, Manchester and Newcastle upon Tyne, England, 1955, 23 × 32 cm. (oblong), 9 pages. Copies may be purchased from the Royal Soc. Depository of Unpublished Mathematical Tables, Burlington House, London W. 1, and also from the Hospital Physicists Association, Diagrams and Data Scheme, Mount Vernon Hospital, Northwood, Middlesex, England. Two copies have been deposited in UMT File.

These tables are, apart from details of arrangement, the same as the tables computed in 1948 by S. Johnston, 81 Fountain Street, Manchester, 2, England, in response to queries by R. D. Evans in *MTAC*, and briefly described in UMT **103** (*MTAC*, v. 4, 1950, p. 163). It may be recalled that Sievert's integral

$$\int_0^x e^{-A \sec \theta}\, d\theta$$

is used in radio-therapy (see *MTAC*, v. 2, 1946, p. 196). The copies now reviewed were made in 1955 by the Radiotherapy Department, Royal Victoria Infirmary, Newcastle upon Tyne, England, although this fact is not stated on them. The copying was done from a typed manuscript by photo-lithography, some of the headings being printed in at the same time. Values of the integral are given to about 5S, without differences, for $A = 0(.5)10$, $x = 0(1°)90°$. More precisely, the number of decimals depends upon the value of A, as follows:

5D	$A = 0.0$ to 1.5
6D	$A = 2.0$ to 4.0
7D	$A = 4.5$ to 6.0
8D	$A = 6.5$ to 8.0
9D	$A = 8.5$ to 10.0

In the version described in UMT **103**, explicit tabulation was not made beyond a value of x after which the integral remains unchanged to 5S; but in the present version all values are explicitly tabulated. It may be added that the National Bureau of Standards is producing more extended tables and that S. Johnston has computed many thousands of values of related integrals.

A. F.

35[L, S].—C. E. Fröberg, "Numerical treatment of Coulomb wave functions," *Rev. Mod. Phys.*, v. 27, 1955, p. 399–411.

Coulomb wave functions are particular forms of confluent hypergeometric functions which are of importance in nuclear physics. They are the solutions of the differential equation

$$\frac{d^2y}{d\rho^2} + \left\{ 1 - \frac{2\eta}{\rho} - \frac{L(L+1)}{\rho^2} \right\} y = 0,$$

in which L (an integer) and η are parameters. The most extensive tables of solutions [1] cover the ranges $0 \leq L \leq 21$, $-6 \leq \eta \leq 6$, $0 \leq \rho \leq 5$. The purpose of the present paper is to give formulae from which isolated values lying outside the range of these tables may be computed to an accuracy of at least five or six significant figures.

The formulae are drawn from the many scattered papers on the subject and include recurrence relations in the L-direction, integral representations, ascending series in ρ, asymptotic series in $1/\rho$, expansions at the transition point $\rho = 2\eta$, and some expansions in terms of Bessel and Airy functions. Useful charts are included which indicate recommended methods in various regions.

The disadvantage of most of the expansions given is that they are useful in only relatively small regions. The theory of the asymptotic solution of differential equations containing a parameter can be applied to the Coulomb wave equation to determine expansions for large η, L in terms of Bessel, Airy or exponential functions, which are uniform with respect to *unrestricted* ρ, unlike those given in the paper. Leading terms of such expansions have been given in more recent publications [2, 3, 4] and higher terms can be obtained, for example, by application of the theory of the reviewer [5, 6].

F. W. J. OLVER

National Physical Laboratory
Teddington, Middlesex
England

1. NBS Applied Mathematics Series, No. 17, *Tables of Coulomb Wave Functions*, v. 1, U. S. Gov. Printing Office, Washington, D. C., 1952.
2. A. ERDÉLYI, M. KENNEDY, & J. L. MCGREGOR, "Asymptotic forms of Coulomb wave functions I," California Institute of Technology: Tech. Report No. 4 NRO43–121, 1955.
3. A. ERDÉLYI & C. A. SWANSON, "Asymptotic forms of Coulomb wave functions II," California Institute of Technology: Tech. Report No. 5 NRO43–121, 1955.
4. N. D. KAZARINOFF, "Asymptotic expansions for the Whittaker functions of large complex order *m*," Amer. Math. Soc., *Trans.*, v. 78, 1955, p. 305–328.
5. F. W. J. OLVER, "The asymptotic solution of linear differential equations of the second order for large values of a parameter," Roy. Soc., *Phil. Trans.*, Ser. A, v. 247, 1954, p. 307–327.
6. F. W. J. OLVER, "The asymptotic solution of linear differential equations of the second order in a domain containing one transition point," Roy. Soc., *Phil. Trans.*, Ser. A, v. 249, 1956, p. 65–97.

36[L].—E. E. ALLEN, "Polynomial approximations to some modified Bessel functions," *MTAC*, v. 10, 1956, p. 162–164.

Polynomial approximations for the following functions: $I_0(x)$, $-3.75 \leq x \leq 3.75$, 8D; $I_1(x)/x$, $-3.75 \leq x \leq 3.75$, 8D; $I_0(x)x^{\frac{1}{2}}e^{-x}$, $3.75 \leq x < \infty$, 7D; $I_1(x)x^{\frac{1}{2}}e^{-x}$, $3.75 \leq x < \infty$, 7D; $K_0{}^{(x)} + \log_e (.5x)I_0(x)$, $0 < x \leq 2$, 7D; $[K_1(x) - \log_e (.5x)I_1(x)]x$, $0 < x \leq 2$, 7D; $K_0(x)x^{\frac{1}{2}}e^x$, $2 \leq x < \infty$, 7D; $K_1(x)x^{\frac{1}{2}}e^x 2 \leq x < \infty$, 7D.

C. B. T.

37[L, S].—D. K. C. MACDONALD & LOIS T. TOWLE, "Integrals of interest in metallic conductivity," *Canadian J. of Physics*, v. 34, 1956, p. 418–19.

This note gives a table of $J_r(x)$,

$$J_r(x) = \int_0^x \frac{z^r dz}{(e^z - 1)(1 - e^{-z})}$$

for $r = 2, 3, 4$, and 6; $x = 0.1, 0.25, 0.5, 1.0, 1.2, 1.5, 2(1)6, 8, 10, 13, 20$, and ∞; for 4–6S.

This table contains the following errors, together with some one or two digit errors in the last significant figure:

x	$J_4(x)$ *is*	*should be*
2	2.201*6*	2.2011
3	5.96*32*	5.9648
4	10.7*293*	10.7319
5	15.3*671*	15.3598
6	19.12*10*	19.1230
8	23.58*74*	23.5840
10	25.2*812*	25.2737
13	25.*9273*	25.8860
20	25.9*639*	25.9754

x	$J_6(x)$ *is*	*should be*
3	29.*69*	29.58
6	295.*40*	295.43
8	507.*00*	506.81
10	63*9.21*	638.78

The computation on SEAC was done using Simpson's Rule and integrating from zero. The end values from SEAC were checked by the asymptotic formula developed below.

As pointed out in the note $J_4(\infty) = 4\pi^4/15$ and $J_6(\infty) = 16\pi^6/21$, from this we see

$$J_4(x) = \frac{4\pi^4}{15} - I_4(x)$$

where

$$I_4(x) = \int_x^\infty z^4e^{-z}\left\{1 + \frac{e^{-z}}{1 - e^{-z}}\right\}^2 dz$$

$$I_4(x) = \int_x^\infty z^4e^{-z}dz + E_4(x)$$

$$I_4(x) = e^{-x}\{x^4 + 4x^3 + 12x^2 + 24x + 24\} + E_4(x)$$

where

$$|E_4(x)| < (2e^{-x} + e^{-2x})I_4(x) \quad x > 0.$$

From the above formula for $J_4(x)$ and a similar formula for $J_6(x)$, $J_4(20)$ and $J_6(20)$ computed by the integration on SEAC were found to be accurate to within one unit in the eight-figure. The corrections listed for $J_4(x)$, $x = 10, 13, 20$, and $J_6(x)$, $x = 10$, were verified by the asymptotic formulas as well as the computation on SEAC.

WILLIAM F. CAHILL

National Bureau of Standards
Computation Laboratory
Washington, D. C.

38[S, X].—R. A. SPARKS, R. J. PROSEN, F. H. KRUSE, & K. N. TRUEBLOOD, "Crystallographic calculations on the high-speed digital computer SWAC," *Acta Cryst.*, v. 9, 1956, p. 350–358.

The paper contains descriptions and flow diagrams for structure-factor calculations, Fourier summations, differential Fourier summations, and a least-squares refinement used in crystallographic calculations on SWAC. The codes are constructed to accept any space group and to be suitable for crystals containing up to two hundred different anisotropic atoms or one thousand isotropic atoms. The discussion of the differential Fourier summations is not as complete as that of the other parts of the calculation.

The authors note that, in addition to these major routines, they have complied several short routines for special purposes. These include correction factors for Weissenberg intensity data, calculation of interatomic distances and angles, and location of maxima on Fourier syntheses.

So far as the reviewer knows the largest crystal treated by the method so far was the vitamin B_{12} fragment [1]. The routines have also been used to analyze about thirty other crystals.

C. B. T.

1. D. C. HODGKIN, J. PICKWORTH, J. H. ROBERTSON, J. G. WHITE, K. N. TRUEBLOOD, & R. PROSEN, "The crystal structure of the hexacarboxylic acid derived from B_{12} and the molecular structure of the vitamin," *Nature*, v. 176, 1955, p. 325–328.

39[T, Z].—T. R. NORTON & A. OPLER, *A Manual for Coding Organic Compounds for Use with a Mechanized Searching System*, The Dow Chemical Co., Pittsburg, Calif., 1953 (Revised 1956), 56 p., 28 cm.;

A. OPLER & T. R. NORTON, *A Manual for Programming Computers for Use with a Mechanized System for Searching Organic Compounds*, The Dow Chemical Co., Pittsburg, Calif., 1956, 23 p., 28 cm.; and "New speed to structural searches," *Chem. and Eng. NEWS*, v. 34, 1956, p. 2812–2816.

One of the many applications of modern computers which particularly interests the chemist is the mechanization of searching the ponderous accumulation of literature in his field. It will be some time before the ultimate electronic library is achieved, but significant steps are being made and the work of Opler and Norton is one of them. They have devised a technique for "For Coding Organic Compounds" and put it to practice with two recent computers, the DATATRON and the I.B.M. 701. Their purpose was to rapidly search the thousands of known organic chemical compounds for correlating physical, chemical, or biological properties with structure. The results of their efforts to date are summarized in an article in Chemical and Engineering News [1] and the techniques are elaborated in the two manuals available from The Dow Chemical Company. The article, "New Speed to Structural Searches," provides a good introduction to and review of the problems and how they are being solved. The heart of the rapid searching technique is the coding system employed to translate the organic structure into numerical form. While a number of systems have been devised, the system of Opler and Norton represents a relatively complete reduction of organic structure notation to numerical form. This, in effect, makes the coding of searching and related routines a straightforward task for most computers.

The coding system detailed in the first manual is built on a sequence of seven-digit numbers. There will be as many of these numbers in a sequence as there are chemical groups in the compound being coded. The system is designed to handle most types of organic compounds now known and is sufficiently flexible to accommodate some of the special classes of compounds currently omitted. The coding manual is clear, concise, and adequately stocked with examples. However, in the words of the authors, "The ultimate value of this convention (of coding) can only be shown by operating experience."

The manual for programming the computer gives a general outline of the type of program which the authors prepared to process the coded compounds and to search the file. Due to the various modes of programming now employed in current computers the manual was written very generally. It is difficult to decide how general or specific to make such a manual. From the point of view of those chemists who have done some coding, the compound notation system would become more acceptable if a few extra pages of detailed example were appended to the coding manual. Aside from that small point the manual very well outlines the structure of the programs.

Opler and Norton are to be commended for their fine work in this and other applications of computers to chemical problems and The Dow Chemical Company should also be commended for making this information generally available.

F. H. KRUSE

759 Warren Street
Westfield, New Jersey

40[T, Z].—THOMAS B. DREW & JOHN W. HOOPES, JR., editors, *Advances in Chemical Engineering*, Academic Press, Inc., 1956, x + 448 p. Price $10.00.

It is most amazing that the use of computers has not been appreciated by the chemical industry, in contrast, say, with the aircraft industry which has been using them for decades. Nobody would build an airplane without the aid of a computer, yet plants are still built with a slide rule. It is a pity that a book of this title should not have a chapter on the use of computers in chemical engineering, for there are innumerable places where automatic computing methods would advance engineering analysis very profitably. One can quote, for example, where the use of the equation of state instead of the perfect gas law has revolutionized certain engineering designs. Perhaps it is the wide variety of possible applications which has failed to arouse the interest of the chemical engineer.

This book has one chapter on computers, written by Robert Schrage, devoted to their application to the control and planning of manufacturing operations. One gets the impression that this is an apology for computers, and that the applications have been very spotty. No large industry relies on a computer for control of its operation.

With this background it is not surprising that this chapter is very general in its discussion of computing equipment, and serves merely as a source of references. The manufacturer or engineer who wants guidance in getting started will find no facts about specific machines in regard to capability, speed, reliability, etc. Not one of the large computers is mentioned by name or number, and it is to be hoped that the chemical industry is not, at this late date, going to be founded on the IBM, CPC, which is the only machine mentioned.

Four sections of the chapter are devoted to more or less detailed descriptions of certain applications. No information is given regarding the time or cost of carrying out these applications, and whether they were economically a success. Indeed, most of the examples were merely exploratory. The author misses an important point, if he is attempting to stimulate interest in computers, among chemical engineers, in his discussion of blending. The model given is so naive that it has no practical application. There are "technological complications." Why not point out that an automatic computer can permit the introductions of great complexity, to make a model realistic, far beyond the capabilities of manual methods?

Most of the examples, Monte Carlo method, factorial designs, method of steepest descents, and linear programming are more truly problems in operations research, and are more fully treated in books on that subject. This chapter is not specific as to how one actually puts these problems on a computer.

For general review of what is being done with computers in chemical engineering, the chapter in this book is readable, concise, and clear. The young engineer will not find much specific or stimulating. The computer will find no unusual applications, but the account of the computing approach to the method of steepest descents in sequential (statistical) design, and optimization in the presence of restrictions, are quite readable introductions.

GILBERT W. KING

International Telemeter Corp.
Los Angeles, California

41[W, X, Z].—M. V. WILKES, *Automatic Digital Computers*, John Wiley & Sons, Inc., New York, 1956, x + 305 p., 22 cm. Price $7.00.

In the rapidly changing field of automatic digital computers an occasional summing up and surveying of the prospect is necessary. This book introduces the reader to the principles underlying the design and use of present-day computers. This is done by examples of the construction, design, and application of existing machines. A general treatment of logical design and programming is coupled with a description of many types of storage mediums and arithmetic units. Numerical analysis is not treated.

The subject is introduced by a historical chapter on the development of computers, but even at this stage the author introduces logical elements and describes flipflops, gates, and counters in some detail as applied in the circuits of ENIAC and EDVAC. The principles of logical design he derives from a detailed study of EDSAC, the serial machine at Cambridge. Indeed, throughout the book, his emphasis in each instance is on the serial machine, followed by a discussion of the corresponding parallel logic. This is not a fault, but undoubtedly stems from his experience with serial computers.

A chapter is devoted to relay computers. The author admits that these have probably reached their ultimate in development due to the physical limitations of the relays themselves, but feels that a study of relay computing circuits will aid in the general field of computing machine design. Here again, logical principles are demonstrated by their application in existing machines: the Bell Telephone Laboratory computers and the Harvard computers.

Storage forms discussed in detail include: ultrasonic, both solid and liquid, electrostatic, and magnetic (wire, tape, drum, and core). Electronic switching circuits are treated systematically in a separate chapter, although descriptions of computer circuits occur at many places throughout the text.

The treatment of programming is quite complete, with examples again derived from EDSAC material. It includes machine language coding, master and interpretive routines, subroutines, and symbolic, or relative address coding. It is a thorough general discussion of basic coding practice and present trends toward more elaborate programs. A final chapter discusses the problems of choosing a design for a computer and the organization of a computing center. The author's opinions are clearly derived from extensive experience with both computers and operating personnel. He has devoted a few pages to an Appendix, entitled, "Machinery and Intelligence," which settles, as far as he is concerned, the question as to whether machines can "think."

The book is not intended as a textbook. However, with the aid of the annotated bibliography to provide additional subject matter in connection with certain details of machine construction or program arrangement, as needed, it might prove acceptable as such in an introductory course on digital computers.

One criticism may be launched against the discussion of storage devices. Punched paper tape, punched cards, and various forms of printers and photographic storage devices are discussed under the heading of logical design. They could well have been treated in more detail in the chapter on storage devices, or separately, as input and output devices.

Another difficulty lies in the heavy emphasis on EDSAC, particularly in the chapter on program construction. The problem of having to specialize the subject matter in favor of one computer instead of another is one which any author in this field must face. The reader, familiar though he may be with some computer, will generally be forced to learn the language of another. The choice of machine for illustration will be made, then, dependent on the author's experience, as in this case.

But these are minor criticisms of what is, in fact, a very complete account of the subject. For the layman, and for most mathematicians and engineers, this book presents a lucid, readable summary of computer design and operating principles. An adequate number of diagrams supports the descriptive material.

The list of chapters follows: 1. The Development of Automatic Digital Computers, 2. The Principles of Logical Design, 3. The Principles of Programme Construction, 4. Relay Computers, 5. Storage, 6. Electronic Switching and Computing Circuits, 7. The Design and Operation of Digital Computers, Appendix: Machinery and Intelligence, Annotated Bibliography, and Index.

F. H. HOLLANDER

University of California
Los Angeles, California

42[X].—NBS Applied Mathematics Series, No. 12, *Monte Carlo Method. Proceedings of a Symposium held on June 29, 30, and July 1, 1949*, in Los Angeles, California, under the sponsorship of the RAND Corporation, and the National Bureau of Standards, with the cooperation of the Oak Ridge National Laboratory (A. S. Householder, editor). U. S. Gov. Printing Office, Washington, D. C., 1951, vii + 42 p, 26.0 cm. Price 30¢.

Contents: Preface by J. H. Curtiss; Foreword by A. S. Householder; "Showers produced by low-energy electrons and photons," by R. R. Wilson; "An alignment chart for Monte Carlo solution of the transport problem," by B. I. Spinrad, G. H. Goertzel, and W. S. Snyder; "Neutron age calculations in water graphite, and tissue," by A. S. Householder; "Methods of probabilities in chains applied to particle transmission through matter," by W. C. DeMarcus and Lewis Nelson; "Stochastic methods in statistical mechanics," by G. W. King; "Report on a Monte Carlo calculation performed with the ENIAC," by Maria Mayer; "Calculation of shielding properties of water for high energy neutrons," by P. C. Hammer; "A Monte Carlo technique for estimating particle attenuation in bulk matter," by B. A. Shoor, L. Nelson, W. DeMarcus, and R. L. Echols; "Estimation of particle transmission by random sampling," by H. Kahn and T. E. Harris; "History of RAND'S random digits—Summary," by G. W. Brown; "The mid-square method of generating digits," by P. C. Hammer; "Generation and testing of random digits at the National Bureau of Standards, Los Angeles," by G. E. Forsythe; "Various techniques used in connection with random digits," by J. von Neumann; "Round table discussion" (chaired by J. W. Tukey); summary by H. H. Germond.

Four of the papers deal with random or pseudo-random digits and most of the others deal with the progress of elementary particles through matter.

Although the symposium was held more than seven years ago these proceedings give a good impression of the ideas and uses of the Monte Carlo method. The article by H. Kahn and T. E. Harris (pages 27–30) is especially informative, and describes three techniques for reducing sampling variance. These techniques are: (i) the splitting technique, in which important particles are conceptually split into two; (ii) importance sampling in which the probability distributions of the original model are altered so as to spend more of the sampling effort in important regions of phase space; and (iii) the technique of making a Monte Carlo method as little like Monte Carlo as possible. "That part of the problem that is hard to do analytically is done by sampling, and that part that is easy to do analytically, is so done." J. von Neumann's influence in all these techniques is acknowledged.

The paper on the generation and testing of random digits at Los Angeles contains a slip in the application of the serial test. This slip is one of principle and had been made before and will probably be made again. The principle has since been corrected: see the reviewer's paper [1], and C. B. Tompkins's review of RAND's Million Random Digits [*MTAC*, v. 10, Rev. 11, 1956, p. 40 and 42].

I. J. Good

25 Scott House
Princess Elizabeth Way
Cheltenham, England

1. I. J. Good, "The serial test for sampling numbers and other tests for randomness," Camb. Phil. Soc., *Proc.*, v. 49, 1953, p. 276–284.

43[X].—H. A. Meyer, Editor, *Symposium on Monte Carlo Methods*, held at the University of Florida, March 16 and 17, 1954. John Wiley & Sons, Inc., New York, 1956, p. xvi + 382, 28.5 cm. Price $7.50.

Contents: "An introductory note," by A. W. Marshall; "Generation of pseudo-random numbers," by O. Taussky and J. Todd; "Phase shifts—middle squares—wave equation," by N. Metropolis; "A general theory of stochastic estimates of the Neumann series for the solution of certain Fredholm integral equations," by G. E. Albert; "Neighbor sets for random walks and difference equations," by T. S. Motzkin; "Monte Carlo computations," by Nancy M. Dismuke; "Applications of Monte Carlo methods to tactical games," by S. Ulam; "Conditional Monte Carlo for normal samples," by H. F. Trotter and J. W. Tukey; "Monte Carlo techniques in a complex problem about normal samples," by H. J. Arnold, B. D. Bucher, H. F. Trotter, and J. W. Tukey; "An application of the Monte Carlo method to a problem in gamma ray diffusion," by M. J. Berger; "Stochastic calculations of gamma ray diffusion," by L. A. Beach and R. B. Theus; "Application of multiple-stage sampling procedures to Monte Carlo problems," by A. W. Marshall; "Questionable usefulness of variance for measuring estimate accuracy in Monte Carlo importance-sampling problems," by J. E. Walsh; "Experimental determination of eigenvalues and dynamic influence-coefficients for complex structures such as airplanes," by C. W. Vickery; "Use of different Monte Carlo sampling techniques," by H. Kahn; "A theoretical comparison of the efficiencies of two classical methods and a Monte Carlo method for computing one component of the solution of a set of linear algebraic equations," by J. H. Curtiss; "A description of the generation and testing of a set of random normal deviates," by E. J. Lytle, Jr.; "Machine sampling from given probability distributions," by J. W. Butler; "A Monte Carlo technique for obtaining tests and confidence intervals for insurance mortality rates," by J. E. Walsh; "Experiments and models for the Monte Carlo method," by A. Walther; Bibliography. The bibliography extends from page 284 to 370 and contains many abstracts.

The expression, "Monte Carlo method," has changed its meaning somewhat during the last eight years. At first it described a method of approximating the numerical solutions of ordinary mathematical problems by probabilistic methods. A one-person game or artificial sampling experiment is performed in which the expected score is the required answer and the observed score is an estimate of it. An example is to go for a random walk in order to invert a matrix, or to throw needles on a grating in order to evaluate π. Judging by most of the applications to date the name now describes a similar method for attacking problems that for the most part originate in a probabilistic setting. The methods now use several techniques for the reduction of the variance of the answer. Some of these techniques, such as systematic and stratified sampling, have long been known to statisticians; others are at least partly new. Often the technique can be expressed by saying that the probability model used in the Monte Carlo method is not quite the same as the model from which the model arose. A useful summary of six of these techniques is given in the paper by H. Kahn (p. 146–190). This paper is very readable, and so are most of those following it in the book. But most of the preceding ones are almost in note form and have signs of hurried preparation.

The introductory article contains the following passage: "The Trotter and Tukey paper, with its companion applied paper $\cdots$ [by Arnold, Bucher, Trotter, and Tukey], is probably the most exciting paper in the volume. First, the authors introduce a new variance reducing technique (conditional Monte Carlo) and second, in the applied paper they show that it works very well. It is heartening to see mathematical statisticians really becoming interested in Monte Carlo techniques and using them on their own problems. This paper deserves close study (and needs it) to get the conditional Monte Carlo trick straight. It seems to be more complicated than, for example, the methods discussed by Kahn."

Curtiss's paper gives a detached comparison of the Monte Carlo method, with classical methods, for solving a set of linear algebraic equations. The Monte Carlo method does not compare favourably on the whole. [Cf. J. Todd, "Experiments on the inversion of a 16×16 matrix," NBS Applied Mathematics Series No. 29, 1953, p. 113–115.] On pages 192–3 Curtiss gets close to noticing that Hausdorff summation is in effect a Monte Carlo process.

Four of the papers relate to the generation of pseudo-random numbers and other random variables. One of these methods, the mid-square method, exemplifies how the Monte Carlo method may lead indirectly to new developments in mathematics and technology. For eight-digit binary numbers the mid-square method may be defined by the iteration $u_{n+1} = [u_n^2/16] \pmod{256}$. This example is illustrated (page 31) by means of an oriented linear graph with one vertex for each of the 256 numbers (except that two vertices seem to be missing). As pointed out by Metropolis and Ulam (*Amer. Math. Monthly*, v. 50, 1953, p. 252–253) the frequencies of the numbers of predecessors of the vertices seem to obey a Poisson distribution. Thus the oriented linear graph can be regarded from at least one point of view as a pseudo-random network. This fact may be of interest in mathematical biophysics or in robotology for constructing mathematical models of the central nervous system. If robots are mass-produced it would be advisable for their brains to be pseudo-random networks, and not random ones, so that the study of the behaviour of prototypes would be a reliable guide to the behaviour of marketed models. This advantage of pseudo-random networks over random ones is analogous to the checking advantage that pseudo-random numbers have in ordinary Monte Carlo.

On page 33 there is a table of the number of trees in the oriented linear graphs corresponding to the mid-square method using a binary base and 6(2)20 digits. On page 34 there is a table of the number of trees when there are two and four digits and the base is 2(1)16; six digits, base = 2(1)8; eight digits, base = 2(1); and ten digits, base = 2, 3.

On page 224 there is a table, by Curtiss, of the favourable sizes of matrices for three methods of inversion, namely the Gauss elimination method, the linear iterative method, and a Monte Carlo method. Results are given for the two cases when the maximum sum of the moduli of the elements of a row of the original matrix is 0.5 and 0.9, and for various numbers of significant decimal places in the elements of the inverse.

On pages 239 and 240 there are tables, by Lytle, of the frequencies of the ranges 1.1(0.1)5.9 (sample size 10) and 2.2(0.1)6.4 (sample size 25) for samples of random normal deviates. These frequencies were obtained by a crude Monte Carlo method or artificial sampling experiment. The total frequencies for the two experiments were respectively 2500 and 1000.

I. J. Good

25 Scott House
Princess Elizabeth Way
Cheltenham, England

44[X, Z].—*Numerical Analysis*, v. 6, *Proceedings of the Sixth Symposium in Applied Mathematics of the American Mathematical Society*, held in Santa Monica, Calif., Aug. 26–28, 1953, edited by John W. Curtiss (Editorial Committee: R. V. Churchill, Erich Reissner, A. H. Taub), McGraw-Hill Book Co., New York, 1956, vi + 303 p. Price $9.75.

This book contains papers presented at the Sixth Symposium in Applied Mathematics of the American Mathematical Society. The subject of this Symposium was Numerical Analysis, the Symposium being held at Santa Monica City

College, Santa Monica, California, August 26 to 28, 1953. The National Bureau of Standards was a cosponsor, John H. Curtiss, the editor.

With its 19 papers the book gives a vivid survey of large parts of modern applied mathematics. It is a demonstration of the rapid changes having taken place in the field of applied mathematics during the past fifteen years. All papers refer more or less closely to computations using modern high-speed computing devices. The methods of numerical analysis are evaluated in respect to their applicability to High Speed Computers. Several papers deal with the problem of the accuracy of the results obtained, a question which is of special importance for more extensive computations. We can here of course only give a short survey of the comprehensive contents.

Systems of linear equations—a very important subject—are dealt with in several contributions. David Young gives a summary of the development for iteration methods in particular and proposes two improvements. The increase in speed of convergence theoretically to be expected is confirmed by computations on ORDVAC. Joseph W. Fischbach brings some applications of the gradient method and applies the methods of steepest descent to ordinary differential equations, Magnus R. Hestenes uses the cg method (conjugate gradient method) for systems of infinitely many linear equations in Hilbert space, and Paul Rosenbloom gives a comprehensive theory covering partial differential equations, too. Another publication about parabolic equations will follow.

A number of contributions deal with approximation theory. Cecil Hastings, Jr., Jeanne Hayward, and James P. Wong, Jr. report about examinations lasting several years aiming to approximate given functions not only by polynomials but by rational or more general parametric forms (containing, for example, roots) for easier feeding of the functions into computing machines. J. L. Walsh deals with Chebychev's polynomial approximation and Arthur Sard with approximations for derivatives and integrals. Sard includes functions of two independent variables, where the residues are constructed as linear functionals in Banach space and estimations for them thus obtained.

For some time now problems of number theory have been treated on computing machines, e.g., combinatorical problems by R. H. Bruck, and problems of algebraic number theory by Olga Taussky. Progress in the theory of algebraic number fields in particular is hindered by the greatly increased difficulties of numerical examples. Olga Taussky deals with factorization of rational primes, ideal classes, and a problem of Hilbert. Emma Lehmer gives three classes of problems suitable for computing machines: special problems, testing of hypotheses, and research problems. T. S. Motzkin deals with the interesting assignment problems, with normalization, approximate solution, vertex approach and face approach methods, equidistribution problem and related problems. C. Tompkins examines problems whose variables are permutations and reports about the treatment of problems with SWAC. His report includes systematic machine generation of permutations, embedding of permutations in continuous spaces, and the problem of economic computing. Richard Bellman deals with the theory of dynamic programming and obtains a functional equation the approximate solution of which is obtained by successive approximation.

Of great importance is further research in differential equations and related fields. Stefan Bergman points to the kernel functions for solving boundary value problems. They are useful when more then a single boundary value problem has to be solved.

With their aid one can estimate how the solution of a linear partial differential equation changes with changes of the domain for which it is considered, of the coefficients of the differential equation or of the boundary values. The methods are well suited for High Speed Computers. The paper includes numerous very instructive examples. S. P. Frankel reports about the stability of the finite difference method for partial differential equations in special cases. D. R. Clutterham and A. H. Tabu examine the shock configuration in Mach reflection; the method worked out by them stood the test on ILLIAC; two pages of tables give results.

S. E. Warschawski collects different methods of approximative conformal mappings, integral equation methods, mapping of nearly circular domains according to M. S. Friberg, the Rayleigh-Ritz-Method, and the "perturbation method." These methods are theoretically examined, in some cases estimations of error are given and tested numerically for ellipses.

Helmut Wielandt gives an important methodical estimate of the eigenvalues for second kind integral equations with symmetric kernel. An error estimate is carried out for approximations obtained by performing the integration approximately by a formula of numerical quadrature. Special integration formulas are examined and numerical examples given. Wolfgang Wasow deals with the problem of asymptotic transportations of certain distributions into the normal distribution. His examinations include Student's and the χ^2 distribution and possibly other distributions of interest.

Thus the volume presents very interesting aspects of a number of modern research areas.

L. COLLATZ

University of Hamburg
Hamburg 19, Harvestehuderweg 10
Germany

45[X, Z].—CLARENCE L. JOHNSON, *Analog Computer Techniques*, McGraw-Hill Book Co., New York, 1956, 280 p. Price $6.00.

This book was written as a textbook for a course in the operation of an electronic differential analyzer. Thus, its objectives are highly specialized; it does not treat network analyzers, electro-mechanical differential analyzers, or any of the other analog computers. Furthermore, the author is interested in techniques of using analog computers, and only passingly in techniques of building such computers; in particular, he limits his discussion of engineering to the information which he feels is essential in proper application of the computer. In this application, however, he is insistent that the operator "feel" the way in which the machine actually functions analogously to the operation being simulated. Thus, he would like to have the operator understand why a differential equation describes a system being studied and why the same differential equation describes the operation of the analog computer used to find a solution; by bypassing the differential equation, then, the operator understands how the analog computer simulates the system to be studied.

In order to get on with his task, the author limits himself bibliographically to those works which are of almost immediate importance. It is somewhat shocking to find a book on analog computation in whose index the name of Vanevar Bush appears only once. The Massachusetts Institute of Technology is likewise listed only once, and the name of S. H. Caldwell is not in the index. Thus, explicit reference to the great pre-war development of differential analyzer techniques at M. I. T. is almost completely lacking. However, the reviewer can see no real necessity for recalling old times in a strictly utilitarian textbook; the bibliographical omissions were mentioned above to illustrate the utilitarian character of the book and not to criticize the author's scholarship.

The one departure from the restrictions to electronic differential analyzers occurs in the last chapter when incremental digital computers are discussed (the so-called digital integrating differential analyzers). The thought here is that their use is more similar to that of the electronic differential analyzers than to that of the other digital machines.

The material in the text is presented in a lucid and elementary way. It is accompanied by problems for the student and numerous illustrations. Each chapter has a selected list of references, short but seemingly adequate for the material involved.

The reviewer feels that the book well attains its fundamental purpose of presenting the material that must be known to a person who intends to operate a standard electronic differential analyzer effectively.

The table of contents is: Chapter 1. Introduction; Chapter 2. The Linear Computer Components; Chapter 3. Time- and Amplitude-scale factors; Chapter 4. The Synthesis of Servomechanism Systems; Chapter 5. Multiplying and Resolving Servos; Chapter 6. Additional Computer Techniques; Chapter 7. The Representation of Nonlinear Phenomena; Chapter 8. Multipliers and Function Generators; Chapter 9. Miscellaneous Applications of the Electric Analog Computer; Chapter 10. Analog Computer Components and Computer Control; Chapter 11. The Checking of Computer Results; Chapter 12. Repetitive Analog Computers; Chapter 13. The Digital Integrating Differential Analyzer.

C. B. T.

46[X].—A. S. HOUSEHOLDER, Supplementary "Bibliography on Numerical Analysis," A. C. M. *Journal*, v. 3, 1956, p. 85–100.

Contains 321 titles supplementing those in the author's book, *Principles of Numerical Analysis.*

C. B. T.

47[X, Z].—GEORGE E. FORSYTHE, "Selected references on use of high-speed computers for scientific computation," *MTAC*, v. 10, 1956, p. 25–27.

See also the author's "A numerical analyst's fifteen-foot shelf," *MTAC*, v. 7, 1953, p. 221–228.

C. B. T.

48[Z].—R. K. RICHARDS, *Arithmetic Operations in Digital Computers*, D. Van Nostrand Co., Inc., New York, 1955, iv + 397 p., 23 cm. Price $7.50.

So far as this reviewer is aware, this important book contains the most complete existing description of the application of Boolean algebra to the logical design of digital computers. It is already a standard handbook of computer design used with profit by design engineers, students of the logic of computers, and students of the logic of such operations as microcoding.

The author prepared the contents as a set of notes to be used in connection with a course offered to engineers in the laboratories of the International Business Machines Corporation. As such it is strictly utilitarian. Its bibliography is incomplete. In the index one does not find the name of von Neumann, and the name of Claud Shannon occurs just once; the Shannon reference credits him with appearing to have pointed out the adaptability of Boolean algebra to the design of switching circuits. Similarly, the index lists thirteen references to IBM, but none to any of the groups which have formed Sperry Rand.

The reviewer finds no quarrel with this detachment and biased reference. However, the lack of complete references precludes use of the book as a reliable basis for estimating the complete state of the science at the time the book was written. The author had a goal which he met well. He wanted to describe design of computing instruments and he turned to the material at hand. Where it was convenient and where it served his purpose he took material from other sources, and to the extent that he had information readily available concerning the ultimate source he seemed to give it. Thus he gives no fewer than a dozen examples of binary adders in block diagram form (using Boolean elements) and discusses their relative merits. He goes into considerable detail concerning the difficulties of designing decimal components and again he illustrates his arguments with many block diagrams. In similar fashion he examines the other every-day problems of the designer of circuits for computers. He claims no new results and hence he feels little need for discussing relations between his results and those of other workers.

There is no discussion of the practical limitations of circuit design. Thus the deterioration of pulses on transmission lines and the effects of such deterioration on the logical functioning of the circuits (with the implications which can be drawn concerning the speed of computation which is safe) are not discussed in detail. Indeed, only the most idealized engineering realizations of the Boolean elements are discussed at all. These realizations are limited to vacuum tube circuits with no discussion of logical elements built of transistors and magnetic cores.

Similarly, the reduction of Boolean expressions to most economical form is discussed by implication only and no thorough analysis is attempted.

These omissions are not shortcomings, for the book is a thorough exposition by example of much of the formalism which is necessary for an effective application of knowledge in these omitted fields to the design of computers. Thus, there is no better place for an engineer or other person not already well informed concerning symbolic logic and its application to switching circuits to learn how to combine the components which may be developed from new electronic devices (such as transistors) into effective computing instruments. Similarly, there is no other source which seems to give as easy access to the ideas involved in studies of fast and complex circuitry (such as [1], for example).

The author has included a chapter on computer organization and control and one programming. These are fairly superficial accounts suitable for project engineers but certainly not sufficiently complete to serve as serious expositions of these complicated subjects. Presumably these chapters were added to describe the general machine to the engineer working on its components. However, the subjects treated are not completely within the scope of Boolean algebra (as the author notes on p. 339) and hence not completely within the scope of this book.

In short, the author has prepared a well-directed set of notes for use as a practical handbook for anyone interested in the inside of an electronic computing instrument. The book contains no problems for solution by the reader, but otherwise it is entirely suitable for use as a textbook for a course covered by the material accorded attention.

The table of contents follows. Chapter 1. Symbolic Representation of Quantities, Chapter 2. Boolean Algebra Applied to Computer Components, Chapter 3. Switching Networks, Chapter 4. Binary Addition and Subtraction, Chapter 5. Binary Multiplication and Division, Chapter 6. Decimal Codes, Chapter 7. Counting, Binary and Decimal, Chapter 8. Decimal Addition and Subtraction, Chapter 9. Decimal Multiplication and Division, Chapter 10. Miscellaneous Operations, Chapter 11. Computer Organization and Control, Chapter 12. Programming, Bibliography, and Index.

C. B. T.

1. A. Weinberger & J. L. Smith, "A One-Microsecond Adder Using One-Megacycle Circuitry," IRE Trans. on Electronic Computers, v. EC-5, 1956, p. 65–73. This article also appears under the title, "The Logical Design of a 1-Microsecond Parallel Adder using 1-Megacycle Circuitry," in Western Joint Computer Conference, *Proc.*, Feb. 7–9, 1956, San Francisco, California, sponsored by The Am. Inst. of Elec. Engineers, The Assn. for Computing Machinery, and the Inst. of Radio Engineers. Pub. by Am. Inst. of Elec. Engineers, New York, 1956, p. 103–108.

TABLE ERRATA

Reviews in this issue mention errata in the following works:

D. K. C. MacDonald & Lois T. Towle, "Integrals of interest in metallic conductivity," Review **37**, p. 38–39.

Michio Takashima, "Tables for testing randomness by means of lengths of runs," Review **27**, p. 33.

NOTES

Societa Italiana per il progresso delle scienze

The 46th congress of this Society met in Sicily, 15–21 September 1956. During the meeting the fiftieth anniversary of its foundation was celebrated. In the inaugural session, which was attended by Prime Minister Segni, Professor Mauro Picone paid an eloquent tribute to Vito Volterra, who was one of the original group which founded the SIPS. Professor Picone, who is the Director of the Istituto Nazionale del Applicazioni del Calcolo, organized the Mathematics Section of the Congress, the theme of which was the progress in mathematical analysis due to automation; the Secretary of the Mathematics Section was Dr.