TECHNICAL NOTES AND SHORT PAPERS

The Formula-Controlled Logical Computer "Stanislaus"

By F. L. Bauer

The evaluation of a formula of propositional calculus is considerably simplified if this formula is written in the parenthesis-free notation of the Warsaw School, [1]. The Warsaw notation may be formulated in the following way:

There are symbols for operations, e.g.: N for negation; K for conjunction; A for disjunction; E for equivalence; C for implication; and symbols p, q, r, s, t for variables.

A variable is a formula. A formula preceded by the symbol N is a formula. Two juxtaposed formulas preceded by any one of the symbols, K, A, E, C are a formula. Evaluation of such a formula is done in the following way: Each of the variable symbols $p, q, r, \cdots$ has a value 0 or 1. The operation symbol acts on the value of the one or two formulas governed by it giving the value of the compound formula.

It was remarked in 1950 by H. Angstl, [2], that a mechanical evaluation of a formula, written in Warsaw notation without brackets, can be done in the following easy way: Each of the variable symbols is represented by a box with one output, the negation by a box with one input and one output, and the other operation symbols by a box with two inputs and one output. The meaning of a formula in the Warsaw notation is given by Angstl's rule: The first input of each operation symbol is to be connected with the output of the next following symbol, either variable or operation. The symbol N excepted, the second input of each operation symbol is to be connected with the first remaining free output of a symbol going from left to right. This may be demonstrated by an example, which uses Stanislaus present capacity of eleven symbols: the tautology of transitivity of the implication $[(p \rightarrow q) \ \& \ (q \rightarrow r)] \rightarrow (p \rightarrow r)$, written in Warsaw notation

$$C\ K\ C\ p\ q\ C\ q\ r\ C\ p\ r$$

and represented according to Angstl's rule by the diagram Fig. 1.

Following the Angstl rule, there is an easy way to build a mechanism for the evaluation of a logical formula, especially for a switching network which does the necessary connections of the logical units corresponding to the boxes mentioned above. Of course, these units can be built up in the usual way, e.g., with relays. But the proper switching may be done automatically by punching the formula on a keyboard.

We found building such an apparatus highly instructive on account of our interest in formula-translation, because it is a model of a computer which immediately obeys formulas in a common language. This means that, apparently in contrast to the apparatus of Kalin and Burkhart, [3], and later modifications, logical formulas are directly "written" (e.g., typed on a keyboard) rather than wired manually by connecting outputs and inputs on a switchboard.

It seems quite clear that this idea of direct formula control is not restricted to

Received February 9, 1959; in revised form May 1, 1959.

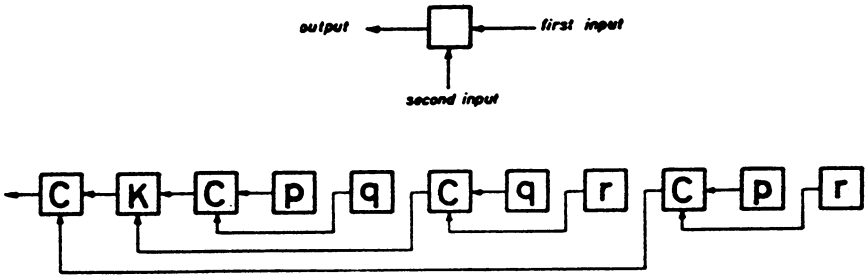

FIG. 1.—Example of a mechanical evaluation of a formula.

logical operations and the values 0, 1 only. Indeed, our interest is primarily devoted to arithmetical formulas, and we have obtained (in collaboration with Dr. K. Samelson, of Mainz) some results we believe to be remarkable. They will be published in the near future.

The wiring design of Stanislaus was done in December 1950. With the friendly help of colleagues, the toy was built in the following years, mostly from surplus material. The work suffered long interruptions, since it had a very low priority in our working program. Stanislaus was finished at the end of 1956 and presented in a lecture by H. Angstl, "Vorführung eines logistischen Rechengerätes für den Aussagenkalkül," held on January 8, 1957, in the Logistisches Seminar der Universität München. Stanislaus is now sometimes used for demonstration purposes. There might be possibilities of serious use outside the scope of our interest.

In the language of computer engineering we constructed, influenced by the material provided to us, something like a parallel-in-formula computer, but serial-in-formula operation would have been possible too, and the essential idea would have been the same. Indeed, the Burrough Truth Function Evaluator as described in 1954 by Burks, Warren and Wright, [4], is realized in this way.

For reasons of simplicity of the design we were going as far as providing each column in the keyboard with its own logical unit. The logical units are built up conventionally by relays. Also the truth values are represented by zero or working voltage on the output of the five switches corresponding to the values 0 or 1 of the variables p, q, v, s, t. Our interest was not devoted to these points, but to an automatic connection of the logical units by a mechanized switching system which works according to Angstl's rule. This switching system contains an additional feature which shows whether the string of symbols on the keyboard is a formula or is not well-formed.

Details may be seen from the wiring diagram, Fig. 2. Application of the formula-controlled logical computer is most simple. One only has to push buttons on the keyboard which operates like a desk calculator keyboard and therewith to "write in" the formula. The truth value of the variables involved may be set on switches. A red or yellow light flashes on according to the value of the formula being "wrong" or "right" for these values. A special switch provides the test whether the string of symbols "written in" is a formula (blue light) or not well-formed.

The keyboard scheme may be seen from Fig. 3. The formula written-in is $KNpNq$ or $\bar{p} \wedge \bar{q}$ in common notation. The switch at the bottom of the keyboard is in position "Check for meaningfulness." After changing it to the other

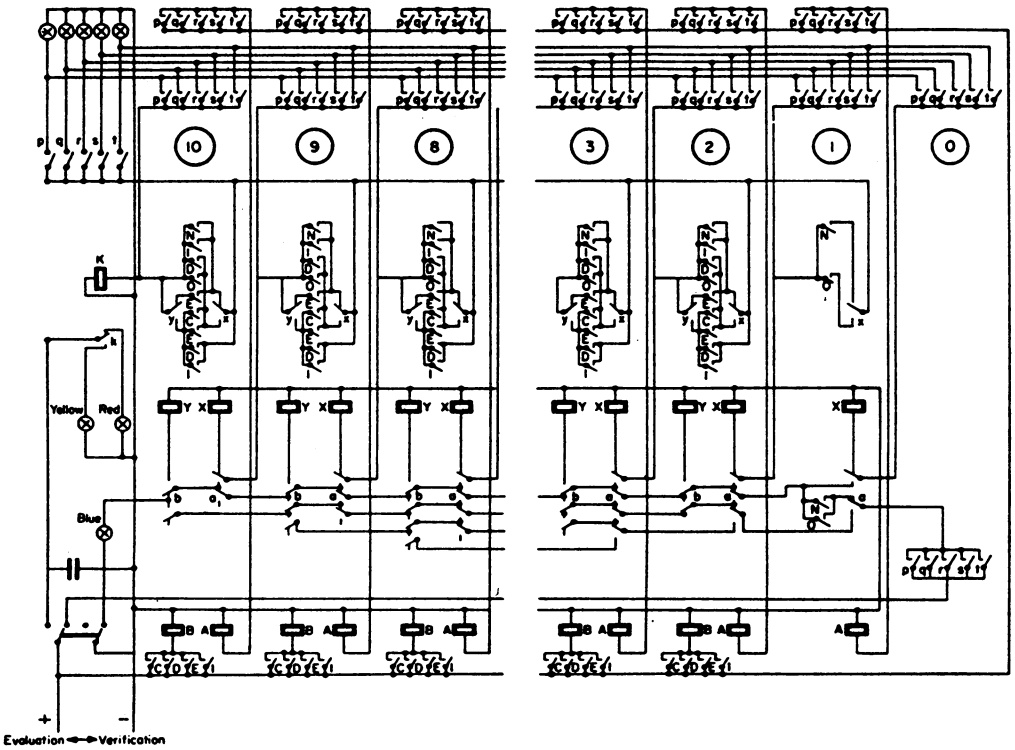

FIG. 2.—Wiring diagram.

Pushbuttons on the formula keyboard, columns 0–10; *O*, blank; *N*, negation; *C*, conjunction; *D*, disjunction; *E*, equivalence; *I*, implication; *p*; *q*, corresponding; *r*, variables; *s*; *t*.

Relays, columns 0–10; *X*, *Y*, logical circuits; *x*, *y*, corresponding contacts; *A*, *B*, transmission lines; *a*, *b*, corresponding contacts.

Relays and switches, left side: *p*, *q*, *r*, *s*, *t*, switches for truth values input.

Evaluation-Verification switch for evaluation or check; *K*, evaluation relay; *k*, corresponding contact.

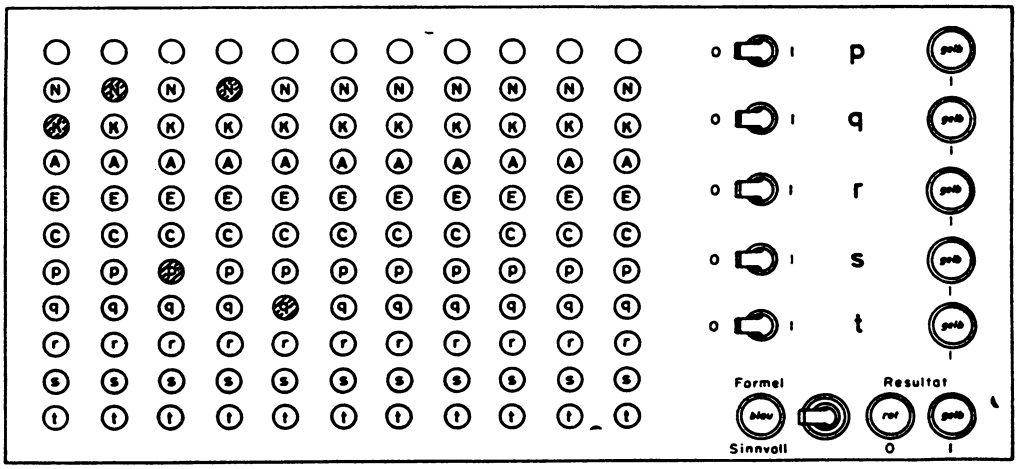

FIG. 3.—Keyboard scheme.

side, the yellow result light will flash on, indicating the truth value of $KNpNq$ for p = wrong and q = wrong according to the left position of the switches for the variables p and q on the right side of the keyboard.

Johannes Gutenberg-Universität in Mainz,
Institut für Angewandte Mathematik,
Mainz, Saarstrasse 21, Germany

1. J. Lukasiewicz & A. Tarski, "Untersuchungen über den Aussagenkalkül," *Comptes Rendus des Séances* de la Société des Sciences et des Lettres de Varsovie, Cl. III, 23, p. 31–32, 1930; and Paul Rosenbloom, *The Elements of Mathematical Logic*, New York, 1950.
2. Private Communication, not published. Angstl reported on his results in a seminar on logistics held in 1950 at the University of Munich by Professor W. Britzelmayr.
3. W. Burkhart, "Electrical analysis of truth functions," Thesis.
4. Arthur W. Burks, Don Warren & Jesse B. Wright, "An analysis of a logical machine using parenthesis-free notation," *MTAC*, v. 8, 1954, p. 53–57.

Evaluation at Half Periods of Weierstrass' Elliptic Function with Rectangular Primitive Period-Parallelogram

By Chih-Bing Ling

The purpose of this paper is to evaluate the following Weierstrass' elliptic function at half periods [1],

$$e_1 = \wp(\omega_1), \qquad e_2 = \wp(\omega_2), \qquad e_3 = \wp(\omega_3), \tag{1}$$

where $2\omega_1$ and $2\omega_2$ are double periods of the function and ω_3 is defined by

$$\omega_1 + \omega_2 + \omega_3 = 0. \tag{2}$$

This paper tabulates only the values of the function whose primitive period-parallelogram is a rectangle with $2\omega_1 = 1$ and $2\omega_2 = ai$, where $a \geqq 1$.

The three functions in (1) form a set of distinct roots of the cubic [1]

$$x^3 - px - q = 0, \tag{3}$$

where

$$p = 15\sigma_4, \qquad q = 35\sigma_6, \tag{4}$$

and

$$\begin{aligned} \sigma_{2k} &= \sum_{m,n=-\infty}^{\infty}{}' \frac{1}{(2m\omega_1 + 2n\omega_2)^{2k}} \\ &= 2\sum_{m=1}^{\infty} \frac{1}{m^{2k}} + 2\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} \frac{1}{(m + nai)^{2k}}. \end{aligned} \tag{5}$$

The accent on the summation sign denotes the omission of simultaneous zero values of m and n from the double summation.

The cubic (3) indicates that

Received April 7, 1958; in revised form, August 7, 1959.