

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[B, C, K, S].—A. H. WAPSTRA, G. J. NIJGH & R. VAN LIESHOUT, *Nuclear Spectroscopy Tables*, North Holland Publishing Company, Amsterdam, 1959, viii + 136 p., 27 cm. Price \$8.90.

This book consists of a well-selected collection of tables and graphs of interest to nuclear physicists. A slightly condensed list of contents follows:

### Chapter I. *Mathematical Data.*

Common logarithms. Powers of 10 and 2, for exponents 1.00(0.01)9.99, 4D. Cube roots of integers 1(1)999, 5D. Brief discussion of least-squares method, and a graph for checking the consistency of least-squares computations. Table of Gaussian distribution and its integral for arguments 0.00(0.02)3.48, 4D.

### Chapter II. *Tables of Atomic Constants.*

### Chapter III. *Elements and Isotopes.*

Names, atomic numbers, and symbols of elements. Atomic weights and abundances of isotopes.

### Chapter IV. *Heavy Particles.*

Range energy curves for protons, deuterons, and alpha particles. Straggling of protons in air (graph). Magnetic rigidity of protons, deuterons, and alpha particles (table). Half-lives for alpha disintegration and spontaneous fission of heavy elements (graphs).

### Chapter V. *Electrons.*

Range energy curves for beta-particles. Saturation backscattering coefficient (graph). Magnetic rigidity of electrons (table). Discussion of shapes of continuous beta-spectra. Reduced Fermi function  $f(z, p)$  and beta-decay functions  $L$  and  $M$  for negatrons and positrons (tables). Screening correction for negatrons and positrons (graphs). Discussion of beta-decay transition probabilities and their values. Ratios of electron capture in different electron shells. Computation of  $\log ft$  values (nomogram and graphs).  $K$  capture/positron emission ratios (table and graphs).

### Chapter VI. *Gamma Rays.*

Half-thickness of some substances for absorption of gamma rays vs energy (graph). Photon absorption cross-sections vs energy (table). Energy of Compton scattered gamma rays (discussion and graph). Gamma-decay half-lives (discussion, nomogram, and graphs).

### Chapter VII. *X-rays and Auger Electrons.*

Electron binding energies in different shells for all elements (table). Relative intensities of  $K$  X-ray components and of  $K$ -Auger electrons (table).

### Chapter VIII. *Angular Distributions and Correlations.*

Brief discussion of angular distributions and correlations involving photons, alpha particles, beta rays and  $K$  conversion electrons. Tables of  $F_v(L, L', j_i, j_j)$  coefficients.

### Chapter IX. *Nuclear Models.*

Discussion of nuclear mass formula, nuclear shell model, collective model, magnetic moments, quadrupole moments, and gamma- and beta-decay

probabilities. Graphs of Nilsson level scheme of single particle orbits in spheroidal potential. Table of measured ground state spins of odd-A and odd-odd nuclei. Tables of Clebsch-Gordan coefficients.

Chapter X. *Calibration Standards.*

Tables of standard gamma and electron lines and of standard alpha rays. Gamma-ray absorption coefficient in NaI crystals. Table of standard nuclides for calibration of gamma-ray spectrometer.

The tables and graphs have been presented so as to be easily read, and the quality of the printing is good. Much of the material is used frequently by nuclear physicists but is widely scattered in the literature. Thus, this book should prove very helpful to people in the field of nuclear physics, and this reviewer recommends it highly.

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2[D, L].—L. K. FREVEL, J. W. TURLEY & D. R. PETERSEN, *Seven-Place Table of Iterated Sine*, The Dow Chemical Company, Midland, Michigan, 1959. Deposited in UMT File.

Following a detailed description of the method of computation employed, the authors give a 7D table of the  $n$ th iterated sine function of  $x$  for  $n = 0(.05)10$ , and  $x = k(\pi/20)$ , where  $k = 1(1)10$ . It is stated that the computations were performed on a Datatron 204, and the results are considered correct to within  $5 \cdot 10^{-7}$ .

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3[G].—K. M. HOWELL, *Revised Tables of 6j-Symbols*, U. Southampton Math. Dept., Research Report 59-1, 1959, xvi + 181 p., 33 cm.

The Wigner 6j-symbol has been defined by Wigner in general in connection with the reduction of the triple Kronecker product of any simply reducible group. In these tables this group is taken to be either the three-dimensional rotation group or the two-dimensional unitary group. The symbols are denoted by

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{Bmatrix}$$

where the quantities  $j_1, \dots, k_3$  are integers or half-integers. If we let

$$J_0 = j_1 + j_2 + j_3, \quad J_1 = j_1 + k_2 + k_3, \quad J_2 = j_2 + k_1 + k_3$$

$$J_3 = j_3 + k_1 + k_2$$

$$K_1 = j_1 + j_2 + k_1 + k_2 \quad K_2 = j_1 + j_3 + k_1 + k_3$$

$$K_3 = j_2 + j_3 + k_2 + k_3,$$

then the explicit expression for the 6j-symbol is

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{Bmatrix} = \left\{ \prod_{r,s} (K_r - J_s)! / \prod_s (J_s + 1)! \right\}^{\frac{1}{2}} \cdot \sum_t (-1)^t (t+1)! / \left\{ \prod_r (K_r - t)! \prod_s (t - J_s)! \right\}.$$

These symbols are invariant under the 144 products of separate permutations of the  $K$ , alone and the separate permutations of the  $J$ , alone. The symmetries of the  $6j$ -symbols are made use of in the organization of the tables. In the present (revised) edition of the tables a wider group of symmetries is exploited than in the earlier edition.

In the tables the  $6j$ -symbols are classified in terms of a set of six ordered parameters, and the tables are arranged in descending "speedometer" order of these parameters. Rules for determining the values of these parameters from given  $j_1, \dots, k_3$  are included.

The square of the value of the  $6j$ -symbol is printed in the table, together with its correct sign. In addition the entries are written as rational fractions in terms of their prime factors. Thus the fraction  $-4/5\sqrt{21}$  is written as  $-2 \cdot 2 \cdot 2 \cdot 2/3 \cdot 5 \cdot 5 \cdot 7$ . A composite member, all of whose factors are greater than 103 is printed as if it were a prime.

The tables were duplicated from stencils cut directly from the output tape of a Ferranti Pegasus Computer. The program used in the computer was essentially the same as that used for the earlier version of the tables. It is stated that, "It seems reasonably sure that these tables are as reliable as, and more comprehensive than, the first set of tables of the  $6j$ -symbol."

A. H. T.

4[H, M].—H. J. GAWLIK, *Zeros of Legendre Polynomials of orders 2-64 and weight coefficients of Gauss quadrature formulae*, A. R. D. E. (Armament Research and Development Establishment) Memorandum (B) 77/58, Fort Halstead, Kent, December 1958, 25 p.

The Gauss quadrature formula is  $\int_1^1 f(x) dx = \sum_{r=1}^n A_r f(a_r)$  with  $A_r, a_r$  chosen so that the equality is valid whenever  $f(x)$  is any polynomial of degree  $2n - 1$  or less. The quantities  $a_r$  are the zeros of the Legendre polynomial  $P_n(x)$ , while

$$A_r = \frac{1}{P_n'(a_r)} \int_1^1 \frac{P_n(x)}{x - a_r} dx.$$

The memorandum here reviewed gives 20-decimal values of  $a_r$  and  $A_r$  for all zeros  $a_r$  of each polynomial  $P_n(x)$ , for  $n = 2(1)64$ , and is by far the most extensive and accurate of such tables available.

The most extensive of the previously published tables is that of Lowan, Davids, and Levinson [1], which gives a similar table to 15 decimals for  $n = 2(1)16$ . This table has been reproduced several times, in whole or in part [2]. It has been supplemented by a table by Davis and Rabinowitz [3], which gives 20-decimal values for  $n = 2, 4, 8, 16, 20, 24, 32, 40, 48$ . Discrepancies between Davis & Rabinowitz and Gawlik amount to only a unit in the 20th decimal in several places, with the probability that Gawlik is the more accurate; both tables would thus appear to be reliable.

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1. AMER. MATH. SOC., *Bull.*, v. 48, 1942, p. 739-743; v. 49, 1943, p. 939 (errata).
2. See, for example, NBS Applied Mathematics Series, No. 37, *Tables of Functions and of Zeros of Functions*, 1954.
3. NBS, *Jn. of Research*, v. 56, 1956, p. 35-37.

5[I].—A. N. LOWAN, *The Operator Approach to Problems of Stability and Convergence of Solutions of Difference Equations and the Convergence of Various Iteration Procedures*, Scripta Mathematica, New York, 1957, x + 104 p., Office of Technical Services, Department of Commerce, Washington 25, D. C. 26 cm. Price \$3.00.

In the numerical solution of linear second-order differential equations by difference methods, one has to solve

$$(1) \quad A_k u_{k+1} + B_k u_k + C_k u_{k-1} = D_k$$

where  $A_k, B_k, C_k, D_k$  are sparse matrices with regular structure, the  $u$ 's are vectors, and the integer subscripts refer to time-steps, iteration cycles, etc. In many important cases  $A_k, B_k, C_k$  are independent of  $k$  and closely related. Stability and mesh-convergence of "stepping-ahead" solutions (parabolic, hyperbolic equations) and convergence of iterative solutions (elliptic equations) can be discussed in terms of the operators  $A_k^{-1}B_k$  and  $A_k^{-1}C_k$ . In the von Neumann technique, as formalized and extended by the reviewer, one essentially takes the Fourier transform (in the extended sense) of (1), thus introducing immediately the eigenvectors and eigenvalues of these operators. This is equivalent to a change of coordinates in vector space under which  $A_k, B_k, C_k$  take very simple forms. Various authors, however, have retained the original coordinates and worked directly with (1); prominent among these are S. Frankel, D. Rutherford, A. Mitchell, P. Lax, R. Richtmyer, J. Douglas, J. Todd, and (unpublished) C. Leith. Professor Lowan here gives a detailed, connected account of this second method which he calls the "operator approach". He covers the usual parabolic, elliptic, and hyperbolic partial differential equations, homogeneous and non-homogeneous, with some attention to equations with variable coefficients. He discusses stability and mesh-convergence for parabolic and hyperbolic equations, and convergence of the various iterative schemes for elliptic equations. Original contributions, besides the organization of the material, include a discussion of iteration schemes for solving "implicit" difference approximations to parabolic and hyperbolic equations and a novel "second-order" Richardson scheme for elliptic difference equations. In addition, Professor Lowan has written down a number of "folk theorems", rather widely known but unpublished. There are eight "Sections" and six "Appendices". Most of the typographical errors have been detected by the author and listed on the "Errata" sheet. However, the figures on page 5 and at the top of page 33 should be corrected; in the second line from the bottom of page 44, read  $(-1)^{k+1} \sqrt{2} \sin rh\pi/M$ ; page 26, line 5 and page 47, line 18 should show the respective qualifications " $r > 1$ " and " $r \leq \frac{1}{2}$ ". Moreover, the reader should be aware that the estimates of truncation error in difference solutions given during the discussion of mesh-convergence require that various functions be "sufficiently continuous." Those of us interested in the numerical solution of partial differential equations are indebted to Professor Lowan for a very worthwhile addition to the literature of this field.

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6[K].—CHARLES W. DUNNETT, "Tables of the bivariate normal distribution with correlation  $1/\sqrt{2}$ ," 1958, 28 cm. Deposited in UMT File.

The bivariate normal probability distribution,

$$L(h, k; r) = \frac{1}{2\pi\sqrt{1-r^2}} \int_h^\infty \int_k^\infty \exp\left[-\frac{1}{2}\left(\frac{x^2 - 2rxy + y^2}{1-r^2}\right)\right] dx dy,$$

has been tabulated by Karl Pearson [1] for  $r = -1.0(.05)1.0$ . The present tables were prepared to avoid the necessity of interpolating in Pearson's tables when  $r = \pm 1/\sqrt{2}$ . This case arises in certain double sampling procedures in which probability statements concerning  $X$  and  $X + Y$  jointly are required, where  $X$  and  $Y$  are independent normal chance variables with the same variance.

The tables were computed on a Royal McBee LGP-30 electronic computer, by numerical quadrature, using the relation

$$L(h, k; r) = \int_h^\infty \left[1 - F\left(\frac{k - rx}{1 - r^2}\right)\right] f(x) dx$$

where  $f(x) = (1/\sqrt{2\pi}) \exp(-x^2/2)$  and  $F(x) = \int_{-\infty}^x f(x) dx$ . The function is tabulated for  $r = 1/\sqrt{2}$  in Table I and for  $r = -1/\sqrt{2}$  in Table II for positive values of its arguments,  $h$  varying in steps of 0.1, and  $k$  varying in steps of  $0.1\sqrt{2}$ . All entries are given to six decimals and should be correct to this number of places.

The function can be determined for negative values of its arguments by using the relationships

$$L(-h, k; r) = L(-\infty, k; r) - L(h, k; -r)$$

$$L(h, -k; r) = L(h, -\infty; r) - L(h, k; -r)$$

$$L(-h, -k; r) = L(h, k; r) + 1 - L(h, -\infty; r) - L(-\infty, k; r)$$

where  $L(h, -\infty; r) = L(-\infty, h; r) = 1 - F(h)$ , which is the right-hand tail area of the univariate normal distribution. In order to avoid the necessity of consulting tables of  $F(h)$ , these values are included in the table.

Tables III, IV and V were computed from Tables I and II using these relationships. The error in the entries in Tables III and IV should be no greater than a unit in the sixth decimal place. The error in the entries in Table V should be no greater than two units in the sixth decimal place. All tables have been deposited in the Unpublished Mathematical Tables repository.

AUTHOR'S ABSTRACT

1. KARL PEARSON, *Tables for Statisticians and Biometricians*, Part II, Cambridge University Press, 1931.

7[L].—E. A. CHISTOVA, *Tablitsy funktsii Bessela ot deistvitel'nogo argumenta i integralov ot nihh* (*Tables of Bessel functions of real argument and of integrals involving them*), Izdatel'stvo Akademii Nauk SSSR, Moscow, 1958, 524 p. + loose card, 28 cm. Price 45 rubles.

This important volume in the now familiar series of Mathematical Tables of the Computational Center of the Academy of Sciences was initiated by V. A. Ditkin.

The main table occupies pages 23–522 and gives values for

$$x = 0(.001)15(.01)100.$$

of the eight functions

$$J_n(x), \quad Ji_n(x) = \int_x^\infty \frac{J_n(u)}{u} du,$$

$$Y_n(x), \quad Yi_n(x) = \int_x^\infty \frac{Y_n(u)}{u} du,$$

where  $n = 0$  and  $1$ . The values are to 7D or, near singularities at the origin, 7S. Auxiliary functions (detailed below) are provided for  $x = 0(.001).150$ . Over the range  $x = 1.350(.001)15$  there are no differences, and linear interpolation provides results correct within two units of the last place. Second differences are required for some functions in parts of the ranges  $x = .150(.001)1.350$  and  $x = 15(.01)100$ ; the quantities printed (when greater than about 16) are sums of two consecutive second differences, for use in Bessel's formula.

The values of  $J_0(x)$  and  $J_1(x)$  are rounded to 7D from the well-known Harvard tables, and are included for convenience; the values of the other six quantities result from original computations, and may be checked only partially against previous, less extensive, tables which are listed in a bibliography. The integrals  $Ji_0(x)$  and  $Ji_1(x)$  were computed by Simpson's rule on an electronic computer and other machines. The functions  $Y_0(x)$ ,  $Y_1(x)$ ,  $Yi_0(x)$ , and  $Yi_1(x)$  were evaluated on the electronic computer BESM, using Taylor series and asymptotic expansions. All values were checked by differencing. By means of formulas given on pages 11–12, the integrals of  $J_0(u)$ ,  $J_1(u)$ ,  $Y_0(u)$ , and  $Y_1(u)$ , *not* divided by  $u$ , may be simply expressed in terms of the tabulated functions.

The nine auxiliary functions given on pages 17–19 are all tabulated for  $x = 0(.001).150$  to 7D without differences. The functions are:

$$Li_0(x) = Ji_0(x) + \ln \frac{1}{2}x$$

$$C_0(x) = Y_0(x) - (2/\pi)J_0(x) \ln x \quad E_0(x) = (2/\pi)\{Ji_0(x) + \ln x\}$$

$$C_1(x) = x\{Y_1(x) - (2/\pi)J_1(x) \ln x\} \quad E_1(x) = (2/\pi)\{Ji_1(x) - 1\}$$

$$D_0(x) = (2/\pi)J_0(x) \quad F_0(x) = Yi_0(x) - (2/\pi) \ln x\{Ji_0(x) + \frac{1}{2} \ln x\}$$

$$D_1(x) = (2/\pi)J_1(x) \quad F_1(x) = x\{Yi_1(x) + (2/\pi) \ln x[1 - Ji_1(x)]\}$$

It is stated that the errors do not exceed 0.6 final unit, except that they may attain one final unit in the case of the auxiliary functions  $C_0(x)$ ,  $C_1(x)$ ,  $F_0(x)$ , and  $F_1(x)$ .

A table of the Bessel coefficient  $\frac{1}{4}t(1-t)$  for  $t = 0(.001)1$  to 5D without differences is given on page 523 and also on a loose card.

A. F.

8[L].—OTTO EMERSLEBEN, "Werte einer Zetafunktion zweiter Ordnung mit Argument  $s = 2$ ," Bearbeitet in der Abteilung Angewandte Mathematik der Universität Greifswald, Greifswald, 1956.

This paper contains a tabulation of the function

$$Z(x, y) = \sum_{k=-\infty}^{\infty} \sum'_{l=-\infty}^{\infty} \frac{e^{2\pi i(kx+ly)}}{k^2 + l^2}$$

where the prime denotes the fact that the term with  $k = l = 0$  is omitted.

The table is given for values  $x$  and  $y$  in the range

$$\frac{1}{2} \geq x \geq y \geq 0,$$

where  $x$  and  $y = 0(0.01)0.5$ . The entries are given to six and sometimes seven decimal places, and are said to be accurate to at least two units in the last decimal place.

In the calculation of this table, use was made of the seven-place tables of the exponential integrals published in 1954 by the U.S.S.R. Academy of Science, Institute of Exact Mechanics and Computation.

A. H. T.

9[L].—HERMAN H. LOWELL, *Tables of the Bessel-Kelvin Functions Ber, Bei, Ker, Kei, and their Derivatives for the Argument Range 0(0.01)107.50*, Technical Report R-32, National Aeronautics and Space Administration, Washington, D. C., 1959, 300 p., 26 cm.

These tables provide an elaborate and attractively arranged compilation of decimal values of the Bessel-Kelvin functions (frequently referred to simply as the Kelvin functions) of the first and second kinds of order zero, together with their first derivatives. Approximations to ber and bei and their first derivatives appear in floating-point form to generally 13 or 14 significant figures. On the other hand, the number of significant figures given for ker and kei and their first derivatives vary from 9 to 13, according to a pattern explained in the detailed introduction, which also describes the construction of these tables and the checks applied to the tabular entries. The calculations were performed on an IBM 650 calculator using the Bell Telephone Laboratories Double-Precision (16-figure) Interpretive System.

In addition to the checks applied by the author, the reviewer collated the values of ber  $x$  and bei  $x$  with similar data given by Aldis [1] to 21D, for the range  $x = 0.1(0.1)6.0$ . No discrepancies were detected.

The range, precision, and accuracy of the tables under review establish them as the definitive tables of the Kelvin functions at the present time.

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1. W. STEADMAN ALDIS, "On the numerical computation of the functions  $G_0(x)$ ,  $G_1(x)$ , and  $J_n(x\sqrt{i})$ ," *Roy. Soc. London, Proc.*, v. 66, 1900, p. 32-43.

10[L].—NUMERICAL COMPUTATION BUREAU, Report No. 11, *Tables of Whittaker Functions (Wave Functions in Coulomb Field) Part 2*, The Tsuneta Yano Memorial Society, 1-9 Yuraku-cho, Chiyoda-Ku, Tokyo, Japan, 1959, 52 p., 26 cm. Price \$3.00.

The first part of these tables was reviewed in *MTAC*, v. 12, 1958, p. 86-88. The earlier review contains some errors and fails to give complete information.

The functions considered are defined thus:

$$G_{\xi,l} + iF_{\xi,l} = \exp\left(-\frac{x}{2}\xi\right) \exp\left[-i\left(\frac{l}{2}\pi - \sigma_l\right)\right] W_{i\xi, l+1/2}(-ix).$$

The earlier review has  $\frac{1}{2}$  in place of  $l + \frac{1}{2}$  in the subscript of the Whittaker function,  $W$ , and incorrectly uses  $G_{\xi,1/2}(x)$  and  $F_{\xi,1/2}(x)$  for the case  $l = 0$ . Both the previous review and the present tables fail to define  $\sigma_l = \arg \Gamma(l + 1 + \frac{1}{2}ix)$ .

The table now reviewed is concerned with values of  $F_{\xi,0}(x)$  for large  $\xi$ . Values of this function are presented to five decimal places, together with  $\delta_r^2$  and  $\delta_k^2$ , where  $k = 1/\xi$  and  $r = x/(2k)$ , corresponding to  $r = 0(.1)10$  and  $k = 0(.01)1$ , that is,  $\xi \geq 1$ .

It is observed that for  $k = 0$ , that is,  $\xi \rightarrow \infty$ ,

$$F_{\xi,0}(x) = \sqrt{2r}J_1(2\sqrt{2r}); \quad G_{\xi,0}(x) = \sqrt{2r}Y_1(2\sqrt{2r}),$$

which are identifiable as Bessel-Clifford functions multiplied by  $2r$ . This relation allows comparison with appropriate data in a publication [1] of the National Bureau of Standards; such a comparison has revealed 24 errors (all due to rounding) in the tables under review, thereby suggesting a general accuracy therein within one unit of the fifth decimal.

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1. NBS Applied Mathematics Series, No. 28, *Tables of Bessel-Clifford Functions of Orders Zero and One*, U. S. Government Printing Office, Washington, D. C., 1953.

11[L, M].—W. W. GERBES, G. E. REYNOLDS, M. R. HOES, & C. J. DRANE, JR., *Table of  $S(x)$  and its First Eleven Derivatives*, Vol. 1, 2, 3, Air Force Cambridge Research Center, Bedford, Massachusetts, 1958, 27 cm.

The tabulated function  $S(x)$  defined by

$$S(x) = \int_0^x \left( \frac{\sin \frac{u}{2}}{u/2} \right)^2 du$$

is related to the sine integral  $Si(x)$  by

$$S(x) = 2 \left[ Si(x) - \frac{1 - \cos x}{x} \right].$$

For ease in computation in the design of antennas, the function  $S(x)$  and its first eleven derivatives are tabulated to six decimal places for

$$x = 0^\circ(1^\circ)18,000^\circ.$$

The introduction gives the characteristics of the functions, reduction formulas, power series representations, asymptotic expressions, integral representations, differential equations, transforms, addition formulas, etc., and the method of computation.

The tables were computed using the IBM 650 calculator.

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12[L, M].—R. HENSMAN & D. P. JENKINS, *Tables of  $\frac{2}{\pi} e^{z^2} \int_z^\infty e^{-t^2} dt$  for Complex  $z$* , Royal Radar Establishment, Malvern, Worcestershire, England. Deposited in UMT file.

The function  $\frac{2}{\pi} e^{z^2} \int_z^\infty e^{-t^2} dt$  has been tabulated to 6 decimal places for  $0(0.02)2.00$  in the real part of  $z$  and  $0(0.02)4.00$  in its imaginary part, and also for  $0(0.1)10.0$  in both real and imaginary parts. Second differences are given and second-difference interpolation in the appropriate table gives 6-decimal accuracy for the whole range covered. The error in using linear interpolation need not exceed a unit in the fourth decimal.

#### AUTHORS' SUMMARY

13[L, M].—A. V. HERSHEY, *Computing Programs for the Complex Exponential Integral*, NAVORD Report No. 5909, NPG Report No. 1646, U. S. Naval Proving Ground, Dahlgren, Virginia, June 1959, iii + 16 p. Figures and tables. 27 cm. Astia Document Service Center, Armed Services Technical Information Agency, Arlington Hall Station, Arlington 12, Va.

In this report there appears a detailed discussion of the use of asymptotic series with remainder in the evaluation of the complex exponential integral on the Naval Ordnance Research Calculator (NORC). Brief descriptions are also given of two NORC subroutines for the calculation of the exponential integral and the sine and cosine integrals of a real argument.

A series expansion of the remainder of the asymptotic series is presented, and is used to evaluate the remainder with improved accuracy.

The author also describes the construction of a rational approximation to the exponential integral that is valid over the negative half of the complex plane outside the unit circle. In appended tables appear approximations to 13S of the coefficients of two polynomials of the fourteenth degree whose quotient gives values of the function  $ze^{-z} Ei(z)$  to within a maximum relative error in the absolute value of  $2.2 \times 10^{-13}$ .

The appendix also includes a table of values to 13S of the real and imaginary parts of  $Ei(z)$ , corresponding to  $x = -20(1)20$  and  $y = 0(1)20$ . These results were obtained on the NORC by means of double-precision arithmetic, using sixteen-point Gauss integration over each unit interval, beginning with  $x = -100$ , where the value of the integral was considered to be negligibly small.

Comparison by the reviewer of these data with corresponding entries in an extensive earlier set of tables [1], carried to 6D and 10D, revealed only three instances of rounding errors in the latter, all of such size as to lie within the guaranteed limit of a unit in the last decimal place.

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1. NBS Applied Mathematics Series, No. 51, *Tables of the Exponential Integral for Complex Arguments*, U. S. Government Printing Office, Washington, D. C., 1958. See also *MTAC*, v. 13, 1959, p. 57-58.

14[L, M].—K. A. KARPOV, *Tablitsy funktsii  $F(z) = \int_0^z e^{x^2} dx$  v kompleksnoi oblasti*

(*Tables of the function  $F(z) = \int_0^z e^{x^2} dx$  in the complex domain*), Izdatel'stvo Akademii Nauk SSSR, Moscow, 1958, 518 p. + 2 inserts, 27 cm. Price 61 rubles.

This is a companion volume to the tables reviewed in *MTAC*, vol. 12, p. 304–305, and completes the tabulation of the error function in the complex plane. The present volume contains 5D or 5S values of the real and imaginary parts of the function

$$F(z) = \int_0^z e^{x^2} dx = u + iv$$

for  $z = \rho e^{i\theta}$ ,  $0 \leq \rho \leq \rho_0$ ,  $\pi/4 \leq \theta \leq \pi/2$  and  $\theta = 0$ . The quantity  $\rho_0$  depends on  $\theta$  and decreases from  $\rho_0 = 5$  for  $\theta = \pi/4$  to  $\rho_0 = 3$  for  $\theta = \pi/2$ . An exception is  $\theta = 0$ , for which  $\rho_0 = 10$ . In the introduction, a diagram is given representing the intervals in  $\theta$  and the value of  $\rho_0$  for each  $\theta$ , and a table indicates the intervals in  $\rho$  in various parts of the volume. As in the earlier volume, the diagram is reproduced on a cardboard inset, which serves also as an index to the numerical tables.

The introduction gives integral representations and series expansions for  $u$  and  $v$ , graphs of  $u$  and  $v$  as functions of  $\rho$  for selected values of  $\theta$ , relief diagrams of  $u$  and  $v$  over the sector of tabulation, a description of the tables and numerical examples showing their use, some useful numerical values, values of  $\cos 2\theta$ ,  $\sin 2\theta$ , and values of  $(2n + 1)\theta$  in radians for  $n = 0(1)5$  and for those values of  $\theta$  included in the tables. There is also a one-page auxiliary table of  $t(1 - t)/4$  for  $0 \leq t \leq 1$ . This table, together with a nomogram for finding  $\tilde{\Delta}^2 t(1 - t)/4$ , where  $\tilde{\Delta}^2$  designates the sum of two consecutive second differences for use in Bessel's interpolation formula, is reproduced also on a cardboard inset.

Using the symmetry properties of  $F(z)$ , this function can now be evaluated on the real axis and in a sector of half-angle  $45^\circ$  to both sides of the imaginary axis. Between them, Karpov's two volumes contain a very satisfactory tabulation of the error function in the complex plane.

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15[L, M, S].—Y. NOMURA & S. KATSURA, "Diffraction of electric waves by circular plate and circular hole," *Sci. Rep. Ritu, B-(Elect. Comm.) 10*, No. 1, 1958, 43 p.

The problem of the diffraction of a plane electromagnetic wave by an infinitely thin, perfectly conducting, circular disk of radius  $a$ , and the problem of the diffraction of such a wave by a circular hole of radius  $a$  in a plane conducting screen are discussed. The method of solution involves the expansion of the two-component Hertz vector in terms of hypergeometric polynomials. The solution is valid for all frequencies. However, convergence is poor when  $ka = 2\pi a/\lambda$  becomes large. Tables of values are included of the real and imaginary parts of

$$G_{n\nu}^m = (2m + 4\nu + 1) \int_0^\infty \frac{J_{m+2n+1}(\xi) J_{m+2\nu+1}(\xi) d\xi}{\sqrt{\xi^2 - (ka)^2}}$$

for  $m = 0(1)4$ ,  $n, \nu = 0(1)5$ , and  $ka = 0(.25)5$ , where  $J_p(\xi)$  is the Bessel function of the first kind of order  $p$ . The function  $X_{\nu}^m$  is defined in terms of the equations

$$\sum_{r=0}^{\infty} G_{r,n}^m X_{r,n}^m = \delta_{r,n}.$$

Tables for  $X_{\nu}^m$ , for  $n = 0(1)4$ ,  $\nu = 0(1)2, 5$ ,  $m = 0(1)4$ , over the same range of  $\xi$ , are also included.

In addition, tables are given which are useful for the calculation of the field distribution at large distances, and tables are given which enable one to determine the current distribution on the plate and the electric field distribution on the whole.

All table entries are given to four decimal places. However, no indication is given as to the numerical method of evaluating the table entries nor as to their actual accuracy.

A. H. T.

16[P].—CHARLES J. THORNE, *Temperature Tables: Part 1. One-Layer Plate, One-Space Variable, Linear*, NAVORD Report 5562, U. S. Naval Ordnance Test Station, California, 1957, iv + 711 p., 28 cm.

This table is concerned with listing the solution of the heat conduction equation in a plate of finite thickness, with heat transfer at both faces. In mathematical form

$$\begin{aligned} U_{xx} &= kU_t & 0 < X < L, & \quad t > 0 \\ KU_x &= -h_i(U_i - U) & X = 0, & \quad t > 0 \\ KU_x &= -h_0(U - U_0) & X = L, & \quad t > 0 \\ U &= U_0 & t = 0, & \quad 0 < X < L \end{aligned}$$

where the conductivity  $K$ , density  $\rho$ , specific heat  $c$ , diffusivity  $h = c\rho/K$ , heat transfer coefficients  $h_i$  and  $h_0$ , and stagnation temperatures  $U_i$  and  $U_0$  are assumed to be constant.  $L$  is the plate thickness,  $x$  is the distance, and  $t$  is the time. For the tables a dimensionless system of variables is adopted, i.e.,  $x = X/L$ ,  $kL^2\tau = t$ ,  $u = (U - U_0)(U_i - U_0)$ ,  $\alpha_0 = h_0L/K$ ,  $\alpha_i = h_iL/K$ . Then the above problem becomes

$$\begin{aligned} u_{xx} &= u_\tau & 0 < x < 1, & \quad \tau > 0 \\ u_x &= -\alpha_i(1 - u) & x = 0, & \quad \tau > 0 \\ u_x &= -\alpha_0u & x = 1, & \quad \tau > 0 \\ u &= 0 & \tau = 0, & \quad 0 < x < 1 \end{aligned}$$

The analytical solution of this problem is

$$u(x, \tau) = 1 - \frac{\alpha_0(1 + \alpha_i x)}{\alpha_0 + \alpha_i + \alpha_0\alpha_i} + 2 \sum_{n=1}^{\infty} \frac{e^{-\beta_n^2\tau}}{\beta_n D'(\beta_n)} \cdot \{\alpha_i[\beta_n \sin \beta_n - \alpha_0 \cos \beta_n] \sin \beta_n x + \alpha_i[\beta_n \cos \beta_n + \alpha_0 \sin \beta_n] \cos \beta_n x\},$$

where

$$D'(\beta) = -\beta(2 + \alpha_0 + \alpha_i) \sin \beta + (-\beta^2 + \alpha_0 + \alpha_i + \alpha_0\alpha_i) \cos \beta,$$

and the  $\beta_n$ 's are the positive roots of

$$(\alpha_0\alpha_i - \beta^2) \sin \beta + \beta(\alpha_0 + \alpha_i) \cos \beta = 0.$$

The table gives the dimensionless temperature  $u$ , where  $0 < u < 1$ , for the following ranges of parameters

$$x = 0(.01).02(.03).05(.05).3(.1).7(.05).95(.03).98(.01)1$$

$$\tau = .001(.0005).002(.001).008(.002).01(.01).08(.02).1(.1).8(.2)1(1)8(2)10(10)$$

$$80(20)100(100)800(200)1000$$

$$\alpha_i = .001, .002, .004, .006, .01, .02, .04, .06, .1, .2, .4, .6, 1, 2, 4, 6, 10, 20, 40, 60, 100, 200, 400, 600, 1000.$$

$$\alpha_0 = 0, .001, .004, .01, .04, .1, .4, 1, 4, 10, 40, 100, 400, 1000.$$

The tabular entries are given to five figures, with better than three of the figures being accurate. For the most part, the error appears to be one or two units in the fifth figure.

The table should be very useful to those people who are engaged in design work involving heat transfer, as, for example, rocket nozzle design. The introduction also contains a generalization of the heat conduction problem defined above which can be solved by means of the tables.

The tabular entries are printed with reasonable clarity. There are, however, a few obvious misprints in the introduction.

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17[S].—G. A. BARTHOLOMEW & L. A. HIGGS, *Compilation of Thermal Neutron Capture Gamma Rays*, Report AECL No. 669, 1958, 146 p., 27 cm. Available from Scientific Document Distribution Office, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada. Price \$2.50.

This report presents a compilation of energy, absolute intensities, and spectral distribution of gamma rays produced by capture of thermal neutrons, together with a complete bibliography of information on this subject through June 1, 1958. The results obtained from measurements at the Chalk River Laboratories over several years using the pair spectrometer have been reviewed. These results have been modified where necessary such that all intensity determinations are presented on a uniform basis. The accuracy of pair spectrometer intensity measurements is discussed.

Included in the tables are the energies and intensities (photons per hundred captures) of resolved gamma rays obtained from experiments in which absolute intensities were determined. References to other data not tabulated is also given. Where an appreciable portion of the gamma ray spectrum is unresolved, a spectral distribution curve is given. Most of the curves plot the number of gamma rays per capture per Mev as a function of energy. Results published by the Moscow group are included. A rough measure of accuracy and completeness of the results is given with each tabulation and with most of the curves.

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18[S].—M. E. ROSE, *Internal Conversion Coefficients*, Interscience Publishers, Inc., New York, 1958, xxii + 173 p., 31 cm. Price \$6.25.

When a nucleus is in an excited state for which the excitation energy is insufficient for the emission of nuclear particles, the de-excitation will proceed predominantly by either one of two competing mechanisms. Either a  $\gamma$ -ray will be emitted or the nuclear excitation will be transferred to one of the orbital electrons resulting in the ejection from the atom of this electron. The latter process is referred to as internal conversion. If  $N_e$  is the number of conversion electrons emitted per second and  $N_\gamma$  is the number of photons emitted per second, the internal conversion coefficient  $\alpha$  is defined as

$$\alpha = N_e/N_\gamma.$$

This book gives a comprehensive account of a ten-year program devoted to the calculation of internal conversion coefficients. It contains a ten-page introduction which gives a precise and thorough account of the physical and numerical approximations made in the course of the calculations of the tables. These in turn constitute the bulk of the work, 164 pages.

The conversion coefficients are strongly dependent on  $k$ , where  $kmc^2$  is the transition energy,  $Z$  the atomic number,  $L$  the angular momentum change, and on  $\Delta\pi$ , the parity change. The tables list values of  $\alpha_L$  and  $\beta_L$ , ( $L = 1, 2, 3, 4, 5$ ), the coefficients for  $2^L$  electric pole and  $2^L$  magnetic pole conversions, respectively, for  $k = 0.05(0.05)0.2(0.2)1.0(0.5)2.0$  and  $Z = 25(1)95$ .

Also included in the tables are values of certain radial matrix elements  $R_k(m)$  and  $R_k(e)$  for the  $K$  shell. These are uncorrected for screening and for finite nuclear size.

The author lists the sources of error in determination of the radial wave functions, which are a fundamental set of intermediate quantities in the calculation of the tables, and expresses the view that all of these errors are small and amount at most to 1–3 per cent. He expresses the view that the irreducible minimum error involved in any calculation of internal conversion coefficients, aside from nuclear structure effects, is just smaller than the experimental error in the best measurements now available.

The author states that, "interpolation in the tables will be necessary only in the energy variable  $k$ . For this purpose interpolation on a  $\log \alpha$  or  $\log \beta$  versus  $\log k$  is advisable since the plots of the conversion coefficients on a log-log scale show very little curvature."

A. H. T.

19[S].—C. H. WESTCOTT, *Effective Cross Section Values for Well-Moderated Thermal Reactor Spectra*, Report AECL No. 670, 1958, 27 p., 27 cm. Available from Scientific Document Distribution Office, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada. Price \$1.00.

This report is concerned with the determination of an effective neutron absorption cross section, which cross section is recommended for calculations involving reaction rates. The effective cross section is defined in terms of a neutron density distribution per unit velocity. The neutron spectrum assumed consists of a Maxwellian distribution at a temperature  $T^\circ\text{K}$  plus an admixture of a  $1/E$  distribution

of flux per unit energy interval, the admixture being controlled by an epithermal index  $r$ . For  $r = 0$ , the spectrum is a pure Maxwellian.

Using this spectrum, the effective cross section,  $\sigma$  can be written as

$$\sigma = \sigma_{2200}(g + rs)$$

where  $\sigma_{2200}$  is the microscopic absorption cross section at 2200 m/sec. The  $g$  and  $s$  factors depend on the shape of the absorption cross section as a function of neutron energy. Specifically for nuclides obeying the  $1/v$  law over the entire energy range,  $g = 1$  and  $s = 0$ .

The accuracy of the results obtained depends on the input data used which, in general, has been taken from the 1958 revision of the Brookhaven *Neutron Cross Section Compilation*. Tables of  $\sigma$  and  $g$  and  $s$  factors in 20 centigrade degree steps from 20°C to 760°C are listed for  $r = 0.03$  or  $r = 0.07$ . These values of  $r$  correspond to average parameters appropriate for the moderator and fuel rods respectively of the NRX Reactor. In some instances  $\sigma$  for  $r = 0$  are given. Elements which follow the  $1/v$  law fairly closely in the thermal region do not usually have  $g$  values listed. The applicability of the compilation is limited to well moderated reactors and to thin samples in which self shielding has been neglected.

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20[T].—ALVIN GLASSNER, *The Thermochemical Properties of the Oxides, Fluorides, and Chlorides to 2,500° K*, Argonne National Laboratory ANL-5750, U. S. Government Printing Office, Washington, D. C., 1957, vi + 74 p., 27 cm. Price \$45.

This table gives empirical equations for the thermodynamic properties per mole (heat capacity  $C_p$ , enthalpy  $H$ , entropy  $S$ , and gives free energy  $F$ ) in the form of power series in the absolute temperature  $T$ . The heat capacity at constant pressure is fitted to the equation:

$$C_p = a + (b \times 10^{-3})T + (c \times 10^{-6})T^2 + \frac{d \times 10^5}{T^2}$$

Only three parameters are evaluated; either  $c$  or  $d$  is set equal to zero. Integration of the heat capacity to give  $H$ ,  $S$ , and  $F$  requires two additional constants of integration,  $A$  and  $B$ . The coefficients are tabulated for solid, liquid, and gaseous states of the elements (Table I—3 p.), the oxides (Table II—4 p.), the fluorides (Table III—5 p.) and the chlorides (Table IV—5 p.). Each of these tables includes, in addition, the heat and entropy of the phase transitions, the entropy at 298°K, and appropriate references to the source material. Tables V, VI, and VII give the enthalpy and free energies of formation of each substance from the elements at 298°K, as well as the coefficients  $\Delta\alpha$ ,  $\Delta b$ ,  $\Delta c$ ,  $\Delta d$ ,  $\Delta A$ , and  $\Delta B$  needed to calculate values at other temperatures.

The publication concludes with 21 pages of graphs showing the temperature dependence of  $\Delta F_f$  from 300° to 2500°K.

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- 21[W].—KENNETH J. ARROW, SAMUEL KARLIN & HERBERT SCARF, *Studies in the Mathematical Theory of Inventory and Production*, Stanford University Press, Stanford, California, 1958, x + 340 p., 25 cm. Price \$8.75.

This work is the initial volume in the Stanford Mathematical Studies in the Social Sciences. The reviewer joins the principal authors in recommending careful attention to the first two chapters: in Chapter I, Arrow presents a remarkably concise and enlightening discussion which, more constructively than anything else the reviewer has read, relates inventory theory to economics; in Chapter II, this useful survey is continued as the principal authors treat common features of many inventory models after placing them within a realistic framework for decision models. The Introduction ends with summaries of results of the remaining three parts: Optimal Policies in Deterministic Inventory Processes, Optimal Policies in Stochastic Inventory Processes, and Operating Characteristics of Inventory Policies. This book is judged to devote reasonable attention to computing problems both for calculation of solutions and for illumination. Reading of individual chapters has convinced the reviewer that the general promises on computing made by the authors on pages 16–19 were honestly kept. The frequent graphs and tables are uniformly helpful and pleasing. A bibliography of four pages covering mainly the years 1955–1957 is also included. It is to be hoped that subsequent volumes of this Stanford Series will push forward into the wide reaches of inventory problems including, for example, areas of demand prediction, measures of utility for satisfying differing demand patterns, and even seemingly prosaic questions such as how to maintain records of extensive inventory systems. In summary, this book is a substantial contribution to the mathematics of inventory and production problems. Since it provides a sound exposition over quite a broad range, it should serve as a valuable source for further research.

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- 22[W, Z].—DANIEL D. McCracken, HAROLD WEISS, & TSAI-HWA LEE, *Programming Business Computers*, John Wiley & Sons, Inc., New York, 1959, xvii + 510 p., 24 cm. Price \$10.25.

Here is a well-written book about the elements of programming high-speed electronic computers. It is particularly written for those people who are interested in the programming of management data problems. The book touches upon down-to-earth details such as verifying the program accuracy, input and output programming, and rerun techniques. It discusses the advantages and disadvantages of machine-aided coding. In general, *Programming Business Computers* is a comprehensive survey of programming with special emphasis on business data processing.

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- 23[X].—J. N. GOODIER & P. G. HODGE, JR., *Elasticity and Plasticity*, v. 1, Surveys in Applied Math. Series, John Wiley & Sons, Inc., New York, 1958, ix + 152 p., 23 cm. Price \$6.25.

The articles of this series are said to be "aimed at a broad, mathematically literate audience looking for an up-to-date account of modern progress in applied mathematics and an appraisal of future promising research directions." The first author of Volume 1, J. N. Goodier, who contributed the article entitled, "The Mathematical Theory of Elasticity," assumed that the reader was well versed in the theory of elasticity, and made no effort to make his article self-contained. He contented himself with a brief survey of "those significant recent developments believed least known to readers whose first language is English." However, even this intention is not fully carried out and a list of topics omitted is given at the end of the article. Three pages of bibliography are given, which include only those books and papers actually discussed or cited.

The major portion of the discussion deals with work of Russian authors. Great stress is given to the work of Muskhelishvili and to investigations inspired by it.

There is no mention in this article of the application of numerical methods to problems in elasticity, aside from a reference to the survey of numerical methods in conformal mapping given by G. Birkhoff, D. M. Young and H. Zarantonello, in *Proc. Symp. Appl. Math*, v. 4, 1953, p. 117.

The second article written by Phillip G. Hodge and entitled, "The Mathematical Theory of Plasticity," is practically self-contained and satisfies, very well, the needs of the member of the audience described in the first paragraph of this review.

Chapters 1 to 4 of this article are theoretical in nature, and Chapters 5 to 7 are concerned with applications of the theory. In particular, Chapter 5 is a well written, moderately exhaustive treatment of the behavior of a simply supported circular plate under a uniform normal pressure. In Chapter 6 other problems are discussed more briefly in an attempt to illustrate the current state of development in plasticity problems.

Particular attention is paid to significant Russian contributions. Chapter 7 contains transcription of parts of a report by W. Prager on Russian contributions up to 1949, and a section by the author entitled, "Contributions from 1949 to 1955."

There is no mention made in this article of the applications of numerical methods to problems in plasticity.

A. H. T.

24[X].—L. V. KANTOROVICH & V. I. KRYLOV, *Approximate Methods of Higher Analysis*, Translated from the third Russian edition by Curtis D. Benster, Interscience Publishers, Inc., New York, 1958, xv + 681 p., 24 cm. Price \$17.00.

In the April 30, 1959 issue of *Le Monde*, on page 5, there is a description of the organization of scientific activities in Russia, in the course of which the following remark is made: "Contrairement aux Américains, les Russes paraissent parfaitement au courant de la littérature mondiale." The author is Maurice Letort, "président du comité consultatif de la recherche scientifique et technique."

One would like to be indignant, but unfortunately the gibe is deserved. In fact, many Americans who visit Russia, or otherwise make contacts with Russian scientists, are amazed at how up-to-date their acquaintance is with American literature, which implies that their own acquaintance with Russian literature is much less so. However, the article in *Le Monde* also provides a partial explanation by describing the extensive Russian facilities for translating and abstracting (2000 full time



“collaborators,” plus many part time). It is hardly necessary to comment on the meagerness of our own facilities.

Until recently, even reviews and abstracts of Russian literature were pitifully sparse, although here there has been vast improvement. The original Kantorovich-Krylov has been known and appreciated by a few Americans, probably largely due to informal publicity given it by George Forsythe, who encouraged the making and publishing of the present translation. But *Mathematical Reviews* has no review of the second edition, published in 1941, and for the third edition it listed chapter headings and remarked, in an unsigned article, only that the edition differed very little from the previous one.

At any rate, we can be grateful to translator and publisher for the present volume. The book itself is concerned mainly with the numerical solution of partial differential equations, as the title to the first edition (1936) indicated. The first chapter deals with expansion in series, both orthogonal and nonorthogonal, with a section on the improvement of convergence. Next come methods of solution of Fredholm integral equations with applications to the Dirichlet problem. Then comes a chapter on difference methods, and one on variational methods. This accounts for slightly more than half of the book. There follow two chapters, for a total of nearly 250 pages, on conformal methods, and finally about 50 pages on Schwarz's method. Throughout, the presentation is extremely readable, with the inclusion of numerous examples, but no exercises. Unfortunately there is no index, either, although the table of contents is fairly detailed (5 pages).

In organization the translation deviates from the original only in collecting the references at the end, with footnotes referring to author and number. This I consider to be desirable. In detail the translation is faithful and quite clear. At times the phraseology is too faithful for elegance, and on rare occasions the translator is even led astray. One such example occurs on page 7: “Just as there, we may separate the problem into two, and moreover in each case the conditions are null on two sides.” While the reader should understand what is meant, there are two faults to find here. First, “pričem” should be translated as “where,” not “and moreover.” Second, a condition cannot be null. I confess, I do not understand the construction in the original, which is “usloviya nulevye,” and perhaps the translator can be forgiven for assuming the adjective to be in predicate form in spite of the ending. Perhaps the authors themselves were careless.

But one could always find fault with details, whereas the important thing is that the book is now available to readers of English. Again our thanks to publishers and translator.

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25[Z].—FRANZ L. ALT, *Electronic Digital Computers*, Academic Press, New York, 1958, x + 336 p., 23 cm. Price \$10.00.

In the preface to his book, Alt addresses himself primarily to “physicists, chemists, engineers and others in similar occupations who have occasion to require the solution of computational problems by means of digital computing machines.”

Alt's primary motivation with respect to this audience is to provide sufficient introductory information to improve communication between the "originator of a computing problem and the team around the machine."

The Introduction, Part I, leads the reader gracefully toward Alt's objectives with a well formulated statement of the stages through which a problem goes on its way to the number factory. Having outlined the required process of problem formulation to problem analysis to programming to coding to machine computation, the author proceeds to discuss the process in reverse. Thus, Part II provides a functional survey of automatic digital computers and Part III discusses programming and coding. By the time the object audience reaches more familiar ground in Part IV (Problem Analysis) and the statement of computer applications in Part V, it has gained the hindsight necessary to make modern computing methods more palatable.

The historical survey of automatic digital computers in Part II is somewhat weakened by the selection of the memory type as a principal classification characteristic. The more significant factor, of operation time, or conceptual factors such as the stored program are therefore undermined. However, the bulk of Part II adequately introduces the key factors differentiating one machine from another, that is, number representation and memory, arithmetic, control and input-output organs. The reviewer felt a lack of graphical presentation of the material in Part II—the inclusion of so much data in the same form as the normal prose is likely to lose the reader for whom this book is written.

Part III remains consistent in its reverse discussion of programming and coding, by describing coding first and then programming. A 4-address instruction set is defined and the coding of a simple arithmetic expression and trigonometric function is used as a vehicle for explaining coding operations. Single address coding is also described in the terminology of IBM manuals. The sections on programming demonstrate the use of flow charts and define factors and terminology significant to computing tactics.

Part IV, covering problem analysis, is by far the strongest part of the book. It collects, in very readable form, methods of computation used in numerical solution of ordinary differential, partial differential, and algebraic equations. As mentioned in the preface, there is no attempt at rigor in this presentation. The discussion of numerical methods is well seasoned with qualitative comments reflecting computational experience and with references to the 210 papers listed in the appended bibliography.

This book cannot by itself serve as a text for the classroom. This book will not serve as a reference textbook in the sense of its cataloging completeness. However, it is this reviewer's feeling that *Electronic Digital Computers* will be found on reference shelves for many years, by virtue of its very readable presentation of Alt's extensive experience in high-speed computation. The goal, established in the preface, is very adequately achieved.

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26[Z].—JAMES T. CULBERTSON, *Mathematics and Logic for Digital Devices*, D. van Nostrand Co., New Jersey, 1958, x + 224 p., 23 cm. Price \$4.85.

This text is intended for students with a background of college algebra who plan to enter the computer field. As the author points out in his first sentences, it “presents neither a course in programming computers nor a mathematical analysis of computing mechanisms. Preliminary to these things, it provides a course in some mathematics which the student will find useful later”. Specifically, the material covered includes some combinatorics and probability, Boolean algebra and propositional calculus, with applications to switching circuits. These topics are covered by a text written in a highly readable colloquial style, plentifully illustrated by examples and by the tremendous total of 1262 numbered exercises. (The latter are “dressed-up”, provided with continuity and some attempt at humor in a manner which the reviewer finds slightly repellent—but this is a matter of taste. The device does have the merit of permitting problems which are essentially identical to appear in radically different formulations.)

The unifying concept of the first part of the text is that of the neuron model. Various successive refinements of the receptor-central-effector system are introduced, leading to adders and other complicated input-output systems. Chapter I is introductory, presenting the summation and product notations and the ideas of an algorithm and iterative approximation. Unfortunately, two of the five examples of the summation notation are incorrect, and there is no clear distinction between an algorithm and an iteration. After correctly defining the former as concluding in a finite number of steps, the author introduces “Newton’s algorithm” for the square root, which is an example of the iteration process. Further, he states the completely false result that  $B = \frac{1}{2}(A + N/A)$  is always a better approximation to  $\sqrt{N}$  than  $A$ . These points will illustrate that statements in the text must be carefully watched; accuracy has frequently been sacrificed to simplicity of statement.

Chapters II and III present the basic facts about combinations, permutations, and elementary frequency probability. Chapter IV is an excellent elementary account of arithmetic in various radix systems and of conversion from one system to another. The various mechanical procedures, such as subtraction by complementation and division by subtraction, are considered in detail. Except for minor matters of choice of language, this chapter may be highly recommended.

The second portion of the book deals with logic. The larger part of Chapter V is an exposition of the syllogistic logic in its full mediaeval pattern, including even the vowel notation of the scholastics, and omitting only the traditional mnemonics, Barbara, Darii, etc. The reviewer finds this portion of the book utterly astounding. It is as if one were to come across a long commentary on *De Rarum Natura* in a text on modern atomic physics. Neither mathematicians nor computer engineers use syllogisms; what purpose can this chapter serve? The latter portion discusses relations, omitting reflexivity and giving much more stringent definitions of one-many and many-one than customary.

The final three chapters develop successively Boolean algebra, the propositional calculus, and the model of the latter in terms of switching circuits. The reduction to normal form and initial simplification are well presented. Too much reliance is placed on checking by means of Venn diagrams. The student is not adequately

warned that, while a Venn diagram illustration of Boolean inequalities is always available, equality in a particular Venn diagram does not necessarily imply general equality.

In summary, the text covers matters of algebra, arithmetic, and logic that students should know before taking advanced courses in logical programming or component design. It may be recommended as a text or for collateral reading if the instructor will warn the student of possible pitfalls and inaccuracies.

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27[Z].—DEPT. OF THE ARMY, *Catalog of Commercially Available Automatic Data Processing Systems*, Department of the Army Pamphlet No. 1-250-4, 1958, 107 p., 26 cm.

This pamphlet is an excellent compilation and description of automatic data processing systems which are commercially available as of July, 1959. It contains the descriptions of twenty-five digital data processing systems ranging in size from the Bendix G15D and the Royal McBee L'GP30 to the UNIVAC 1105 and the DATAmatic 1000. A photograph is included with each computer description.

This compilation is subdivided into categories entitled, respectively, General Description, System Components, Programming, Personnel Requirements, and Site Preparation.

The systems component category describes a typical computer configuration consisting of central processor, arithmetic unit, input-output control, high speed memory, magnetic tape units, paper tape units, card readers and punches, and high speed printers. This category lists some pertinent characteristics such as word length, numeric characters per word, timing, pulse repetition rate, size of memory, checking, and error correcting features. A rather complete list of specifications and characteristics of input and output media is included. The instruction word structure is also presented.

The personnel requirements category recommends a programming and operating complement of personnel but excludes maintenance personnel. Manufacturer's training of operators and programmers is discussed, with an option of training at the manufacturer's premises or at the installation site.

The over-all floor space, floor loading, and air conditioning requirements are also given in the site preparation category.

However, the major contributions of this pamphlet are the tables of cost, power requirements, and physical characteristics of each unit. These tables also present rental, purchase, and maintenance costs.

This reviewer considers this pamphlet an excellent guide to all computer users who require a ready reference in the field.

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28[Z].—MARTIN GARDNER, *Logic Machines and Diagrams*, McGraw-Hill Book Company, Inc., New York, 1958, ix + 157 p., 23 cm. Price \$5.00.

Only on rare occasions does one come across a book written on a technical subject which is entertaining as well as informative. It is also unusual to find a book on the mechanistic aspects of formal logic which does not begin with either Venn diagrams or a description of Boolean algebra. The author has presented an historical survey of the subject in a somewhat narrative fashion, beginning with an almost complete biography of Ramon Lull and ending with speculations on the future of logic machines. The "References" after each chapter are considerably more than just references, and make as interesting reading as the text. The book is by no means devoid of the author's opinions and no attempt has been made at concealment. In fact, it is amusing to note that, even though some of the artifacts and methods described in the book are treated somewhat modestly by the author, he cannot resist the temptation to devote a chapter to a method which he, himself, has devised. This, I am sure, is understandable to any person who has worked in the field.

Persons interested in the field of logic, either as a subdivision of philosophy or as an aid to digital computer design, will find the book well worth its reading time. My only complaint is that he has not included the work done in the area of computer design, which could be interpreted as legitimate subject matter under this title.

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