side of eq. (4). The hypergeometric function appearing in eq. (4) can be computed as described above but convergence is slow for values of x close to unity, and in this case it may be desirable to compute the functions differently [4].

A subroutine [5] has been written for the IBM 704 to calculate the toroidal harmonics using the methods described here. For given m and x a table is obtained of the first twenty values of $P_{n-\frac{1}{2}}^m(x)$ and $Q_{n-\frac{1}{2}}^m(x)$ i.e. $n = 0, 1, \cdots, 19$ as well as their derivatives using the formula [2]:

$$(5) \qquad (x^2 - 1) \frac{d}{dx} R_{n-\frac{1}{2}}^m(x) = (n - m + \tfrac{1}{2}) R_{n+\frac{1}{2}}^m(x) - x(n + \tfrac{1}{2}) R_{n-\frac{1}{2}}^m(x)$$

Because the functions increase very rapidly with both m and x, it is convenient to make the restriction $x < 40$, $m \leqq 21$. Where tabulated values exist [2], the code is found to give full agreement. In some other cases, the equation for the Wronskian [3] of the solutions was checked and found to be very accurately satisfied. The code computes correctly the toroidal harmonics to at least six significant figures.

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1. M. Goldstein & R. M. Thaler, "Calculation of Bessel functions," *MTAC*, v. 13, 1959, p. 102–108.
2. NBS Mathematical Tables Project, *Tables of the Associated Legendre Function*, Columbia University Press, New York, 1945.
3. P. M. Morse & H. Feshbach, *Methods of Theoretical Physics*, McGraw-Hill Book Company, New York, 1953.
4. J. P. Auffray (private communication).
5. A. Rotenberg, *NU ASLG, Share Distribution 711*, Share Program Librarian, IBM, 590 Madison Avenue, New York 22, New York.

Transcendental Equation for the Schrödinger Equation

By J. R. M. Radok

The problem of the determination of the energy levels of single particles in cylindrical wells of different dimensions reduces to the transcendental equation

$$\mathfrak{h}_\lambda^{(1)}(i - \sqrt{k^2 - \epsilon^2}) + \mathfrak{y}_\lambda(\epsilon) = 0$$

for the Schrödinger equation, where $\mathfrak{h}_\lambda^{(1)}$, $\mathfrak{y}_\lambda$ are modified quotient Bessel functions for which a table has been published recently by Morio Onoe [1] and the variables

$$k^2 = \frac{2mUa^2}{\hbar^2}, \qquad \epsilon^2 = 2m \frac{(E + U)a^2}{\hbar^2}$$

involve the quantities

$$m = \text{mass},$$

$$U = \text{potential energy},$$

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Table 1

	$\lambda = 0$					$\lambda = \pm 1$					$\lambda = \pm 2$				$\lambda = \pm 3$			
k	ϵ^k_{10}	ϵ^k_{20}	ϵ^k_{30}	ϵ^k_{40}	ϵ^k_{50}	ϵ^k_{11}	ϵ^k_{21}	ϵ^k_{31}	ϵ^k_{41}	ϵ^k_{51}	ϵ^k_{12}	ϵ^k_{22}	ϵ^k_{32}	ϵ^k_{42}	ϵ^k_{13}	ϵ^k_{23}	ϵ^k_{33}	ϵ^k_{43}
1	0.97	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
2	1.52	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3	1.80	—	—	—	—	2.71	—	—	—	—	—	—	—	—	—	—	—	—
4	1.95	3.94	—	—	—	2.99	—	—	—	—	3.90	—	—	—	—	—	—	—
5	2.03	4.41	—	—	—	3.13	—	—	—	—	4.16	—	—	—	—	—	—	—
6	2.10	4.61	—	—	—	3.23	5.75	—	—	—	4.31	—	—	—	5.32	—	—	—
7	2.13	4.77	—	—	—	3.32	5.99	—	—	—	4.43	—	—	—	5.49	—	—	—
8	2.17	4.84	7.47	—	—	3.39	6.13	—	—	—	4.51	7.28	—	—	5.60	—	—	—
9	2.20	4.93	7.63	—	—	3.45	6.24	8.82	—	—	4.60	7.45	—	—	5.70	8.56	—	—
10	2.21	4.99	7.78	—	—	3.49	6.32	9.04	—	—	4.68	7.58	—	—	5.78	8.73	—	—
11	2.23	5.02	7.84	10.50	—	3.52	6.40	9.20	—	—	4.72	7.66	10.40	—	5.84	8.83	—	—
12	2.25	5.05	7.92	10.69	—	3.56	6.46	9.30	11.89	—	4.75	7.72	10.57	—	5.89	8.92	11.72	—
13	2.27	5.08	8.00	10.80	—	3.59	6.52	9.38	12.12	—	4.78	7.78	10.68	—	5.93	9.00	11.88	—
14	2.29	5.09	8.04	10.89	13.56	3.60	6.56	9.45	12.25	—	4.80	7.83	10.75	13.52	5.96	9.05	11.99	—
15	2.30	5.10	8.09	10.98	13.71	3.62	6.60	9.51	12.36	14.98	4.81	7.88	10.82	13.68	5.99	9.10	12.07	14.84

$$E = \text{total energy},$$
$$2\pi h = \text{Planck's constant},$$
$$a = \text{radius of the well}.$$

Using this table, the first few roots have been obtained graphically and are recorded in Table 1 to three significant digits. For most practical purposes, these values should be satisfactory. If necessary, they can be improved by use of Newton's method.

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1. M. Onoe, *Tables of Modified Quotients of Bessel Functions of the First Kind for Real and Imaginary Arguments*, Columbia University Press, New York, 1958.

A Note on Factors of $n^4 + 1$

By A. Gloden

The factorizations enumerated in this note form a sequel to my published factor table [1] of integers $n^4 + 1$. They have been obtained by means of my table of solutions of the congruence $x^4 + 1 \equiv 0 \pmod{p}$ for primes lying between $8 \cdot 10^5$ and 10^6 [2].

The following numbers are primes:

$n^4 + 1$ for n = 912, 914, 928, 930, 936, 952, 962, 966, 986, 992, 996.

$\frac{1}{2}(n^4 + 1)$ for n = 1071, 1087, 1101, 1119, 1123, 1125, 1135, 1163, 1173, 1183.

$\frac{1}{17}(n^4 + 1)$ for n = 1562, 1726, 1732, 1834.

$\frac{1}{41}(n^4 + 1)$ for n = 1818, 1848, 1982, 2006, 2012, 2064, 2088, 2094, 2228, 2340, 2364.

$\frac{1}{73}(n^4 + 1)$ for n = 2346.

$\frac{1}{89}(n^4 + 1)$ for n = 2262, 2302, 2544, 2682.

$\frac{1}{113}(n^4 + 1)$ for n = 2468.

$\frac{1}{137}(n^4 + 1)$ for n = 2476.

$\frac{1}{233}(n^4 + 1)$ for n = 2808.

$\frac{1}{2 \cdot 17}(n^4 + 1)$ for n = 1709, 1715, 1759, 1787, 1827, 1845, 1855, 1879, 1895, 1963, 2015, 2021, 2031, 2093, 2185, 2229, 2259, 2287, 2303, 2327, 2331.

$\frac{1}{2 \cdot 41}(n^4 + 1)$ for n = 2211, 2299, 2651, 2761, 2791, 2815.

$\frac{1}{2 \cdot 73}(n^4 + 1)$ for n = 2533, 2577, 2691, 2723, 2857.

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