REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

13[D, Z].—E. G. Kogbetliantz, "Computation of Arcsin N for $0 < N < 1$ using an electronic computer," IBM, *Journal of Research*, v. 2, No. 3, 1958, p. 218–222.

All known subroutines for Arcsine are based on the relation Arcsin N = Arctan $N/(1 - N^2)^{1/2}$. Therefore, Arcsine is not computed as such, but as an Arctangent.

To avoid the loss of machine time caused by the computation of $N/(1 - N^2)^{1/2}$, a direct computation of Arcsine is proposed. A subroutine yielding the first six correct significant digits in only five multiplications and divisions is described in detail to illustrate the new method's rapidity. The same number of five operations is necessary to compute, knowing N, the number $N/(1 - N^2)^{1/2}$.

Author's Summary

14[D, Z].—E. G. Kogbetliantz, "Computation of Arctan N for $-\infty < N < +\infty$ using an electronic computer," IBM, *Journal of Research*, v. 2, No. 1, 1958, p. 43–53.

Rational (R) and polynomial (P) approximations to Arctan N are studied with the aim of computing this function, to any prescribed accuracy and without unduly increasing the number PC of stored constants, in a minimum number M of multiplications (and divisions for R approximations). The number Dg of first correct significant digits in principle is not bounded. The results corresponding to the values 8, 10, 18 and 20 of this number are as follows:

Approximation	Point (Computation)	Single Precision			Double Precision		
		Dg	*M*	*PC*	*Dg*	*M*	*PC*
Rational (R)	Floating	8	4	21	18	6	19
			5	9		7	17
	Fixed	10	5	14	20	6	30
			6	9		7	18
Polynomial (P)	Floating	8	5	10	17	8	21
			6	8	18	9	14
	Fixed	10	6	11	20	9	22
			7	9		10	15

If M is increased, subroutines with smaller PC are easily deduced from our general results. Thus, for instance, rational approximations with $Dg = 6$ can be obtained in three multiplications only, if $PC = 19$, but the same accuracy $Dg = 6$ characterizes also the cases $M = 4$ with $PC = 11$, and $M = 5$ with $PC = 7$.

If polynomial approximations are used, $Dg = 6$ is obtained for $M = 5$, $PC = 7$, but also for $M = 4$ and $PC = 11$. No subroutines with a stored table of values of Arctan x are considered.

Author's Summary

15[E, Z].—E. G. Kogbetliantz, "Computation of e^N for $-\infty < N < +\infty$ using an electronic computer," IBM, *Journal of Research*, v. 1, No. 2, 1957, p. 110–115.

Rational (R) and polynomial (P) approximations to the exponential function e^N are studied. They allow e^N to be computed for any value of the exponent N in the infinite range from minus infinity to plus infinity in a minimum number M of multiplications (and divisions, for the rational approximation). This minimum is attained without unduly increasing the number PC of precomputed and stored constants and also without limiting the number Dg of the first correct significant digits. The main results are presented in the following table.

Machine	Approx.	Computation	Single Precision			Double Precision		
			M	PC	Dg	M	PC	Dg
Binary	R	floating point	3	19	9	5	21	19
			4	8	10	6	8	18
		fixed point	3	35	10	6	10	21
			4	8	10			
Decimal	P	floating point	5	17	8	8	30	18
						9	24	17
		fixed point	5	25	10	9	32	20
			6	19	10			

Author's Summary

16[F].—A. Gloden, *Table de Factorisation des Nombres $N^4 + 1$ dans l'intervalle $10000 < N \leqq 20000$.* Manuscript of 50 leaves deposited in UMT File.

The present table completes that of the same title reviewed in *MTAC*, v. IX, p. 26. In the present one, thanks to the collaboration of Dr. N. G. W. H. Beeger, a considerable number of large factors less than $64 \cdot 10^{10}$, identified as prime numbers, have been entered.

The machine calculation was performed in the following manner. The number $N^4 + 1$ or $(N^4 + 1)/2$ was divided by the product of the known prime factors, then the quotient was divided successively by each of these factors to ascertain if any was a multiple factor. If none of these last divisions was exact, the quotient was identified as a prime number.

Some typographical errors appearing in the previous manuscript table have been eliminated.

My "Table des solution de la congruence $x^4 + 1 \equiv 0$, mod p, pour $8 \cdot 10^5 < p < 10^6$" is in preparation. This will permit the entry in the present table of the prime factors occurring in that interval.

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17[F].—Hansraj Gupta, C. E. Gwyther & J. C. P. Miller, *Tables of partitions*, Royal Society Mathematical Tables, v. 4, Cambridge University Press, 1958, xxxix + 132 p., 28 cm. Price $12.50.

The partition function $p(n, m) = p_1(n, m)$ is defined as the number of partitions of n into parts not exceeding m. More generally $p_s(n, m)$ for $s > 1$ is the number of partitions of n into parts where there are s varieties of each of the integers $1, \cdots, m$ and $s - 1$ varieties of the integers greater than m.

These tables are four in number, giving respectively the values of $p(n, m)$, $p_2(n, m)$, $p_3(n, m)$ and $p_4(n, 0)$. Each table provides a partial check on the preceding table. Thus, the formula

$$P(n, m) = \sum_{r=0}^{R} (-1)^r p_2 \left(n - r\left(m + \left(\frac{1}{2}r\right) + \frac{1}{2}\right), r\right)$$

where R is the largest value of r for which the first parameter is not negative, provides a check on Table I from Table II, and may be used to calculate values of $p(n, m)$ outside the range of Table I.

The tables are preceded by a fairly long introduction and an extensive bibliography. The introduction describes the tables, gives historical background, the basic recursions, and exact formulae for $p(n, m)$, $m = 1, \cdots, 12$. The exact formula for $p(n, m)$ is a polynomial of degree $m - 1$ in n, plus terms of lower degree depending on the residues of n with respect to moduli not exceeding m. The exact formulae involve inconveniently large numbers as is illustrated by the calculation of $p(100, 12) = 167\ 13148$. Theoretical discussions and some asymptotic formulae are included.

Table I gives $p(n, m)$ for n from 1 to 200 with m ranging from 0 to min $(n, 100)$, and from $n = 201$ to 400 with m ranging from 0 to 50. Table II gives $p_2(n, m)$ for n from 1 to 1000 with m ranging from 0 to the value indicated by max m below:

Range of n	1–50	50–100	100–150	
max m	n	23	20	
Range of n	150–200	200–250	250–300	
max m	12	11	7	
Range of n	300–350	350–400	400–450	450–500
max m	6	5	4	3
Range of n	500–550	550–1000		
max m	1	0		

Table III gives $p_3(n, m)$

for $n = 1$ to 50 with $0 \leqq m \leqq n$
for $n = 50$ to 100 with $0 \leqq m \leqq 19$
for $n = 100$ to 150 with $0 \leqq m \leqq 6$
and for $n = 150$ to 200 with $m = 0$.

Table IV gives $p_4(n, 0)$ for $n = 1$ to 200.

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18[I, X].—HER MAJESTY'S NAUTICAL ALMANAC OFFICE, *Subtabulation, A Companion Booklet to Interpolation and Allied Tables*, Her Majesty's Stationery Office, London, England, 1958, 54 p., 24.5 cm. Price 7s.6d. net.

Interpolation and Allied Tables was reviewed recently in this journal [1], and it was then pointed out that it contained material essential for every computer and numerical analyst, whether practicing or aspiring. This booklet, in which the printing and layout are excellent, contains material concerning methods of systematically interpolating a function, tabulated at an interval h, to obtain its values at a smaller interval, h/p, where p is an integer. Since this is a problem which is almost certain to be encountered by computers early in their career, they should be aware of efficient ways of dealing with it. Three types of methods are described, and their merits under various conditions, principally with reference to equipment available, are discussed.

I. *Direct Methods*

These methods are the most straightforward: one simply interpolates using his favorite method (or the one most suitable in the circumstances). We describe the tables which are provided, using the notation of the booklets, which is summarized in the earlier review [1].

1. *Bessel's Formula*

The coefficients to $6D$ of the double second difference, and of the third and fourth difference corrections are given in the cases $p = 20$, $p = 24$. [These cases, of course, cover those when p is a factor of 20, or 24.]

2. *Lagrange's Formula*

Six-point coefficients, to $8D$, are given in the cases of $p = 20$, $p = 24$.

3. *Everett's Formula*

Again for $p = 20$, $p = 24$, the coefficients E_2 and F_2 are given to $7D$, M_4 and N_4 to $3D$, E_4 and F_4 to $6D$

4. *Lagrange's Formula*

Ten-point coefficients, to $10D$, are given in the cases $p = 20$, $p = 24$.

5. *High-precision formulae and coefficients*

These cover the cases $p = 2$, $p = 3$ and $p = 10$.

6. *Throw-back formulae*

This is a useful collection of formulae covering various simple, simultaneous and generalized throw backs for the Bessel and Everett formulae.

There are various worked examples and sound, practical advice on detailed points.

II. *Precalculated second differences*

This method is a new development which supersedes the "end-figure" method of Comrie, and is suitable when only desk machines are available and in cases where the sixth difference can be as much as 500,000.

The basic idea is to take the Bessel interpolation in the form

$$f_p = f_0 + {}_p\delta_{\frac{1}{2}} + B_2(\delta_{m0}^2 + \delta_{m1}^2) + B_3\delta_{m\frac{1}{2}}^3 + T_4(\gamma_0{}^4 + \gamma_1{}^4) + T_5\gamma_{\frac{1}{2}}{}^5$$

where, in addition to the notation of the previous review, we have used

$$1000\gamma^5 = \delta^5 - 0.2\delta^7 + \cdots$$

$$T_5 = 10000(B_5 + 0.108B_3).$$

This can be written as

$$f_p = \{f_0 + rA + r(r - n)B\} + \{\tfrac{1}{6}\nu r(r - n)(2r - n)C\} + \{r\nu a + B_2(r\nu)b\} + B_3(r\nu)C + T_4(\gamma_0{}^2 + \gamma_1{}^7) + T_5\gamma_{\frac{1}{2}}{}^5$$

where A, B, C, a, b, c are integers defined by

$$\tfrac{1}{2} = nA + a, \qquad (\delta_{m0}^2 + \delta_{m1}^2) = 4n^2B + b, \qquad \delta_{m\frac{1}{2}}^3 = 2n^3C + c,$$

and where $p = r/n = r\nu$, r being an integer.

Then the terms in the first braces have a constant second difference (with respect to r) of $2B$; the term in the second braces has a constant third difference of $2C$. Tables are given to provide the second differences of the remaining terms, and the leading first differences. From these the values of f_p can be built up. There are worked examples, illustrating two methods of use of the tables.

The tables provided cover the cases $p = 4, 5, 6, 10$.

These methods, although very powerful, are rather complicated, and are not likely to appeal to the more casual user, who will probably be content with the methods described in the first section.

III. *The method of bridging differences*

This is a method which is appropriate for use with multi-register adding machines. There is a discussion of the theory, and a list of formula indicating their limits of applicability and the number of registers needed.

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1. MTAC, Rev. **54**, v. 12, 1958, p.99–103.

19[K].—D. S. Stoller & L. C. Stoller, *Tables of the Coefficients of Certain Linear Predictors.*

Tables are given for the coefficients a_0, a_1, $\cdots$ a_n of the linear predictor $y_{n+1} = a_0 x_0 + \cdots + a_n x_n$, where x_0, x_1, $\cdots$ x_n are observations at $n + 1$ equally spaced points and y_{n+1} is an estimate of the next observation, x_{n+1}. Tables are listed for polynomials of degree 1 to 10 in j of the expected value $E(x_j)$ of x_j and for various numbers of observed points. A detailed description of the assumptions under which these tables are computed and discussion of the method of computation are given in [1]. These tables are on file in the Unpublished Mathematical Tables repository of *MTAC*.

H. P.

1. D. S. Stoller & L. C. Stoller, "Calculating the Coefficients of Certain Linear Predictors," *MTAC*, v. XIII, number 66, April 1959, p. 122–124.

20[L].—Carson Flammer, *Spheroidal Wave Functions*, Stanford University Press, Stanford, Calif., 1957, ix + 220 p., 25.5 cm. Price $8.50.

A condensed, but fairly complete collection of definitions of the angle and radial solutions of the wave equation in prolate and oblate spheroidal coordinates is presented in this book, together with a set of tables relating the notations for the various related functions, which are now in use in the literature. Series expansions and various integral representations for the functions are given. A bibliography of work on these functions is given, and the more important results are brought into line with the definitions used in the present book. A useful chapter is included on the related solutions of the vector wave equation in spheroidal coordinates.

The numerical tables have been adopted from several sources, reworked, and previous errors rectified. Coefficients in the series of powers of c (interfocal distance divided by twice the wave length) for the separation constants λ_{mn} and for the coefficients d_r^{mn} of the expansion of the prolate angle functions in associated Legendre functions $P_{m+r}^m(\eta)$ are given to ten decimal places for $m = 0(1)3$, $n = m(1)9$. The separation constants themselves are given to at least six decimals for $m = 0(1)3$, $n = m(1)3$, $c = 0(.2)5$, and for $n = 1$, $m = 5(2)19$, for a few values of c between 1.2 and 3.2, including $\frac{1}{2}\pi$, $\frac{3}{4}\pi$ and π. A similar tabulation is given of the expansion coefficients d for the angle and radial functions, and also for the coefficients of the expansion of the angle functions in powers of $(1 - \eta^2)$ (η being the angle coordinate in the elliptic coordinates which form the spheroids by axial revolution).

The prolate angle functions are also tabulated, to four decimals, for $m = 0(1)3$, $n = m(1)3$ for $c = 0(.5)5$ and $\theta(= \cos^{-1}\eta) = 0(5°)90°$. Values of the prolate radial functions of the first and second kind, and their derivatives are also given, for these values of m, n and c, for a few values of the radial coordinate ξ near the origin ($\xi = 1$).

Corresponding values for the oblate functions, for a somewhat reduced range of values of c, m and n are also tabulated.

The book should be welcomed by those working in the field. However, this reviewer is sure the author will agree that more work, both analytic and computational, is needed before wave calculations in spheroidal coordinates will be comparatively easy to carry out.

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21[L].—M. E. LYNAM, *Table of Legendre Functions for Complex Arguments*, The Johns Hopkins University Applied Physics Laboratory Report, TG-323, 1958, 4 p., Tables.

The Legendre functions $P_\nu(x)$ and $Q_\nu(x)$ and their first derivatives are tabulated to either 8D or 8S for $x = 0\ (.05)\ .95$ and for values of ν with negative imaginary part that satisfy the equation $\nu\ (1 + \nu) = -iy$, where $y = 0.01\ (.01)\ .10$, $0.2\ (.1)\ 2.5$, and $3\ (1)\ 24$.

An indication of the relative accuracy of the tabulated values is given in a double column headed "Error," which records the (negative) characteristics of the common logarithms of both the real and the imaginary parts of the difference

$$1 - (1 - x^2)\left[P_\nu(x)\frac{d}{dx}Q_\nu(x) - Q_\nu(x)\frac{d}{dx}P_\nu(x)\right]$$

when evaluated by substituting the computed values in the Wronskian determinant. Spurious symbols in this column are due to printing difficulties arising from characteristics numerically greater than 9.

The author states in the text that the calculations were performed on the Univac Scientific 1103A Computer, employing a subroutine previously prepared for calculations of rocket instability.

J. W. W.

22[L].—MORIO ONOE, *Tables of Modified Quotients of Bessel Functions of the First Kind for Real and Imaginary Arguments*, Columbia University Press, New York, 1958, ix + 338 p., 27.3 cm. Price $12.50.

As is generally known, tables of Bessel functions of integral order for real and pure imaginary arguments are now adequate. But since the "modified quotient" $\mathfrak{J}_n(z) = zJ_{n-1}(z)/J_n(z)$ frequently occurs in mathematical physics and engineering, this book gives tables of $\mathfrak{J}_n(x)$ and $\mathfrak{J}_n(ix)$, in both cases for $n = 1(1)16$ and $x = 0(.01)20$, without differences. Most of the computation was done at the Watson Scientific Computation Laboratory, chiefly on an IBM 607.

The tables of $\mathfrak{J}_n(x)$ occupy pages 17–176. The number of significant figures given is normally between eight and ten, and falls below six only in a few cases at single arguments near zeros (or, in one instance, a pole) of the functions tabulated. On page 174, it may save trouble to note that at $n = 14$, $x = 18.90$, the printed value 92338, followed by two spaces and a decimal point, means $9.2338 \cdot 10^6$.

The tables of $\mathfrak{J}_n(ix)$ occupy pages 179–338. As the functions have no zeros or poles in the range of tabulation, the values are given uniformly to eight decimals, which means that there are either nine or ten significant figures.

The short introductory text includes a number of formulas relating to the modified quotient functions.

A. F.

23[L].—JAMES C. WILTSE & MARCIA J. KING, *Values of the Mathieu Functions*, The Johns Hopkins University Radiation Laboratory, Tech. Report No. AF-53, Baltimore, Maryland, 1958, 78 p.

[L].—JAMES C. WILTSE & MARCIA J. KING, *Derivatives, Zeros, and Other Data Pertaining to Mathieu Functions*, The Johns Hopkins University Radiation Laboratory, Tech. Report No. AF-57, Baltimore, Maryland, 1958, 75 p.

The authors have used the notation and normalizations defined in [1]. In the following summary, notation such as $r = 0(1)2$ will imply that $r = 0$ does not apply to functions associated with odd periodic solutions, such as $So_r(s, x)$, $Jo_r(s, x)$.

The following tables appear in the first report under review:

Table I. Periodic Mathieu functions $Se_r(s, v)$ and $So_r(s, v)$, $r = 0(1)2$; $s = 3, 6, 9, 15, 25, 40$; 7 to 13 angles v between 0° and 90°; for $Se_o(s, v)$, the additional parameters $s = 1(1)9$; $Se_r(-s, v)$ and $So_r(-s, v)$, $r = 0, 1$; $s = 3, 6, 9, 15$, $v = 0(15°)90°$; 4D.

Table II. Radial Mathieu functions of the first kind, $Je_r(s, u)$ and $Jo_r(s, u)$: $r = 0(1)2$; $s = 1, 3, 6, 9, 15, 25, 40$, for 15 to 22 values of u less than 2.01, unequal intervals in u; 3D or 4D.

Table III. Radial Mathieu functions of second kind, $Ne_r(s, u)$, $No_r(s, u)$, s up to and including 25; same range of u and format as Table II.

Table IV. Mathieu-Hankel functions $He_r^{(1)}(-s, u)$ and $Ho_r^{(1)}(-s, u)$, $s = 1$, 3, 6, 9, 15; $r = 0(1)2$; 2D to 4D. Format and range of u about as in Table II: for $r = 0$ and 2, $s = 25$ also.

(In all tabulations in both reports, values of the function at $s = 0$ are given to 5 or more significant figures, since they could be readily obtained from the tables in [1].)

An important feature of the report is a set of 39 pages of good graphs from which one may in fact read the magnitude of the functions or better, in the region covered by the tabulation. Quoting the authors: "The tables contain approximately 1750 calculated values of the various Mathieu functions."

The following tables appear in the second report:

Table I. $Se_r'(s, v)$ and $So_r'(s, v)$, $r = 0(1)2$, $s = 1, 3, 6, 9, 15, 25, 40$; $v = 0(15°)90°$; 4D. Also $Se_r'(-s, v)$, $So_r'(-s, v)$, $s = 1, 3, 6, 9, 15$; $v = 0(15°)90°$; 4D.

Table II. $Je_r'(s, u)$ and $Jo_r'(s, u)$, $r = 0(1)2$; $s = 1, 3, 6, 9, 15$; for $r = 1$, $s = 25, 40$ also; 2D–4D. Similar range of u to that in Table II of the first report.

Table III. $Ne_r'(s, u)$, $No_r'(s, u)$, $r = 0(1)2$, s up to and including 15; 2D to 3D.

Table IV. Derivatives with respect to u of $He_r^{(1)}(-s, u)$ and $Ho_r^{(1)}(-s, u)$; 2D–4D; same format and range of u as Table III.

Table V. $Se_2(-s, v)$ and $So_2(-s, v)$; $s = 1, 3, 6, 9, 15$; $v = 0(15°)90°$; 4D.

Table VI. First Zero of $Je_r(s, u)$ and $Jo_r(s, u)$, other than $u = 0$; $r = 0, 1, 2$; $s = 3, 6, 9, 15, 25, 40$; also $s = 1$ for $r = 0$; 3D.

Table VII. Second Zero of $Je_r(s, u)$ and $Jo_r(s, u)$; $s = 6, 9, 15, 25, 40$; for $r = 0$, also $s = 3$; 3D.

Table VIII. First Zero of $Je_r'(s, u)$ and of $Jo_r'(s, u)$, $r = 0, 1, 2$; $s = 3, 6, 9, 15$; for $r = 1$, also for $s = 1, 25, 40$; for $r = 2$, also $s = 1$; 3D.

The tables are followed by 32 pages of graphs of the various functions.

The periodic Mathieu functions for the parameter s can be found in the very accurate and systematic tabulation of Ince [2]. However, the authors devoted relatively little space to them, and since the normalization and notation are different, there was every justification for including this material. The tables for the parameters $-s$ and the radial functions are new, and form a worthwhile exploratory contribution. The graphs in both pamphlets constitute an especially welcome addition.

The writer spot-checked two values of the radial functions from some systematic tabulations that are being made at WADC. The values are essentially as accurate as claimed, although the authors' argument, such as $u = 0.182$, does not mean that the authors used precisely this value of u, but one close to it, which gave them a convenient value for subsidiary arguments required in their computations. It would be too difficult to check the accuracy of the tables systematically, since the u-arguments are not equally spaced. Some values of $Ne_1\ (s, u)$ on page 29 of the first report have been changed by pen in the reviewer's copy. However, the tables were not intended for interpolation purposes, and even if some other errors still exist there, the tables serve a useful purpose for exploratory studies. The reviewer noted only a few inaccuracies:

On pages 6 and 7 of the first report, the normalizations as given are not completely defined. A better statement would have been, ". . . the functions are normalized so as to satisfy equations (12)–(17)."

In Table IV of the second report, corresponding to $s = 9$, the "i" was omitted on the first line.

Two sentences on page 9, second paragraph of this report need revision. The radial Mathieu functions approach the corresponding Bessel functions, as s approaches zero, only if u approaches infinity in a special manner. It is not true that the Mathieu functions approach the Bessel functions, as s approaches zero, when u is merely different from zero. The special manner requires that $s^{\frac{1}{2}} \cosh u$ approach a constant value $k\rho$, in the authors' notation. A similar change is necessary on page 9 of the first report, first paragraph.

The authors deserve credit for the careful performance of a very useful task that presented considerable difficulty.

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1. NBS, *Tables Relating to Mathieu Functions*, Columbia University Press, New York, 1951.
2. E. L. Ince, "Tables of Elliptic Cylinder Functions," Roy. Soc. Edin., *Proc.* 52, 1932, p. 355–423.

24[L,S].—W. M. ROGERS & R. L. POWELL, *Tables of Transport Integrals* $J_n(x) = \int_0^x \frac{e^z z^n dz}{(e^z - 1)^2}$. NBS Circular 595, Government Printing Office, Washington, D. C., 1958, ii + 46 p., 26 cm. Price 40 cents.

The transport integrals $J_n(x)$ defined in the title may be expressed in terms of the integrals

$$F_n(x) = \int_0^x \frac{z^n dz}{e^z - 1}$$

by the formula $J_n(x) = nF_{n-1}(x) - x^n/(e^x - 1)$. Tables of one kind or another have been made for various values of n by various authors, but a consolidated table has been badly needed and the appearance of one is welcome.

The main tables (pages 7–45) give values of $J_n(x)$ to 6S without differences for $n = 2(1)17$, $x = 0(.1)X$, where X depends on n in the following way:

n	2(1)5	6, 7	8(1)11	12(1)17
X	25	30	35	40

For $n = 2(1)7$, 6-figure values of $J_n(x)/x^{n-1}$ are also given.

The values of $J_n(\infty)$ may be expressed in terms of the Riemann zeta function by the formula $J_n(\infty) = n!\zeta(n)$. Small tables on page 46 include values of $\zeta(n)$ and $J_n(\infty)$ to 8S for $n = 2(1)17$, as well as values of Bernoulli numbers used in expansions.

The computations were performed on an IBM 650. Details are given regarding accuracy, which varies with n and x. Usually the maximum error appears to be either one or two units of the sixth figure, but it seems that it may be as much as five units for $n = 12(1)17$ in the range of x from 9 to 11. Proper understanding is, however, made difficult by instability of notation, in that series expansions for small, intermediate, and large values of x are respectively referred to sometimes as a, b, c, (for example, in Table 1) and sometimes as c, b, a (for example, on pages 2–3).

The introduction gives a mathematical account of the functions tabulated, along with references to earlier tables. To these it seems worth while to add the tables of Nagai and Umeda [1], which, as far as $J_n(x)$ is concerned, give 6S for $n = 5(2)11$, $x = 0(.1)10$. These tables are in internationally intelligible form, though the accompanying text is in Japanese. Comparison between the two tables shows that

Nagai and Umeda's value at $n = 9$, $x = 0.8$ is entirely wrong. Apart from this, differences rarely exceed two units of the sixth significant figure, and never exceed five units. Such of the larger discrepancies as the reviewer investigated all turned out to be due to errors in the Japanese tables, which date from the pre-automatic age.

A. F.

1. S. NAGAI & K. UMEDA, "Rikwagaku-kenkyū-jo ihō," *Bulletin of the Institute of Physical and Chemical Research*, v. 18, no. 7, 1939, p. 529–536.

25[M, Z].—A. R. DIDONATO & A. V. HERSHEY, *New Formulae for Computing Incomplete Elliptic Integrals of the First and Second Kind*, NPG Report No. 1618, NAVORD Report No. 5906, January, 1959.

The authors study the application of series expansions for efficient evaluation of elliptic integrals of the first and second kind on high speed computers, with special reference to the NORC. To facilitate computation when the modulus is near unity, they derive an appropriate series expansion for each incomplete integral. The formulae are essentially of the same character as previously given by Radon [1], who has made a rather thorough investigation of the subject. Radon gives numerous expansions and also studies the incomplete elliptic integral of the third kind.

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1. BRIGITTE RADON, "Sviluppi in serie degli intergrali ellittici," Accad. Naz. Lincei, *Atti Mem., Cl. Sci. Fis. Mat.*, s. 8, v. 2, 1950, p. 69–109. See also *MTAC*, v. V, 1951, p. 79–80.

26[P].—RENE A. HIGONNET & RENE A. GREA, *Logical Design of Electrical Circuits*, McGraw-Hill Book Company, Inc., New York, 1958, ix + 220 p., 24 cm. Price $10.00.

This book is devoted to a binary (Boolean) analysis of the synthesis of switching circuits. Most attention is given to relays and relay circuits, but there is a chapter on vacuum-tube and diode circuits. In the main, the book is a convincing and realistic discussion by people experienced in relay circuitry.

The notations explained and used include Euler circles, ordinary Boolean algebra (using the plus symbol + for "or", either a multiplication dot or more frequently no mark for "and," and a prime ′ for "not"), and reference to the vertices, edges, and other parts of the boundary of the unit cube in n-dimensional space. It seems unfortunate that books dealing with this subject continue to use the Boolean symbols which can be confused with ordinary arithmetic symbols instead of $\vee$ and $\wedge$ for "or" and "and" respectively, but this continues to be common.

Many examples make the book easy to follow. However, absence of statements of theorems or other displayed punch lines will force most readers at least to skim through the entire book rather than to refer to isolated sections in order to get much from it.

An appendix includes a table of four-relay contact networks prepared by Edward F. Moore.

There are no bibliographic references not immediately incident to the material presented.

The book is a revision and translation of an earlier French edition.

C. B. T.

27[S].—James Riddell, *A Table of Levy's Empirical Atomic Masses*, Report AECL No. 339, 1956, 89 p., 27 cm. Available from Scientific Document Distribution Office, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada. Price $2.00.

Nearly 4,000 atomic masses have been calculated from Levy's empirical mass equation. These, together with the neutron, proton, and alpha-particle binding energies which can be calculated from them, are tabulated in the report.

Author's Summary

28[W, Z].—Department of The Army Pamphlet, *Introduction to Automatic Data Processing*, No. 1-250-3, Headquarters, Department of the Army, Washington 25, D. C., 1958, 88 p.

This pamphlet is an introduction to the concepts, methods, and equipment components of automatic data processing systems applied to management problems. A large part of the material is devoted to applications with emphasis on the utilization of automatic data processing facilities by the U. S. Army.

Various types of data processing equipment are introduced in the first chapter. No previous knowledge of computers on the part of the reader is assumed. After an explanation of the concepts of machine language and the binary number system, a meaningful description of the more common storage devices follows. The pertinent physical characteristics of the different components are concisely stated with informative comments on the advantages and disadvantages of each. This section is replete with illustrations and diagrams of typical computer systems and components.

The fundamentals of programming are also included in the comprehensive first chapter. This presentation is by no means a complete description of the discipline of computer programming, but it is a straightforward explanation in simple terms of the most important principles.

The title of the second chapter is "Management Aspects of Automatic Data Processing Systems". The chief concern of management is neatly stated as the "control of men, money, minutes, materials, and machines". The thesis here is that all this can be done most effectively by the intelligent use of automatic data processing systems. To guide managers who are about to consider such a system, timely advice is given on cost considerations and on the difficulties of evaluating the production gains to be realized once the system is installed. The questions of purchase versus rental, site planning, and personnel requirements are treated realistically.

The problem of communication with the heart of a data processing system (i.e., with the main computer) is considered in the third chapter. Whenever information is to be transmitted to and from widely dispersed operation centers, fast and reliable communication devices must be selected for use in the system. Data transmission devices in current use are described briefly, and examples of how they are incorporated into large systems are to be found in the fourth chapter, entitled "Applications of Automatic Data Processing Systems". The varied applications described are examples of actual systems in operation taken from private industry, government, and the U. S. Army.

The fifth and final chapter elaborates further on the Army utilization of automatic data processing systems. A number of proposed uses indicate the trend the Army is likely to follow in extending automation to its many activities.

This booklet would be more valuable if it included a bibliography of its source material as well as suggested references for additional information, particularly on equipment and programming. Nevertheless, it is a useful presentation of the actual and potential benefits to be gained, as well as the problems to be faced, in employing automatic data processing systems in large-scale organizations whose operations are widespread both geographically and functionally.

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29[X, Z].—K. H. Booth, *Programming for an Automatic Digital Calculator*, Academic Press Inc., New York, 1958, vii + 238 p., 22 cm. Price $7.50.

The value of this book lies in its provision of many examples of codes for common operations using the automatic computer APEXC. Generally, these operations are applicable to other computers. There are several major weaknesses, however, from the point of view of the reviewer. Most important is lack of precision of language and symbolism. Next in importance is lack of an adequate bibliography or reference to methods other than those recommended for the particular machine; this paucity of references has an irritating impact on the reviewer also in that he finds no credit to many of the early workers in the field. A third weakness lies in the completely pervading reference to the APEXC machine, which is not typical of most of the machines presently available throughout the world. The book is a good coding manual for the APEXC machine, but it falls short of being a well rounded work on coding and its logic.

Chapter 1 is introductory in nature, and describes the machine. Chapter 2 describes the techniques of programming (our usual terms of programming and coding, flow chart, and so on are not used, but the author uses the "schematic programme" and the "detailed programme"). Here there is some material that is applicable only to machines with cyclic memory reference and with provision for optimal spacing of commands in the sequence.

Chapter 3 discusses input and output, and again it is highly directed toward the one machine. It contains a table of decimal-binary equivalents for integers $n = 0(1)31$ (decimal). There is a detailed program for input of binary data, an input check program, a conversion program for decimal numbers, a decimal output program, a binary output program, and a program for storing data from decimal input.

Chapter 4 is devoted to division and square root. A division routine which develops the quotient in digits -1 and 1 instead of 0 and 1 (nonrestoring division) and then corrects is included. The usual iteration $x \rightarrow (x + b/x)/2$ for the square root of b and the usual iteration for the inverse square root (including normalization) are coded. Also coded are a method of successive subtraction of odd numbers for square roots, and a bisection method for the extraction of higher roots.

Chapter 5 includes codes for the evaluation of polynomials (using Horner's method), of even polynomials, and of odd polynomials; it contains explicit codes for the evaluation of the sine and cosine functions by Hastings' polynomial approximation [1], a code for tabulating a function with constant n-th differences, and a code for Simpson integration using no more than 31 ordinates.

Chapter 6 contains some "non-arithmetic codes". These include codes for normalizing numbers, selection of the largest modulus from a set of numbers, and collating.

Chapter 7 is devoted to matrix operations. It includes codes for matrix multiplication and for finding the largest real eigenvalue of a matrix by the power method.

Chapter 8 has a code for solving no more than 20 linear algebraic equations in 20 unknowns simultaneously, using the elimination method.

Chapter 9 is devoted to machine translation from French to English, using stored stems and suffixes. Due account is accorded to the possibility that the adjective following the noun in French should precede it in English.

Chapter 10 returns to the interpretive program for decimal tapes. This gives in some detail one of the conversion difficulties which beset the user of a binary machine.

Chapter 11 is devoted to interpretive routines and pseudo-codes. Floating-point arithmetic is the only subject treated in detail, and codes for the elementary operations are included.

Chapter 12 is devoted to checking. It includes ordinary post-mortem routines.

An appendix includes some well selected minor tables. Here one finds the first thirty powers of two expressed decimally, the first nine positive and negative powers of ten expressed in octal digits, the binary equivalents of the simplest decimal fractions, and octal expressions for one hundred twenty-six 1S or 2D decimal numbers.

Thus we have an instructive collection of codes available, and this collection should be valuable to anyone working with a new computer, anyone who has access to a computing center with similar codes not carefully explained, or anyone who wishes to see the current practice of explaining and printing codes. In general, this presentation is good.

Now, without belaboring the point, some of the less agreeable features of the book should be mentioned.

The book is really more a collection of codes than a book on programming, so it may be misnamed. No mention is made of the underlying logical principles of coding; however, these have not been presented successfully anywhere else. Still, the inductive process in coding (or the use of difference equations or recursive relations) is not stressed at all.

There is minimal mention of the work of others. This is important in that the reader is not given clues as to other places to look for information. There seems to be reason to believe that the author is unaware of much pertinent literature; her complete ignoring of the work of others in machine translation makes this chapter almost useless, even though (as she admits) her group is a leading center in this work.

The author's language and notation are imprecise, and it seems that the neophyte coder might deduce that precision is not necessary (a position which seems completely untenable, since coding is nothing more than the precise expression of arithmetic procedures in a restricted basic language). For example, on page 113 there is a careless confusion in subscripts; in Chapter 5 it is falsely stated that the Horner method gives the smallest number of multiplications necessary for the evaluation of any polynomial (it might more appropriately have been described as usually the shortest convenient method); in Chapter 7 she does not note that her matrix must have real eigenvalues, and so on.

The book is not complete. The author does not suggest (for example) a double iteration for taking square roots (using the iteration noted above for the square root and a second iteration mentioned in her book for reciprocals so that no division is required). Only the power method of finding eigenvalues is mentioned, and, in general, only those methods for which codes are mentioned above are considered. There are frequent references to a companion volume on *Automatic Digital Calculators* [2] and another on *Numerical Methods* [3], but these seem to the reviewer to be inadequate reference books upon which to base all the codes of a digital computer.

C. B. T.

1. C. Hastings, *Approximations for Digital Computers*, Princeton, 1955, [*MTAC*, v. 9, 1955, p. 121].
2. A. D. Booth & K. H. V. Booth, *Automatic Digital Calculators*, 2nd edn., Butterworths, London, 1956.
3. A. D. Booth, *Numerical Methods*, 2nd edn., Butterworths, London, 1957, [*MTAC*, v. 10, 1956, p. 105].

30[X, Z].—Gilbert R. Gray, *DTMB Univac Transportation Simplex*, Report 1266, David Taylor Model Basin, Washington, D. C., 1958, ii + 23 p. diagrs., tables, refs.

This report describes the methods and UNIVAC I routines developed to obtain initial solutions for Suzuki's Transportation Simplex Method. The initial solutions considered are the column, row, and matrix minimal (or maximal) and the Northwest Passage solution. At present the $m \times n$ cost matrix is limited by $m \leqq 30$ and $m + n \leqq 719$. The starting routines developed serve two main purposes: (1) to obviate tedious hand calculations and data tape preparations, and (2) to reduce the machine time required to solve a complete transportation problem.

Author's Summary

31[Z].—Rita M. Horbett, Arthur Shapiro, Karen L. Bradley, Sybil E. Jakob & Harriet R. Mallin, *Automatic Routines for Programming Management Data Problems on Univacs I and II*, Report 1241, David Taylor Model Basin, Washington, D. C., 1958, iv + 71 p. Tables.

A collection of automatic programming routines is presented here. These routines are designed for the UNIVAC, a high-speed, automatic, electronic, digital computer. Routines are included which perform the functions of edit, sort, merge and data conversion. This report contains a detailed description of the capabilities and limitations of the routines, and full directions on how to use them.

AUTHORS' SUMMARY

32[Z].—SIDNEY LECHTER, *Survey of analog-to-digital converters*, Report #1257, David Taylor Model Basin, Washington, D. C., 1958, iv + 145 p.

"Many test facilities are recording measurements of physical phenomena sensed by transducers having an analog output. The analog-to-digital converter can be of assistance to the data reduction field by providing the link, without human intervention, between these measurements and the modern high speed digital computer." So states the author of this report.

This report is a compilation of analog-to-digital converters, cataloged alphabetically by manufacturers. It is to be used as a computer system engineers' handbook. The major contribution of this report is the readily available listing, grouping those analog-to-digital converters which meet the specifications of a particular design. Characteristics of each converter, or encoders as they are usually called, are listed with photographs.

Listed characteristics are: power, space, weight, cost, input and output formats, and typical applications.

In addition, a brief discription of the availability of these converters, their accuracies, additional features and remarks which may be pertinent to each encoder are included. A cross reference list of manufacturers and a list of conversion speeds are also incorporated in this report.

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33[Z].—MONTGOMERY PHISTER, JR., *Logical Design of Digital Computers*, John Wiley and Sons, Inc., New York, 1958, vii + 408 p., 23 cm. Price $10.50.

Logical design is a name given for the planning of the interconnection of circuits, memories, and other units to form a digital system such as a computer. While a large amount of ingenuity and experimental thought may always be required for logical design, it is possible, nevertheless, for the designer to benefit greatly from the effective use of several mathematical techniques. Procedures utilizing some of these techniques are presented and well illustrated by examples and exercises. The presentation is complicated by the fact that these procedures must depend somewhat upon the nature of the circuits and other equipment being interconnected, but the author solves his problem by limiting his treatment, on the one hand, and by indicating procedural variations, on the other. Thus, the examples are principally of serial computers, and all are synchronous, or clocked.

The author follows the methods of the "algebraic equation" school of logical design, although in his use of the Karnaugh-Veitch diagram throughout the process for representing switching functions he has made a significant change from the original approach. His discussion requires no specific background on the part of the reader, but a general familiarity with digital devices would be helpful as well as a willingness to follow the many tedious details which are characteristic of logical design problems. The book should be useful not only to those who have had slight experience and who wish to learn logical design, but also to many who are accustomed to more haphazard and intuitive methods.

In this "how to do it" book one finds that theory is given relatively light treatment; probably to conserve space and increase the number of potential readers. Therefore, the student wishing to deepen his understanding of the ideas which led to the development of the emphasized techniques may find it desirable to supplement his reading. The bibliography is also, understandably, slanted in the direction of practicality and one finds few references to the theory of automata as developed by Burks, Kleene, von Neumann, Turing, McCulloch, and others. True, such material would be of little value in the practical problems of logical design, but it is from this direction that the impetus for further mathematical development has been received.

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