where k is a fixed constant. Such equations are confronted frequently in physics (Shortley et al [8]).

Finally, two problems remain which are worthy of note. Many physical situations allow y to be zero on the boundary S and a solution which satisfies (6.1) in R is desired. An effective technique in the sense that $U \to u$ as $h \to 0$ is needed. Also, a reasonable method for approximating the error in the numerical solution is wanting. The error bound (4.12) involves unknown quantities and the only work which has been done toward establishing error bounds which can be calculated has been for harmonic functions (Walsh and Young [11, 12]).

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Tables of Values of 16 Integrals of Algebraic-Hyperbolic Type

This paper gives tables of values of the following 16 integrals of algebraic-hyperbolic type.

$$(1) \qquad \begin{matrix} I_k \\ I_k{}^* \end{matrix} = \frac{2^k}{k!} \int_0^\infty \frac{x^k dx}{\sinh 2x \pm 2x}, \quad \begin{matrix} (k \geqslant 1) \\ (k \geqslant 3) \end{matrix}$$

$$(2) \qquad \begin{matrix} II_k \\ II_k{}^* \end{matrix} = \frac{2^k}{k!} \int_0^\infty \frac{x^k e^{-2x} dx}{\sinh 2x \pm 2x}, \quad \begin{matrix} (k \geqslant 1) \\ (k \geqslant 3) \end{matrix}$$

(3) $$\begin{matrix} III_k \\ III_k^* \end{matrix} = \frac{2^k}{k!}\int_0^\infty \frac{x^k \tanh x\,dx}{\sinh 2x \pm 2x}, \quad \begin{matrix} (k \geqslant 0) \\ (k \geqslant 2) \end{matrix}$$

(4) $$\begin{matrix} IV_k \\ IV_k^* \end{matrix} = \frac{2^k}{k!}\int_0^\infty \frac{x^k \coth x\,dx}{\sinh 2x \pm 2x}, \quad \begin{matrix} (k \geqslant 2) \\ (k \geqslant 4) \end{matrix}$$

(5) $$\begin{matrix} V_k \\ V_k^* \end{matrix} = \frac{1}{k!}\int_0^\infty \frac{x^k \sinh x\,dx}{\sinh 2x \pm 2x}, \quad \begin{matrix} (k \geqslant 0) \\ (k \geqslant 2) \end{matrix}$$

(6) $$\begin{matrix} VI_k \\ VI_k^* \end{matrix} = \frac{1}{k!}\int_0^\infty \frac{x^k \cosh x\,dx}{\sinh 2x \pm 2x}, \quad \begin{matrix} (k \geqslant 1) \\ (k \geqslant 3) \end{matrix}$$

(7) $$\begin{matrix} VII_k \\ VII_k^* \end{matrix} = \frac{1}{k!}\int_0^\infty \frac{x^k \tanh x \sinh x\,dx}{\sinh 2x \pm 2x}, \quad \begin{matrix} (k \geqslant 0) \\ (k \geqslant 1) \end{matrix}$$

(8) $$\begin{matrix} VIII_k \\ VIII_k^* \end{matrix} = \frac{1}{k!}\int_0^\infty \frac{x^k \coth x \cosh x\,dx}{\sinh 2x \pm 2x}. \quad \begin{matrix} (k \geqslant 2) \\ (k \geqslant 4) \end{matrix}$$

The integral (7) also converges for $k = -1$. It can be shown that

$$\int_0^\infty \frac{\tanh x \sinh x\,dx}{x(\sinh 2x + 2x)} = \ln 3 - \sum_{n=0}^{\infty} \frac{1 - III_{2n}}{(2n+1)2^{2n}} = 0.36236\ 7.$$

With the factors as shown, the integrals tend to unity as the integer k tends to infinity, except II_k and II_k^* both of which tend asymptotically to $2^{-(k+1)}$. The first four integrals, which are called Howland's integrals, arise from the problem of a symmetrically perforated strip or a notched strip (Howland [1],

TABLE 1

k	I_k	II_k	III_k	IV_k	V_k	VI_k	VII_k	$VIII_k$
0	—	—	0.29662 0	—	0.52685 6	—	0.36980 1	—
1	0.76857 5	0.22012 0	0.47443 0	—	0.73884 4	0.91747 7	0.65893 5	—
2	0.76784 7	0.08792 7	0.63084 1	1.09596 7	0.86986 6	0.92259 0	0.83403 0	1.02797 8
3	0.82771 0	0.04334 8	0.75383 6	0.94313 8	0.94005 4	0.95831 1	0.92528 4	0.98312 4
4	0.88350 7	0.02258 3	0.84283 9	0.93632 3	0.97401 0	0.98049 8	0.96827 5	0.98810 2
5	0.92547 6	0.01192 3	0.90323 7	0.95164 4	0.98924 9	0.99154 5	0.98711 6	0.99405 4
6	0.95419 2	0.00628 8	0.94219 2	0.96751 5	0.99571 1	0.99651 5	0.99494 2	0.99736 1
7	0.97269 9	0.00329 5	0.96631 5	0.97953 8	0.99833 7	0.99861 5	0.99806 6	0.99890 2
8	0.98412 4	0.00171 3	0.98077 1	0.98763 5	0.99936 9	0.99946 4	0.99927 5	0.99956 1
9	0.99094 9	0.00088 4	0.98920 7	0.99274 6	0.99976 5	0.99979 7	0.99973 2	0.99983 0
10	0.99492 2	0.00045 3	0.99402 5	0.99583 9	0.99991 3	0.99992 4	0.99990 2	0.99993 5
11	0.99718 9	0.00023 1	0.99673 0	0.99765 4	0.99996 8	0.99997 2	0.99996 5	0.99997 6
12	0.99846 0	0.00011 7	0.99822 7	0.99869 5	0.99998 9	0.99999 0	0.99998 7	0.99999 1
13	0.99916 4	0.00005 9	0.99904 6	0.99928 3	0.99999 6	0.99999 6	0.99999 5	0.99999 7
14	0.99954 9	0.00003 0	0.99949 0	0.99960 9	0.99999 9	0.99999 9	0.99999 8	0.99999 9
15	0.99975 9	0.00001 5	0.99972 9	0.99978 9	0.99999 9	1.00000 0	0.99999 9	1.00000 0
16	0.99987 1	0.00000 8	0.99985 6	0.99988 6	1.00000 0		1.00000 0	
17	0.99993 2	0.00000 4	0.99992 4	0.99993 9				
18	0.99996 4	0.00000 2	0.99996 0	0.99996 8				
19	0.99998 1	0.00000 1	0.99997 9	0.99998 3				
20	0.99999 0	0.00000 0	0.99998 9	0.99999 1				
21	0.99999 5		0.99999 4	0.99999 5				
22	0.99999 7		0.99999 7	0.99999 7				
23	0.99999 9		0.99999 8	0.99999 9				
24	0.99999 9		0.99999 9	0.99999 9				
25	1.00000 0		1.00000 0	1.00000 0				

TABLE 2

k	I_k^*	II_k^*	III_k^*	IV_k^*	V_k^*	VI_k^*	VII_k^*	$VIII_k^*$
1	—	—	—	—	—	—	1.51115 5	—
2	—	—	2.13561 8	—	1.40879 6	—	1.15546 4	—
3	2.03871 1	0.46071 4	1.41506 3	—	1.10522 6	1.20400 4	1.05738 7	—
4	1.35329 4	0.09931 6	1.19555 3	1.70756 9	1.03438 8	1.05121 3	1.02214 3	1.08171 9
5	1.15686 4	0.03241 3	1.10049 3	1.24000 1	1.01212 3	1.01624 8	1.00860 2	1.02146 1
6	1.07673 0	0.01261 7	1.05358 0	1.10538 5	1.00439 8	1.00556 4	1.00332 5	1.00686 4
7	1.03925 1	0.00539 1	1.02903 4	1.05082 2	1.00161 1	1.00196 5	1.00127 3	1.00233 9
8	1.02053 8	0.00243 3	1.01583 4	1.02560 8	1.00059 1	1.00070 3	1.00048 3	1.00081 7
9	1.01087 0	0.00113 6	1.00864 7	1.01319 9	1.00021 6	1.00025 2	1.00018 1	1.00028 9
10	1.00578 5	0.00054 2	1.00471 5	1.00688 6	1.00007 9	1.00009 1	1.00006 7	1.00010 2
11	1.00308 5	0.00026 3	1.00256 4	1.00361 5	1.00002 9	1.00003 2	1.00002 5	1.00003 6
12	1.00164 5	0.00012 8	1.00139 0	1.00190 3	1.00001 0	1.00001 2	1.00000 9	1.00001 3
13	1.00087 6	0.00006 3	1.00075 0	1.00100 3	1.00000 4	1.00000 4	1.00000 3	1.00000 5
14	1.00046 6	0.00003 1	1.00040 3	1.00052 9	1.00000 1	1.00000 1	1.00000 1	1.00000 2
15	1.00024 7	0.00001 6	1.00021 6	1.00027 8	1.00000 0	1:00000 1	1.00000 0	1.00000 1
16	1.00013 1	0.00000 8	1.00011 5	1.00014 6		1.00000 0		1.00000 0
17	1.00006 9	0.00000 4	1.00006 1	1.00007 7				
18	1.00003 6	0.00000 2	1.00003 3	1.00004 0				
19	1.00001 9	0.00000 1	1.00001 7	1.00002 1				
20	1.00001 0	0.00000 0	1.00000 9	1.00001 1				
21	1.00000 5		1.00000 5	1.00000 6				
22	1.00000 3		1.00000 3	1.00000 3				
23	1.00000 1		1.00000 1	1.00000 2				
24	1.00000 1		1.00000 1	1.00000 1				
25	1.00000 0		1.00000 0	1.00000 0				

Howland and Stevenson [2], Ling [3, 4]). The remaining twelve integrals, save VII_k and $VIII_k^*$, arise from the problem of an unsymmetrically perforated strip (Ling [5]).

The four Howland integrals I_k, I_k^*, II_k, II_k^* were tabulated by Howland [1] and Howland and Stevenson [2] to 5D. In a recent paper by Nelson and the present writer [6] these integrals were recalculated and tabulated to 6D. The values of I_k when k is odd were also tabulated by Nelson [7] to 9D. (However our I_k is designated by Nelson as a_k. Nelson has also calculated the remaining Howland integrals to 9D, but this work is unpublished.) For convenience of reference, values of the four Howland integrals are reproduced in Tables 1 and 2. It will be shown that the remaining twelve integrals can be evaluated in terms of the four Howland integrals.

The four integrals III_k, III_k^*, IV_k, IV_k^* may be evaluated by splitting the integrands and then integrating from zero to infinity,

$$\begin{aligned} \frac{x^k \tanh x}{\sinh 2x \pm 2x} &= \frac{x^k}{\sinh 2x \pm 2x} \pm \frac{x^{k-1}(1 - e^{-2x})}{2(\sinh 2x \pm 2x)} \mp \frac{x^{k-1}e^{-x}}{2\cosh x}, \\ \frac{x^k \coth x}{\sinh 2x \pm 2x} &= \frac{x^k}{\sinh 2x \pm 2x} \mp \frac{x^{k-1}(1 + e^{-2x})}{2(\sinh 2x \pm 2x)} \pm \frac{x^{k-1}e^{-x}}{2\sinh x}. \end{aligned} \tag{9}$$

The integrals become

$$\begin{aligned} III_k &= I_k + (I_{k-1} - II_{k-1} - s_k)/k, \\ III_k^* &= I_k^* - (I^*_{k-1} - II^*_{k-1} - s_k)/k, \\ IV_k &= I_k - (I_{k-1} + II_{k-1} - S_k)/k, \\ IV_k^* &= I_k^* + (I^*_{k-1} + II^*_{k-1} - S_k)/k, \end{aligned} \tag{10}$$

where

$$(11)\qquad \begin{aligned} s_k &= \frac{2^{k-1}}{(k-1)!}\int_0^\infty \frac{x^{k-1}e^{-x}dx}{\cosh x} = 1 - \frac{1}{2^k} + \frac{1}{3^k} - \frac{1}{4^k} + \cdots, \\ S_k &= \frac{2^{k-1}}{(k-1)!}\int_0^\infty \frac{x^{k-1}e^{-x}dx}{\sinh x} = 1 + \frac{1}{2^k} + \frac{1}{3^k} + \frac{1}{4^k} + \cdots. \end{aligned}$$

The values of s_k and S_k were tabulated by Glaisher [8].

Next, consider the four integrals V_k, V_k^*, VI_k, VI_k^*. By expanding $\sinh x$ into power series of x, the integral V_k develops into the series

$$(12)\qquad V_k = \sum_{n=0}^{\infty} \binom{2n+k+1}{k} \frac{I_{2n+k+1}}{2^{2n+k+1}}.$$

In order to improve the convergence, Kummer transformation (Knopp [9]) may be used. The series then becomes

$$(13)\qquad V_k = 1 - \frac{1}{3^{k+1}} - \sum_{n=0}^{\infty} \binom{2n+k+1}{k} \frac{1 - I_{2n+k+1}}{2^{2n+k+1}}.$$

Similarly, the integral V_k^* develops into the series

$$(14)\qquad V_k^* = 1 - \frac{1}{3^{k+1}} + \sum_{n=0}^{\infty} \binom{2n+k+1}{k} \frac{I^*_{2n+k+1} - 1}{2^{2n+k+1}}.$$

By expanding $\cosh x$ into power series of x, the following series are obtained in the same way,

$$(15)\qquad \begin{aligned} VI_k &= 1 + \frac{1}{3^{k+1}} - \sum_{n=0}^{\infty} \binom{2n+k}{k} \frac{1 - I_{2n+k}}{2^{2n+k}}, \\ VI_k^* &= 1 + \frac{1}{3^{k+1}} + \sum_{n=0}^{\infty} \binom{2n+k}{k} \frac{I^*_{2n+k} - 1}{2^{2n+k}}. \end{aligned}$$

The last four integrals may be evaluated by splitting the integrands into the following and then integrating from zero to infinity,

$$(16)\qquad \begin{aligned} \frac{x^k \tanh x \sinh x}{\sinh 2x \pm 2x} &= \frac{x^k \cosh x}{\sinh 2x \pm 2x} \pm \frac{x^{k-1}\sinh x}{\sinh 2x \pm 2x} \mp \frac{x^{k-1}}{2\cosh x}, \\ \frac{x^k \coth x \cosh x}{\sinh 2x \pm 2x} &= \frac{x^k \sinh x}{\sinh 2x \pm 2x} \mp \frac{x^{k-1}\cosh x}{\sinh 2x \pm 2x} \pm \frac{x^{k-1}}{2\sinh x}. \end{aligned}$$

The integrals become

$$(17)\qquad \begin{aligned} VII_k &= VI_k + (V_{k-1} - u_k)/k, \\ VII_k^* &= VI_k^* - (V^*_{k-1} - u_k)/k, \\ VIII_k &= V_k - (VI_{k-1} - U_k)/k, \\ VIII_k^* &= V_k^* + (VI^*_{k-1} - U_k)/k, \end{aligned}$$

where

$$(18)\qquad \begin{aligned} u_k &= \frac{1}{2(k-1)!}\int_0^\infty \frac{x^{k-1}dx}{\cosh x} = 1 - \frac{1}{3^k} + \frac{1}{5^k} - \frac{1}{7^k} + \cdots, \\ U_k &= \frac{1}{2(k-1)!}\int_0^\infty \frac{x^{k-1}dx}{\sinh x} = 1 + \frac{1}{3^k} + \frac{1}{5^k} + \frac{1}{7^k} + \cdots. \end{aligned}$$

The values of U_k and u_k were also tabulated by Glaisher [8, 10].

The preceding method of evaluation is satisfactory as far as it goes, except that it does not cover the initial integrals III_0, III_1, III_2^*, III_3^*, VII_0, VII_1^*, and VII_2^*.

The integrals III_0 and III_1 may be evaluated by expanding $\tanh x$ into the following series and then integrating from zero to infinity,

$$(19)\qquad \tanh x = \frac{1}{\sinh 2x}\sum_{n=1}^{\infty}\frac{(2x)^{2n}}{(2n)!}.$$

This leads to

$$(20)\qquad \begin{aligned} III_0 &= \sum_{n=1}^{\infty} (U_{2n} - I_{2n-1})/2n, \\ III_1 &= \sum_{n=1}^{\infty} (U_{2n+1} - I_{2n}). \end{aligned}$$

An alternative expression for III_1 may be obtained as follows. By using the expansion

$$(21)\qquad \sinh x = \tanh x \sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!},$$

the integral V_k develops into the series

$$(22)\qquad V_k = 1 + \frac{1}{3^{k+1}} - \sum_{n=0}^{\infty}\binom{2n+k}{k}\frac{1 - III_{2n+k}}{2^{2n+k}},$$

which gives when $k = 1$,

$$(23)\qquad III_1 = \frac{7}{9} - 2(1 - V_1) + \sum_{n=1}^{\infty}\frac{2n+1}{2^{2n}}(1 - III_{2n+1}).$$

The integrals III_2^* and III_3^* may be evaluated from the following series which is obtained in the same way,

$$(24)\qquad V_k^* = 1 + \frac{1}{3^{k+1}} + \sum_{n=0}^{\infty}\binom{2n+k}{k}\frac{III^*_{2n+k} - 1}{2^{2n+k}}.$$

When $k = 2$ and 3 respectively, this series gives

$$(25)\qquad \begin{aligned} III_2^* &= \frac{23}{27} + 4(V_2^* - 1) - \sum_{n=1}^{\infty}\binom{2n+2}{2}\frac{III^*_{2n+2} - 1}{2^{2n}}, \\ III_3^* &= \frac{73}{81} + 8(V_3^* - 1) - \sum_{n=1}^{\infty}\binom{2n+3}{3}\frac{III^*_{2n+3} - 1}{2^{2n}}. \end{aligned}$$

By expanding $\sinh x$ into power series of x, the integral VII_0 develops into the series

$$(26)\qquad VII_0 = \frac{2}{3} - \sum_{n=0}^{\infty}\frac{1 - III_{2n+1}}{2^{2n+1}}.$$

Similarly, the integral VII_k^* develops into the series

$$VII_k^* = 1 - \frac{1}{3^{k+1}} + \sum_{n=0}^{\infty} \binom{2n+k+1}{k} \frac{III^*_{2n+k+1} - 1}{2^{2n+k+1}}. \tag{27}$$

When $k = 1$ and 2 respectively, it gives

$$\begin{aligned} VII_1^* &= \frac{8}{9} + \sum_{n=1}^{\infty} \frac{2n}{2^{2n}} (III^*_{2n} - 1), \\ VII_2^* &= \frac{26}{27} + \sum_{n=1}^{\infty} \frac{2n(2n+1)}{2^{2n+2}} (III^*_{2n+1} - 1). \end{aligned} \tag{28}$$

Values of the twelve integrals thus computed and rounded off to six decimals, iusng Nelson's tables [7] when necessary, are given in Tables 1 and 2. The followngi formulas are useful for checking purposes:

$$\begin{aligned}
2(1 - III_1) + \sum_{k=1}^{\infty} (III^*_{2k+1} - III_{2k+1}) &= 2, \\
(1 - III_2) + \sum_{k=2}^{\infty} \frac{k}{2} (III^*_{2k} - III_{2k}) &= 1, \\
(1 - IV_2) + \sum_{k=2}^{\infty} \frac{k}{2} (IV^*_{2k} - IV_{2k}) &= 1, \\
(1 - IV_3) + \sum_{k=2}^{\infty} \frac{k(2k+1)}{6} (IV^*_{2k+1} - IV_{2k+1}) &= 1, \\
\sum_{k=1}^{\infty} k(VI_k - V_k) &= \tfrac{1}{2} I_1, \\
\sum_{k=0}^{\infty} (2k+1)(VI_{2k+1} - VII_{2k+1}) &= \tfrac{1}{2} I_1, \\
\sum_{k=1}^{\infty} k(VIII_{2k} - V_{2k}) &= \tfrac{1}{4} I_1, \\
\sum_{k=1}^{\infty} k(2k-1)(VI_{2k} - VII_{2k}) &= \tfrac{1}{4} I_2, \\
\sum_{k=1}^{\infty} \frac{k(2k+1)}{3} (VIII_{2k+1} - V_{2k+1}) &= \tfrac{1}{12} I_2, \\
\sum_{k=3}^{\infty} \frac{k(k-1)(k-2)}{3} (VI_k^* - V_k^*) &= \tfrac{1}{4} I_3^*, \\
\sum_{k=2}^{\infty} \frac{k(k-1)}{2} (V_k^* - VII_k^*) &= \tfrac{1}{4} III_2^*, \\
\sum_{k=4}^{\infty} \frac{k(k-1)(k-2)(k-3)}{24} (VIII_k^* - VI_k^*) &= \tfrac{1}{16} IV_4^*.
\end{aligned} \tag{29}$$

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Tables for the Rapid and Accurate Numerical Evaluation of Certain Infinite Integrals Involving Bessel Functions

Introduction. In a recent paper [1] the author has formulated a method based on Euler's transformation of slowly convergent alternating series for the numerical evaluation of integrals of the form $\int_a^\infty f(x)dx$, where a is a constant and where $f(x)$ oscillates about zero in such a way that the integral over each half-cycle is smaller in absolute magnitude than (and opposite in sign to) that over the preceding half-cycle. The author has had occasion to make much use of this method for the evaluation of integrals of the type

$$\int_0^\infty J_0(x)g(x)dx, \tag{1}$$

and

$$\int_0^\infty J_1(x)h(x)dx, \tag{2}$$

where $g(x)$, $h(x)$ are well-behaved continuous functions which tend to a finite constant value or zero as x tends to infinity; here $J_0(x)$, $J_1(x)$ are Bessel Functions (of the first kind) of orders zero and one, respectively. The present paper gives tables useful in the evaluation of (1), (2).

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