

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[A-F, G-I, K, M]—NINA M. BURUNOVA, *Spravočnik po matematičeskim tablitsam, Dopolnenie N. 1 (Handbook on Mathematical Tables, Supplement No. 1)*, Izdatel'stvo Akad. Nauk SSSR, Moscow, 1959, 184 p., 26 cm. Price 9 rubles.

This is a first supplement to the important work by Lebedev and Fedorova [1], which appeared in 1956. The arrangement is similar to that of the original, with a Part I in fifteen chapters listing the contents of the various tables, which are referred to by number, and a Part II giving the numbered references, separately for each chapter.

With two small exceptions, the titles of the chapters are unchanged. The translated heading of Chapter IV, formerly "Decimal and natural logarithms," is now simply "Logarithms"; this allows the inclusion of a few tables of logarithms to base 2. Chapter XV, besides the former "Prime numbers, factors, products, quotients and fractions," now covers also "Conversion from one system of numeration to another"; this accommodates a few tables which deal with binary-decimal and similar conversions. A three-page list of functions, giving references to both *Handbook* and *Supplement*, also shows a few additional categories not involving alteration of chapter headings.

As in the original Part II, references to works in Russian are naturally given in Cyrillic characters, but the far more numerous references to other works are given in roman characters. A mathematician knowing no Russian could probably contrive to use the greater part of the book.

Like the original work, the supplement is very welcome.

A. F.

1. A. V. LEBEDEV & R. M. FEDOROVA, *Spravočnik po matematičeskim tablitsam*, Moscow, 1956. See RMT 49, MTAC, v. 11, 1957, p. 104-106.

2[A-E, G, J, M, Q]—FRIEDRICH O. RINGLEB, *Mathematische Formelsammlung*, (7th ed.) Sammlung Göschel Band 51/51a, Walter de Gruyter & Co., Berlin, 1960, 320 p., 16 cm. Price DM 3.60.

This little German volume, from the *Sammlung Göschel*, is an enlarged version (42 extra pages, 3 extra figures) of the 1956 edition written by O. Th. Bürkeln. It is a classified collection of formulae and standard theorems from all branches of (essentially) undergraduate mathematics. As such, there is some, but not too much, overlap with similar collections, such as the well-known *Smithsonian Mathematical Formulae*. It is entirely different, however, from the numerical analysis handbook in the *Sammlung Göschel*, *Formelsammlung zur praktischen Mathematik*, by Günther Schulz.

One (probably not anticipated) use to students of this little book is as a pleasant means of picking up a basic mathematical German vocabulary. The sentences are short, factual, and not unduly encumbered by grammar or philosophy.

The sixteen chapter headings are: Arithmetik und Kombinatorik, Algebra, Zahlentheorie, Elementare Reihen, Ebene Geometrie, Stereometrie, Ebene Trigonometrie, Sphärische Trigonometrie, Mathematische Geographie und Astronomie,

Analytische Geometrie der Ebene, Analytische Geometrie des Raumes und Vektorrechnung, Differentialrechnung, Integralrechnung, Funktionentheorie und konforme Abbildung, Differentialgeometrie, and Differentialgleichungen.

D. S.

3[C, L]—L. K. FREVEL & J. W. TURLEY, "Seven-Place Table of Iterated Log_e (1 + x)," The Dow Chemical Company, Midland, Michigan, 1960. Deposited in UMT File.

The n -fold iteration of $\log_e(1 + x)$ is explicitly given by

$$x - \frac{nx^2}{2} + \sum_{t=3}^{\infty} \left\{ (-1)^{t-1} n \left[\frac{1}{t} + \frac{n-1}{2} \sum_{\alpha=0}^{t-3} C_{\alpha} n^{\alpha} \right] x^t \right\},$$

where the C_{α} 's are numerical rational fractions. Using nine terms of this expansion the authors have compiled a 7D table containing 4000 different entries for $n = 0(.05)10$ and $x = 0(.05)1$. All computations were programmed on the DATA-TRON 220, and the output in tabular format was printed directly by a Teletype printer. The recorded values are accurate to $\pm 4 \cdot 10^{-7}$.

AUTHORS' SUMMARY

4[F]—C. L. BAKER & F. J. GRUENBERGER. *The First Six Million Prime Numbers*, The RAND Corporation, Santa Monica, published by The Microcard Foundation, Madison, Wisconsin, 1959, 8 p., 16 x 23 cm. + 62 cards, 7.5 x 12.6 cm. Price \$35.00.

This unusual table is arranged on microcards which present 124 photographs. Each photograph (except the first and last) displays 39 pages of tabulation. Each page lists 1250 prime numbers. The primes range from 1, which is counted as a prime, to 104395289. Each line of a page contains 25 consecutive primes. The first prime in the line is given completely; only the last three digits of the other 24 primes are given. The rank of a prime, once it is located in the table, is given by an obvious formula in terms of its page number, line number, and position in that line. It is just as easy to find isolated values of $\pi(x)$, the number of primes $\leq x$.

It is obvious that the very high condensation of information achieved in this list of primes is not won without some difficulty, namely, the fact that it is invisible to the naked eye. A quite strong pocket magnifying glass or a microcard reading machine is required to read the data. Any use of the table other than finding whether or not a given number is prime or evaluating $\pi(x)$ for isolated values of x is really impractical. For example, to determine the number of twin primes in the 56th million or to calculate a sum involving consecutive primes—problems in which one must carefully keep one's place in the table—would be difficult indeed. Of course, such problems should be done by an electronic digital computer anyway.

There is an interesting description of the IBM 704 program used to generate the list of primes on punched cards, which also contain the differences between consecutive primes.

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EDITORIAL NOTE.—If 2 is counted as the first prime, which is the current practice, then the six millionth prime is 104395301, which happens to be the first member of a prime pair.

5[F].—H. GUPTA, M. S. CHEEMA, A. MEHTA & O. P. GUPTA. Edited by J. C. P. MILLER, *Representation of Primes by Quadratic Forms*, Royal Society Mathematical Tables No. 5, Cambridge University Press, New York, 1960, xxiv + 135 p., 29 cm. Price \$8.50.

The eight tables in this work give solutions a , b and k , n to two related Diophantine equations

$$\kappa p = a^2 + Db^2 \quad \text{and} \quad \kappa kp = n^2 + D,$$

with $\kappa = 1$ and 2 , $D = 5, 6, 10$, and 13 , and for all primes $p < 100,000$ for which solutions exist. In each table there are approximately 2400 primes, that is, about one-fourth of all primes $< 100,000$.

The subtitle indicates that this volume is Part I of a larger work and implies that other values of D will be forthcoming. The significance of these particular values, 5, 6, 10, and 13, is that in all quadratic number fields, $R(\sqrt{-D})$, these are the smallest positive D 's for which unique factorization of the algebraic integers is lacking. In all four cases the class number is 2 (there are two classes of ideals) and this is associated with the two values of κ . For about one-half of the p 's for which $-D$ is a quadratic residue a solution exists for $\kappa = 1$, and for the remaining one-half, for $\kappa = 2$. It may be noted that the $\kappa = 1$ table is always somewhat shorter than its companion $\kappa = 2$ table. This is as expected, since there are *generally* more primes of the form $4Dm + N$ than of the form $4Dm + R$ if R is a quadratic residue of $4D$ and N is not (see *MTAC*, v. 13, 1959, p. 272–284).

There is an interesting, twelve-page introduction to the background ideal and class number theory. It is boldly stated there, p. xii, that unique factorization exists for square-free $D > 0$ only if $D = 1, 2, 3, 7, 11, 19, 43, 67$, and 163 . However, this has never been fully proven. This background theory culminates in several theorems due to H. P. F. Swinnerton-Dyer.

The tables were done by hand, "with the help of two long strips of paper—A and B," and were carefully checked in a variety of ways. Why the solutions to $kp = n^2 + 6$ for $p = 7$ and $p = 31$ are listed, on p. 36, as $k, n = 6, 6$ and $22, 26$ respectively rather than the obvious $1, 1$ and $1, 5$ is not clear to the reviewer. But this slip seems to be exceptional.

The tables may be used as lists of prime ideals in the four fields, $R(\sqrt{-D})$. The dust jacket suggests a second application, that they "may contribute to the understanding of such unsolved questions as 'Is the number of primes of form $n^2 + 5$, or $n^2 + 6$, etc., infinite or finite?'" In this respect, however, it may be remarked that the data here are rather meager since the primes are $< 10^5$, while in 10 minutes an IBM 704 can count such primes up to $3.24 \cdot 10^{10}$ (see *MTAC*, v. 13, 1959, p. 78–86). In fact the 16th prime of the form $n^2 + 5$ is 86441 and is the largest listed in this volume, while the 4368th prime of that form is 32,371, 926, 089. Nonetheless, the data are sufficient to indicate at least rough agreement with the Hardy-Littlewood conjecture. Since

$$L_5(1) = 1.405 > L_6(1) = 1.283 > L_{10}(1) = 0.993 > L_{13}(1) = 0.871$$

we should expect the relative density of primes to increase as we progress from $n^2 + 5$ to $n^2 + 6$ to $n^2 + 10$ to $n^2 + 13$ (*Math. Comp.*, v. 14, 1960, p. 324–326). This is indeed the case.

An apparent anomaly concerned $n^2 + 13$, where there were numerous primes from $n = 0$ to 68 and from $n = 264$ to 298, but none in between. This striking maldistribution was most alarming, and threatened dire consequences to the Hardy-Littlewood conjecture, until the real explanation was found—pages 105 to 120 were missing in the reviewer's copy. Aside from this gross lapse, the volume has the usual elegance of the Royal Society Mathematical Tables.

D. S.

6[F, L].—C. B. HASELGROVE in collaboration with J. C. P. MILLER, *Tables of the Riemann Zeta Function*, Royal Society Mathematical Tables No. 6, Cambridge University Press, New York, 1960, xxiii + 80 p., 29 cm. Price \$9.50.

These important and fascinating tables are concerned primarily with $\zeta(\frac{1}{2} + it)$, the zeta function for real part $\frac{1}{2}$, and with its zeros. This complex function is expressed both in cartesian and polar forms:

$$\begin{aligned}\zeta\left(\frac{1}{2} + it\right) &= \Re\zeta\left(\frac{1}{2} + it\right) + i\Im\zeta\left(\frac{1}{2} + it\right) \\ &= Z(t)e^{-i\theta(t)}.\end{aligned}$$

In the latter, $\theta(t)$ is continuous, with $\theta(0) = 0$, and the signed modulus $Z(t)$, given by

$$Z(t) = \pi^{-1it} \frac{\Gamma\left(\frac{1}{4} + \frac{1}{2}it\right)}{\left|\Gamma\left(\frac{1}{4} + \frac{1}{2}it\right)\right|} \zeta\left(\frac{1}{2} + it\right),$$

changes sign at every zero. The n th zero, γ_n , is the n th solution of $Z(\gamma) = 0$ and the n th Gram point, g_n , is the solution of $\theta(g_n) = n$.

Table I gives $\Re\zeta(\frac{1}{2} + it)$, $\Im\zeta(\frac{1}{2} + it)$, $Z(t)$, and $\theta(t)$ to 6D for $t = 0(0.1)100$. $\Re\zeta(1 + it)$ and $\Im\zeta(1 + it)$ are also listed.

Table II gives $Z(t)$ to 6D for $t = 100(0.1)1000$.

Table III has two parts. Part 1, for $n = 1(1)650$, gives γ_n , g_{n-1} , and $\phi_n = (1/\pi)ph\zeta'(\frac{1}{2} + i\gamma_n)$ to 6D and $|\zeta'(\frac{1}{2} + i\gamma_n)|$ to 5D. Part 2, for $n = 651(1)1600$, gives γ_n to 6D and $|\zeta'(\frac{1}{2} + i\gamma_n)|$ to 5D.

Table IV gives $Z(t)$ to 6D for four other ranges of t ,

$$t = 7000(0.1)7025, \quad 17120(0.1)17145, \quad 100000(0.1)100025, \quad 250000(0.1)250025.$$

Also for these four ranges are given the zeros to 6D and derivatives to 5D. There are 28, 32, 38, and 42 zeros in the four ranges respectively.

Table V gives $(1/\pi)ph\Gamma(\frac{1}{2} + it)$ to 6D for $t = 0(0.1)50(1)600(2)1000$.

The inclusion of g_n and ϕ_n in part 1 of Table III allows the reader to study Gram's "Law" which states that the zeros and Gram points are interlaced:

$$\gamma_{n-1} < g_{n-2} < \gamma_n < g_{n-1} < \gamma_{n+1}.$$

A violation occurs if $|\phi_n| > \frac{1}{2}$. The first violation is for $n = 127$. The first double violation is for $n = 379$ and 380—i.e., there are three Gram points between γ_{379} and γ_{380} . In all, there are 22 violations in these 650 zeros. Gram's "Law" may also be expressed by saying that the complex $\zeta(\frac{1}{2} + it)$ approaches its zero via the 4th or 3rd quadrant. Thus the following statistics for these 650 zeros are of interest: 4th quadrant, 320 cases; 3rd quadrant, 308; 2nd quadrant, 13, and 1st quadrant, 9.

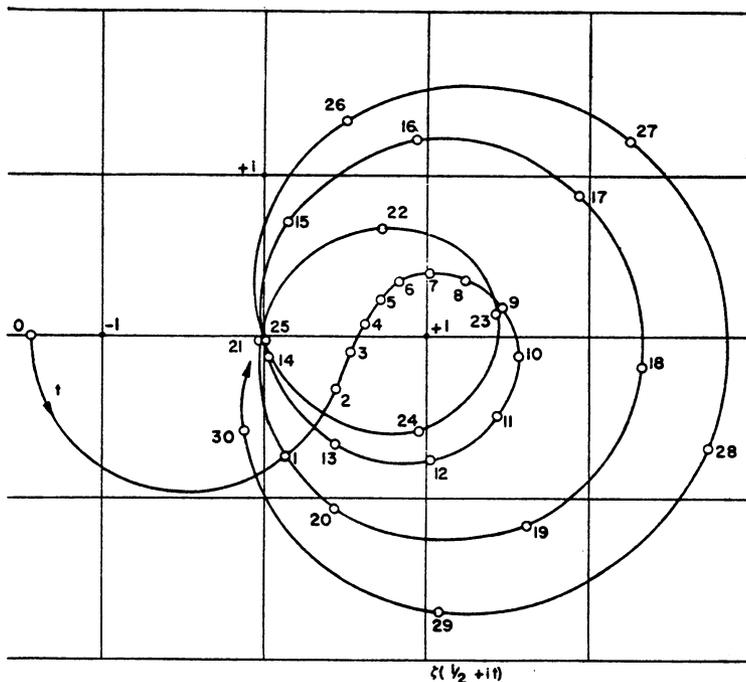


FIG. 1.— $\zeta(\frac{1}{2} + it)$ in the complex plane, for $0 \leq t \leq 30$.

This suggests the conjecture* that $\lim_{N \rightarrow \infty} (1/N) \sum_1^N \phi_n = 0$, that is, that

$$\zeta'(\frac{1}{2} + i\gamma_n)$$

is positive real in the mean. Since violations against the “Law” are associated with exceptionally close zeros or exceptionally large values of $Z(t)$, some extremes discovered by D. H. Lehmer are of interest, and are listed in Table IV:

$$\gamma_m = 7005.062\ 866 \quad \text{and} \quad \gamma_{m+1} = 7005.100\ 565 \quad (m = ?)$$

$$Z(17123.1) = 18.955\ 257.$$

The tables of $\zeta(\frac{1}{2} + it)$ and of $Z(t)$ were both computed on two different machines, EDSAC and the Manchester Mark I, from two different asymptotic formulae, one due to Gram, the second a modification of the Riemann-Siegel formula due to Lehmer. A great deal of difficulty was experienced in eliminating discrepancies between the two methods, and three or four erroneous tables had to be discarded.

* Note added November 10, 1960. For the first 650 zeros this mean value, $\bar{\phi}_n$, shows such a marked trend toward zero that it is even probable that $|\Sigma \phi_n| / \sqrt{N}$ also tends toward zero. Taking only the “worst” points—that is, where $|\Sigma \phi_n|$ reaches a new maximum—sample values of $|\Sigma \phi_n| / \sqrt{N}$ are .0513, .0437, .0380, .0328, .0279, .0251, and .0217 for $n = 8, 33, 64, 126, 256, 379,$ and 606 , respectively. It should be noted that $\bar{\phi}_n$ oscillates as it tends towards zero—up to $n = 650$ it changes sign 265 times. In particular, all 22 of the Gram violations, $|\phi_n| > \frac{1}{2}$, are associated with such sign changes. But this latter is almost inevitable up to $n = 650$, since the extreme values of $\Sigma \phi_n$ up to this limit are only $+5.20578$ for $n = 567$ and $-.533885$ for $n = 606$.

As to uses of the tables, Haselgrove himself has used the values of the first 600 zeros in his disproof of Pólya's conjecture. For later results on this theme, see R. S. Lehman, "On Liouville's function," *Math. Comp.*, v. 14, 1960, p. 311-320.

It is not inconceivable that study of these tables of $\zeta(\frac{1}{2} + it)$ may inspire some investigator to a new approach to the Riemann Hypothesis. Similarly, the table of $\zeta(1 + it)$ can be studied in connection with proofs of the prime number theorem. For both of these "uses," however, a graphical presentation is highly desirable, and it is regretted that a good collection of graphs was not included in this volume. For example, a graph of $\zeta(\frac{1}{2} + it)$ in the complex plane versus the parameter t —say from 0 to 30—is particularly interesting. See Fig. 1. Problem for the reader: If, in Fig. 1, the variable t is thought of as time, explain the initial counterclockwise motion in the orbit and the subsequent clockwise motion with a shorter and shorter mean period. Hint: Consider the formula

$$\zeta(s) = \left(1 - \frac{2}{2^s}\right)^{-1} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$$

for a fixed value of $s = \frac{1}{2} + it$. For what mean value of n does a block of consecutive terms in the series have all its terms in phase; in which block are alternate terms out of phase; and finally, which terms add in an essentially random manner?

Since it is known that Haselgrove has also computed complex zeros of the closely related $L(s)$ and other Dirichlet character series, and since he has not included these results, it would be desirable to issue a companion volume to make these related tables generally available.

Two very minor errors were noted in the Introduction.

Page xii, line 5: change $\zeta(\rho_n)$ to $\zeta'(\rho_n)$.

Page xvi, line 6 from bottom: change (3.7) to (3.32).

With respect to Liénard's table of $\zeta(n)$ for $n = 2(1)167$, which is mentioned on page xx, a recent extension should be noted: J. W. Wrench, Jr., "Further evaluation of Khintchine's constant," *Math. Comp.*, v. 14, 1960, p. 370-371.

D. S.

[G, K, X, Z].—ANTHONY RALSTON & HERBERT S. WILF, *Mathematical Methods for Digital Computers*, John Wiley & Sons, Inc., New York, 1960, xi + 293 p., 27 cm. Price \$9.00.

This book contains contributions from twenty-four research workers in numerical analysis and related fields. Two of the contributors serve as editors, and, as the title implies, all contributors are definitely high-speed-computer oriented. It is interesting to note that ten of the twenty-four authors are from universities.

In the introduction it is stated that "the major purpose of this book is to present many—but by no means all—of the more commonly used tools of the modern numerical analyst along with some of the more promising newly developed methods. The motivation behind this presentation is not only to gather together in one place a partial survey of modern numerical methods but also, in each case, to acquaint the reader with the interplay between computer capabilities and processes of analysis".

The editors have done a good job in carrying out the purpose of the book. They have coordinated the efforts of their colleagues in a remarkable way. About twenty

important and fairly representative problems are discussed by experts in various fields and the presentations, for the most part, are organized in a uniform manner. All but one of the authors describe programs for solving their problems in terms of the following topics:

1. The function of the program
2. A mathematical discussion of the problem
3. A summary of the calculation procedure
4. A complete flow diagram
5. A box by box description of the flow diagram
6. Standard subroutines required by the program
7. A representative sample problem
8. Memory requirements
9. An estimation of the running time
10. A list of references

Actual coding of the programs is omitted, as is any reference to a particular computing machine.

The book is divided into six parts: generation of elementary functions, matrices and linear equations, ordinary differential equations, partial differential equations, statistics, and miscellaneous methods.

Part I contains an interesting discussion of the generation of elementary functions by means of polynomial and rational approximations. Ten of the more common functions are examined in detail, and comparisons of the errors obtained using various approximation methods are included. Because of the nature of its subject matter this part has a format different from that discussed in Paragraph 3. Part II contains six chapters covering matrix inversion, systems of linear equations, and the matrix eigenvalue problem for symmetric matrices. Part III contains four chapters concerned with the numerical solution of ordinary differential equations, and Part IV contains five chapters discussing parabolic, elliptic and hyperbolic partial differential equations. There are four chapters in Part V covering multiple regression analysis, factor analysis, autocorrelation and spectral analysis, and the analysis of variance. The six chapters in Part VI discuss methods for numerical quadrature, Fourier analysis, linear programming, network analysis, and the solution of polynomial equations.

This book should serve as an excellent reference work for those workers who need to solve problems using electronic digital computers. For this reason the format, described above, is especially good. The book is not intended to be a textbook for introducing the subject of numerical analysis, but most students of the subject will find useful information here. Lists at the end of the various chapters contain over two hundred references relating to the problems under consideration.

The reviewer would like to see a supplementary volume appear containing many of the important methods not covered in the present volume. For instance, Part II does not cover the Givens method for finding the eigenvalues of symmetric matrices. Omitted entirely is any discussion of methods for finding the eigenvalues of arbitrary matrices. Other readers will find additional important methods they would like to see covered in a second volume.

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8[G, Z].—FRANZ E. HOHN, *Applied Boolean Algebra, An Elementary Introduction*, The Macmillan Company, New York, 1960, xx + 139 p. \$2.50 (paperback, offset print).

The author gives as much information as it seems possible to include conveniently in a text of this length starting with the veriest elements concerning the use of Boolean algebra in the design of switching circuits—especially for digital computers.

Examples of engineering applications are given, but no complete design of a very extensive arithmetic unit is undertaken. Hence, the ultimate dependence of the designers of computers on Boolean algebra is not completely illustrated, although it is strongly and correctly implied.

The crux of any applications of Boolean algebra to the design of switching circuits lies in computational schemes for writing fairly efficient statements of Boolean propositions. This problem is faced by the author, but only to an extent which permits the reader (and problem worker) to understand the nature of the difficulties which are encountered and some of the procedures which promise to be helpful. In this regard the pamphlet is no less informative than most of the other textbook material available, but additional reference to computational efforts would have been welcome.

The electrical elements to be used are described abstractly in a reasonable way, and altogether the presentation is self-contained, lucid, and reasonably illustrated by problems. No sophistication is required in the reader except for motivation.

Attention is not restricted to circuit design, and the standard applications of Boolean algebra are treated to an extent which is indicated in the chapter headings listed below:

Introduction

Boolean Algebra as a Model of Combinational Relay Circuitry

Boolean Algebra as a Model of Propositional Logic

The Boolean Algebra of the Subsets of a Set

The Minimization Problem

The Binary System of Numeration (Appendix I)

Semiconductor Logic Elements (Appendix II)

It would have been helpful if the author had included various alternate notations. He uses Λ for "or" and no symbol for "and." A short table of notations would be helpful to the neophyte, for not all authors have the thoughtfulness to describe their notation. The bibliography is not extensive.

The printing is by photographic offset process from typed copy, and there was a considerable amount of smearing and a number of ghost images in the review copy. However, these defects did not make reading seriously difficult.

This pamphlet should be a handy introduction to Boolean algebra for many users and a useful adjunct to the texts for several courses which might be offered in colleges.

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9 [K].—B. M. BENNETT & P. HSU, *Significance Tests in a 2×2 Contingency Table: Extension of Finney-Latscha Tables*, July 1960. Deposited in UMT File.

In testing the significance of deviations from proportionality in a 2×2 con-

tingency table

	With Attribute	Without Attribute	Total
Series I	a	$A - a$	A
Series II	b	$B - b$	B
Total	r	$N - r$	N

with fixed marginal totals $A \geq B$, and $a/A \geq b/B$, Finney (1948, *Biometrika* 35, 145) supplied the exact probabilities and the significant values of b at levels .05, .025, .01, .005, respectively for $A = \beta(1) 15$. An extension of the same table for $A = 16(1) 20$ was prepared by Latscha (1953, *Biometrika*, 40; 74). The present tables further extend the exact probabilities (4 decimal places) to the case: $A = 21(1) 25$ and the significant b values at the four levels of the Finney-Latscha tables.

AUTHORS' SUMMARY

10[K].—MORDECAI EZEKIEL & KARL A. FOX, *Methods of Correlation and Regression Analysis*, 3rd edition, John Wiley & Sons, Inc., New York, 1959, xv + 548 p., 24 cm. Price \$10.95.

It is very pleasing to see that Dr. Ezekiel's well-known text, *Methods of Correlation Analysis*, has been modernized. The older excellent book, which for many years stood alone in the desert of statistical literature, has now been joined by a growing array of fine statistical text material. Professor Fox was clearly a wise choice for Dr. Ezekiel to make for the co-author of the present edition.

This book is aimed mainly at the non-mathematical reader, in that algebraic and computational methods are stressed. There is also a wealth of practical material drawn from applied research. However, the mathematical statistician will also find much of great value; for example, the use of digital computers for data analysis is something which not all statisticians are yet well aware of.

In general, the treatment of regression and correlation is quite comprehensive, and results from the recent theoretical literature have been utilized throughout the book. As an example of this point, mention may be made of the inclusion of material dealing with regression and the analysis of variance, time series and errors, and the fitting of simultaneous relations.

The book is divided into seven main sections, with twenty-six chapters in all. The main sections are:

1. Introductory Concepts
2. Simple Regression, Linear and Curvilinear
3. Multiple Linear Regressions
4. Multiple Curvilinear Regressions
5. Significance of Correlation and Regression Results
6. Miscellaneous Special Regression Methods
7. Uses and Philosophy of Correlation and Regression Analysis

There are also three appendices, providing a glossary and important equations, methods of computation, and some technical notes. The inclusion of an author index is also a commendable feature.

Those working in the field of economics, agriculture, and business statistics will find this text of much value, since a good deal of the material is slanted toward

those areas. For teachers, statisticians in general, and members of operations research teams, this revised edition will be found to fill a real need. For applied work in fields like engineering, medicine, sociology, and psychology, the full treatment of the basic concepts of regression and correlation will be of immense value, and some slight reconsideration of the illustrative examples given will often provide insight into the real problems in these other fields.

Thanks are due to both the authors and publishers for making this material available. It is a text that should be in the library of every technical organization.

HARRY WEINGARTEN

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11[K].—E. FIX, J. L. HODGES & E. L. LEHMANN, "The restricted chi-square test," included in *Probability and Statistics*, edited by U. GRENANDER, Almqvist & Wiksell, Stockholm; John Wiley and Sons, New York, 1959, pages 92–108 [See the following review].

This paper contains some new tables of the power function of chi-square tests, i.e., of the non-central chi-square distribution, for small degrees of freedom. As is well-known, the power function $\beta = \beta(\alpha, f, \lambda)$ of a chi-square test depends on three parameters: α , the "level of significance" at which the test of the null hypothesis H_0 is conducted, i.e., the probability of the test falsely rejecting H_0 when it is true; f , the "degrees of freedom" of the test; and λ , the "non-centrality parameter," which measures the "distance" of the alternative $H = H(\lambda)$ under consideration, from the null hypothesis H_0 . The tables of this article give λ to 3D as a function of $\beta = 0.5(0.1)0.9, 0.95$, for $f = 1(1)6$ and $\alpha = 0.001, 0.005, 0.01, 0.05(0.05)0.3, 0.4, 0.5$. The quantity tabulated is that value of the parameter λ which satisfied the equation

$$e^{-(\lambda/2)} \sum_{k=0}^{\infty} \frac{1}{k! 2^{(1/2)f+2k-1} \Gamma(f/2)} \int_{\chi_f(\alpha)}^{\infty} x^{f+2k-1} e^{-(1/2)x^2} dx = \beta$$

where f = number of degrees of freedom and $\chi_f(\alpha)$ is such that

$$\frac{1}{2^{(1/2)f-1} \Gamma(f/2)} \int_{\chi_f(\alpha)}^{\infty} x^{f-1} e^{-(1/2)x^2} dx = \alpha.$$

These tables thus supplement those of E. Fix (1949), reviewed in *MTAC*, v. 4, 1950, p. 206–207.

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12[K].—ULF GRENANDER, Editor, *Probability and Statistics*, Almqvist & Wiksell, Stockholm; John Wiley & Sons, New York, 1959, 434 p., 24 cm. Price, \$12.50.

"Once it had been suggested that a book of studies in probability and statistics should be presented to Harald Cramér in honor of his 65th birthday, the authors

needed little or no persuasion to contribute." This volume, subtitled *The Harald Cramér Volume*, is the result. It constitutes a fitting tribute to Harald Cramér, Professor of Mathematical Statistics and Actuarial Mathematics at the University of Stockholm for over three decades, then President, and now Chancellor of the University of Stockholm—Sweden's outstanding figure in the mathematical theory of probability, mathematical statistics, and actuarial mathematics.

The twenty-one contributors to this volume are from six countries—England (2), Finland (1), France (1), India (1), Sweden (3), United States (13); five are former students of Professor Cramér; and all have enviable international reputations in their respective fields. Their respective contributions are arranged within the volume in alphabetical sequence, with page-lengths indicated in parentheses, and are as follows: T. W. Anderson, "Some scaling models and estimation procedures in the latent class model" (30); M. S. Bartlett, "The impact of stochastic process theory on statistics" (11); J. L. Doob, "A Markov chain theorem" (8); G. Elfving, "Design of linear experiments" (17); W. Feller, "On combinatorial methods in fluctuation theory" (17); E. Fix, J. L. Hodges and E. L. Lehmann, "The restricted chi-square test" (16); U. Grenander, "Some non-linear problems in probability theory" (22); M. Kac, "Some remarks on stable processes with independent increments" (9); D. G. Kendall, "Unitary dilations of Markov transition operators, and the corresponding integral representations for transition-probability matrices" (23); P. Levy, "Construction du processus de W. Feller et H. P. McKean en partant du mouvement Brownien" (13); P. Masani, "Cramér's theorem on monotone matrix-valued functions and the Wold decomposition" (15); P. Masani and N. Wiener, "Non-linear prediction" (23); J. Neyman, "Optimal asymptotic tests of composite statistical hypotheses" (22); H. Robbins, "Sequential estimation of the mean of a normal population" (11); M. Rosenblatt, "Statistical analysis of stochastic processes with stationary residuals" (30); C. O. Segerdahl, "A survey of results in the collective theory of risk" (24); J. S. Tukey, "An introduction to the measurement of spectra" (31); S. S. Wilks, "Non-parametric statistical inference" (24); H. Wold, "Ends and means in econometric model building" (80). Only one of these papers (E. Fix et al) contains a mathematical table of general interest, which is considered separately in the immediately preceding review.

All in all, this volume provides a panoramic and stimulating view of the work at the frontiers of probability and statistical theory and some of their applications. The individual scientist, unless he is intimately concerned with the theory of stochastic processes, will very likely find only a small fraction of its contents of direct interest to him, and, therefore, may consider the volume much too expensive for personal acquisition. On the other hand, it is the type of volume that one expects to find in the library of a university where research in probability and mathematical statistics and their applications is conducted at the post-graduate level, and in the libraries of other organizations where research is carried out in the above and related fields.

Finally, it is to be regretted that the volume does not contain either a photograph or a biography of Professor Cramér at this milestone in his career.

CHURCHILL EISENHART

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13[K].—MASAAKI SIBUYA, "Modal intervals for chi-square distributions," *Ann. Inst. Statist. Math.*, v. 9, 1958, p. 225-236.

Let $f(x)$, $-\infty < x < \infty$, be a continuous unimodal probability density function, and let $L, U (L < U)$ satisfy the conditions: $\int_L^U f(x) dx = 1 - \alpha$ and $f(U) = f(L)$. Then $|L, U|$ is called the $(1 - \alpha)$ -content modal interval for $f(x)$. In this paper $f(x)$ is taken to be the χ^2 density function with ϕ degrees of freedom. Applications are discussed, and a method of computation of modal intervals with given content is developed for this case. Tables are presented of L to 4D, U to 3D, $P_r(x^2 \leq L)$ to 5D, and $(U - L)/(U + L)$ to 4D, for $\phi = 3(1)30(10)100$ and $1 - \alpha = .8, .9, .95, .99$. A previous table [1] gave values of $1 - \alpha$, effectively for $(U - L)/U + L = .01, .05, .10, .20$, and $\phi = 4(4)80$.

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1. M. A. GIRSCHICK, H. RUBIN & R. SITGREAVES, "Estimates of bounded relative error in particle counting," *Ann. Math. Statist.*, v. 26, 1955, p. 276-285.

14[K].—MASAAKI SIBUYA & HIDEO TODA, "Tables of the probability density function of range in normal samples," *Ann. Inst. Statist. Math.*, v. 8, 1957, p. 155-165.

This paper gives details of the calculation of the probability density function $f_n(w)$, for which there are included tables to 4D corresponding to $w = 0(.05)7.65$ and $n = 3(1)20$, where n represents the size of the normal sample. Cadwell's formula [1] is cited as the basis for this calculation.

W. J. DIXON

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1. J. H. CADWELL, "The distribution of quasi-ranges in samples from a normal population," *Ann. Math. Statist.*, v. 24, 1953, p. 603-613.

15[K, S, V].—O. G. SUTTON, *Mathematics in Action*, Harper & Bros., New York, 1960, xvi + 236 p., 20 cm. Price \$1.45 (Paperback).

In the words of its author, "This book, written primarily for the layman, will prove . . . of interest also to students, especially those in the upper forms of schools or in the first years in the university. It is a view of the part played by mathematics in applied science, as seen by a mathematical physicist."

Chapter 1, "The Mathematician and his Task," begins by discussing the meaning of theories in physics and the role of mathematics in the development of these theories. Chapter 2, "The Tools of the Trade," gives special attention to complex numbers and to the development of the calculus and the differential equations of mathematical physics. The approach is essentially that of the physicist ("an infinitesimal quantity [is] one which does not exceed the smallest change of which we can take cognizance in our calculations").

The remaining five chapters, entitled respectively, "Ballistics or Newtonian Dynamics in War," "An Essay on Waves," "The Mathematics of Flight," "Statistics or the Weighing of Evidence," and "Mathematics and the Weather," are essentially independent essays that not only provide illustrations of applied mathematics in

action, but also serve to emphasize fundamentals in both classical and modern physics. An especially notable example is the approach to Heisenberg's uncertainty principle through the problem of the bandwidth required to resolve pulses of short duration. Perhaps the most stimulating chapter is the last, in which the author concludes that "Certain apparently sensible questions, such as the question of weather conditions . . . several days ahead, are in principle unanswerable and the most we can hope to do is to determine the relative probabilities of different outcomes."

There may be some question as to whether the non-mathematical layman will be able to follow all of the development of the last five chapters, and the author is occasionally guilty of extravagance. (Few aerodynamicists, even in the United Kingdom, would be willing to admit that F. W. Lanchester's esoteric volumes *Aerodynamics* and *Aerodometrics* "played a part in aerodynamics not unlike that exercised by Newton's *Principia* in astronomy.") These things notwithstanding, the reviewer believes that the author has succeeded admirably in reaching the goal described in the opening quotation of this review. Indeed, he goes beyond this goal, and the book (especially the individual essays) is warmly recommended to practicing applied mathematicians, as well as to laymen and students.

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16[K, S, W].—C. WEST CHURCHMAN & PHILBURN RATOOSH, Editors, *Measurement: Definitions and Theories*, John Wiley & Sons, Inc., New York, 1959, vii + 274 p., 24 cm. Price \$7.95.

The December 1958 meetings of the American Association for the Advancement of Science included a two-day symposium on Measurement. The dozen papers of this symposium are included in this volume, together with a related paper separately invited but not delivered.

The book is divided into four parts. Part I, "Some Meanings of Measurement," consists of four papers concerned primarily with defining and characterizing the concept of measurement. Part II, "Some Theories of Measurement," contains three papers that approach the definition and characterization of measurement in more formal language—mathematical and logical symbols are more in evidence here—and thus warrant grouping together in a separate class. Part III, "Some Problems in the Physical Sciences," contains three papers that deal with theoretical and practical aspects of measurement in classical and modern physics, plus a paper on "rare events" that is out of place in this volume. Part IV, "Some Problems in Social Science," contains only two papers, one on inconsistency of judgments as a measure of psychological distance and one having to do with experimental tests of a probabilistic theory of economic behavior.

There are in all fourteen contributors, one paper being co-authored. The disciplines they represent are: philosophy (5), psychology (3), psychophysics (1), physics (2), mathematics (1), statistics (1), economics (1), and accounting (1). Philosophy is thus somewhat over-represented; astronomy, the biological and earth sciences, not at all.

In the Preface it is stated that the Symposium "was designed to present contrasts in approaches to the problems of measurement." To this end, "the partici-

pants were chosen from different disciplines" and selected particularly "because it was known that they had different viewpoints on the meaning and significance of measurement." Consequently, this volume "is a book of contrasts," not a unified collection of interrelated essays on measurement, and definitely not a textbook on measurement. At best it presents a broad picture of current thinking on the definition, nature, and functions of measurement, against a background of measurement needs and practices in various disciplines at the middle of the twentieth century. But the "picture" is not all in sharp focus. This is due not so much to differences in expository skill of the authors, as to differences in their objectives. Some seek sharpness of definition at the price of a narrow field of applicability; others demand a broad field of applicability at the sacrifice, if necessary, of sharpness of definition. Furthermore, much of the discussion is at a level of abstraction so far removed from the day-to-day practice of measurement in scientific and industrial laboratories that many who have devoted their lives to measurement of the properties of animate and inanimate things will find a large fraction of the volume very foreign to them, if not entirely unintelligible. Nevertheless, it is a volume that one will expect to find in the library of a university or college where research is conducted at the postgraduate level.

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17[K, Z].—JAPANESE STANDARDS ASSOCIATION, *Random Number Generating Icosahedral Dice (20-face Dice)*, 6-1 Ginza-higashi, Chuo-ku, Tokyo. Price \$2.50 per set of 3 dice + postage \$.70 (up to 9 sets).

This device is a set of three icosahedral dice made of plastic material. The dice are different colors, red, yellow and blue, so that ordered triplets of digits may be generated. Each decimal digit appears on two faces of each die.

The dice were presumably intended to measure 15 millimeters between parallel faces. However, the casting was not particularly good and the measurements listed below were recorded between the ten pairs of faces on the new dice tested. This review was unaccountably lost for several months, and in the intervening period there has been considerable flow of the plastic material so that the measurements are currently considerably worse and actually meaningless, for the faces are clearly no longer plane.

Red	Yellow	Blue
14.74	14.90	15.09
14.97	14.96	14.71
14.82	14.95	14.93
14.88	14.97	14.93
14.94	15.05	14.93
14.76	14.95	14.83
14.85	15.07	15.09
14.96	14.97	14.74
14.83	14.93	14.87
14.79	14.90	15.03

When the dice were new they were tested by 800 rolls each on a level felt surface conforming to the specification of ordinary dice tables. Standard tests [1, 2] applied

to the individual dice and to the group of three detected no bias. Therefore, it seems reasonable to assume that the variations from a true regular icosahedron are minor in terms of the application intended.

Less meticulous samplings have been made with the aged dice, but it seems unlikely that a person generating random decimal digits at a rate which can be met by these dice would notice any serious bias.

The frequencies with which the digits appear can be changed slightly by the usual standard means. These would include weighting to displace the center of gravity (an awkward and cumbersome method at best, and, at worst, one which is difficult to disguise) or applying wax to one or more faces to increase the probability that the waxed face will be on the bottom after the throw.

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1. THE RAND CORP., *One Million Random Digits and 100,000 Normal Deviates*, The Free Press, Glencoe, Illinois, 1955.

2. C. B. TOMPKINS, RMT 11, *MTAC*, v. 10, 1956, p. 39-43.

18[L].—F. M. HENDERSON, *Elliptic Functions with Complex Arguments*, The University of Michigan Press, Ann Arbor, 1960, v + 38 p. + 160 unnumbered p., 29 cm. Price \$8.00.

The tables and charts on the 160 unnumbered pages are in four equal parts, relating in turn to the functions $sn w$, $cn w$, $dn w$, and $E(w) = \int_0^w dn^2 w dw$, each for 19 values of the modular angle $\sin^{-1} k$, namely, 1° , 5° , $5^\circ 85'$, 89° . Each opening has a table on the left and a corresponding chart on the right. If $w = u + iv$, the tables are all for $u/K = 0(.1)1$, $v/K' = 0(.1)1$, and the charts cover the same unit square on a scale such that the side of the square is about 16.2 cm. In Parts I-III, the quantities tabulated, to 4 figures without differences, are the real and imaginary parts x , y of $sn w$, $cn w$, $dn w$, respectively. In Part IV, if $E_R + iE_I = E(w)$, the relations $E(K) = E$, $E(K + iK') = E + i(K' - E')$ have led to the tabulation of the normalized quantities $E_R' = E_R/E$, $E_I' = E_I/(K' - E')$. The lines drawn on the charts are curves of constant x or y in Parts I-III, and curves of constant E_R' or E_I' in Part IV. Seven-figure values of the complete integrals are provided. The information given is sufficient to enable the four functions concerned to be evaluated for any point w in the complex plane.

The well-known tables of Spenceley and Spenceley [1] were used as a source of values for real w , whence the imaginary transformation and the addition formulas were used to compute the values of the functions of $u + iv$. The computation was mostly done on an IBM 650.

The Introduction contains enough information about elliptic integrals and functions to explain the tables and charts to anyone not previously acquainted with the subject. It also contains several applications to potential problems. It is pleasant to find such a valuable contribution to mathematical tabulation made by a civil engineer. The charts were constructed by a group of five Turkish naval officers at the University of Michigan.

One could wish that italic type (available and used in other contexts) had been used for mathematical symbols in the Introduction, but no such minor matter

can obscure the importance of the volume, which cannot fail to be found very useful. Only in the case of Part IV has the reviewer heard of any similar or related table of comparable scope, namely the table of the Jacobian zeta function by Fox and McNamee [2].

A. F.

1. G. W. SPENCELEY & R. M. SPENCELEY, *Smithsonian Elliptic Functions Tables*, Washington, 1947. See *MTAC*, v. 3, 1948, p. 89-92.

2. E. N. FOX & J. MCNAMEE, "The two-dimensional problem of seepage into a cofferdam," *Phil. Mag.*, s. 7, v. 39, 1948, p. 165-203. See *MTAC*, v. 3, 1948, p. 246, 252, also v. 7, 1953, p. 190.

19[L].—L. LEWIN, *Dilogarithms and Associated Functions*, MacDonald & Co. Ltd., London, 1958, xvi + 353 p., 21 cm. Price 65 Shillings.

The functions treated here are for the most part special cases of the Lerch zeta function, which can be defined by the series $\sum_{n=0}^{\infty} z^n / (n+b)^s$, $|z| < 1$, b not a negative integer or zero. To describe the text and tables, it is convenient to give some notation. Let $z = x + iy$, where x and y are real and $i = \sqrt{-1}$. Then

$$(1) \quad Li_2(z) = - \int_0^z t^{-1} \ln(1-t) dt, \quad Li_n(z) = \int_0^z t^{-1} Li_{n-1}(t) dt;$$

$$(2) \quad Ti_2(x, a) = \int_0^x (t+a)^{-1} \overline{\arctan} t dt, \quad Ti_2(x) \equiv Ti_2(x, 0),$$

$$Li_n(iy) = 2^{-n} Li_n(-y^2) + iTi_n(y);$$

$$(3) \quad Cl_2(\theta) = - \int_0^\theta \ln\left(2 \sin \frac{1}{2} t\right) dt, \quad Cl_{2n}(\theta) = \int_0^\theta Cl_{2n-1}(t) dt,$$

$$Cl_{2n+1}(\theta) = Li_{2n+1}(1) - \int_0^\theta Cl_{2n}(z) dz;$$

$$(4) \quad Gl_{2n}(\theta) + iCl_{2n}(\theta) = \sum_{k=1}^{\infty} \frac{e^{ik\theta}}{k^{2n}}, \quad Cl_{2n+1}(\theta) + iGl_{2n+1}(\theta) = \sum_{k=1}^{\infty} \frac{e^{ik\theta}}{k^{2n+1}}.$$

The function $Gl_n(\theta)$ is a polynomial in θ of degree n .

Chapter I deals with the dilogarithm function $Li_2(x)$. The function $Ti_2(x)$ is considered in Chapter II; $Ti_2(x, a)$ in Chapter III. $Cl_2(\theta)$, θ real and positive, is Clausen's integral, and is studied in Chapter IV. Chapters V and VI take up $Li_n(z)$ for $n = 2$ and 3 , respectively, and the analysis of this function for general values of n is the subject of Chapter VII. The general relations in (3) and (4) are also studied in this chapter. Chapter VIII deals with series expansions and integrals which can be expressed in terms of the basic functions in (1)–(4). Chapter IX is very useful. It is a compendium of results derived in the previous chapters. It also contains a survey of mathematical tables.

A description of the tabular material in this volume follows.

Table I. $Li_n(x)$, $n = 2(1)5$
 $x = 0(.01)1.0$, 5D

Table II. $Ti_n(y)$, $n = 2(1)5$
 $y = 0(.01)1.0$, 5D

Table III. $Cl_n(\frac{1}{2}\pi\alpha)$, $n = 2(1)5$
 $\alpha = 0(.01)2.0$, 5D

Table IV. $Gl_n(\frac{1}{2}\pi\alpha)$, $n = 2(1)5$
 $\alpha = 0(.01)2.0$, 5D

Table V. $Li_2(r, \theta) = \text{real part of } Li_2(z)$, $z = re^{i\theta}$.
 $r = 0(.01)1.0$, $\theta = 0(5^\circ)180^\circ$, 6D.

Throughout Table V the symbol x should be replaced by r . No information is supplied for interpolating in the tables.

The volume is replete with striking and curious results, some of which have been rediscovered a number of times, and the book should prevent future duplication of effort. There is a well-detailed table of contents and index. An extensive bibliography is also given.

Y.L.L.

20[L].—VERA I. PAGUROVA, *Tablitsy integro-eksponentsial'noi funktsii*

$$E_\nu(x) = \int_1^\infty e^{-xu} u^{-\nu} du$$

(*Tables of the Exponential Integral Function* $E_\nu(x) = \int_1^\infty e^{-xu} u^{-\nu} du$), Akad.

Nauk SSSR, Vychislitel'nyy Tsent, Moscow, 1959, xii + 152 p., 27 cm. Price 9.60 rubles.

This volume from the Computational Center of the Academy of Sciences of the USSR deals with well-known integrals which depend on the exponential integral when ν is a positive integer n . There are three tables. Table I (pages 3-52) is reproduced, with acknowledgment, from the NBS table calculated for a report of 1946 by G. Placzek and Gertrude Blanch (see *MTAC*, v. 2, 1947, p. 272) and more widely disseminated in 1954 in [1]. The table gives $E_n(x)$ to 7 or more decimals for $n = 0(1)20$, $x = 0(.01)2(.1)10$, and also 7-decimal values of the auxiliary functions $E_2(x) - x \ln x$ and $E_3(x) + \frac{1}{2}x^2 \ln x$ for $x = 0(.01).5$ and $x = 0(.01).1$, respectively; these last two ranges need transposing in the sub-title on page 1.

The other two tables are original. It is not stated what machines were used in computing them. Table II (pages 54-62) gives $e^x E_n(x)$, $n = 2(1)10$ to 7 decimals (6 figures) and e^x to 7 figures, all for $x = 10(.1)20$. Table III (pages 64-151) gives $e^x E_\nu(x)$, $\nu = 0(.1)1$ to 6 or 7 figures and e^x to 7 figures, all for $x = .01(.01)7(.05)12(.1)20$. No table gives differences.

A short introduction contains mathematical formulas and recommendations about interpolation. For integral n , the formula $d^r E_n(x)/dx^r = (-1)^r E_{n-r}(x)$ enables the tabulated function values themselves to be used for interpolation by means of Taylor's series. A table is given showing the accuracy attainable in interpolating various functions linearly or with 3 or 4 Taylor terms or with 3, 4, or 5 Lagrange terms.

A. F.

1. NAT. BUR. STANDARDS, Appl. Math. Ser. No. 37, *Tables of Functions and of Zeroes of Functions*, U. S. Government Printing Office, Washington, D. C., 1954, p. 57-111. See RMT 104, *MTAC*, v. 10, 1956, p. 249.

21[L].—EARL D. RAINVILLE, *Special Functions*, The Macmillan Co., New York, 1960, xii + 365 p., 24 cm. Price \$11.75.

This aptly titled, interesting, extremely well written book is based upon the lectures on Special Functions given by the author at the University of Michigan since 1946. The author's aim in writing the book was to facilitate the teaching of courses on the subject elsewhere. As an instructor in such a course, the reviewer feels certain that this most welcome text is to be accorded a warm reception on many college campuses.

More than fifty special functions receive varying degrees of attention; but, for the sake of usefulness, the subject is not approached on the encyclopedic level. Many of the standard concepts and methods which are useful in the detailed study of special functions are included. There is a great deal of emphasis on one of the author's favorite subjects: generating functions. Two interesting innovations are I. M. Sheffer's classification of polynomial sets and Sister M. Celine Fasenmyer's technique for obtaining recurrence relations for sets of polynomials. Functions of the hypergeometric family hold the center of the stage throughout a major portion of the text. The book concludes with a short current bibliography which should enable the reader to begin a more detailed study of the field.

There are twenty-one chapters in all, and the book may be roughly divided into four distinct parts:

(1) Two short preliminary chapters, 1 and 3, deal separately with infinite products and asymptotic series, respectively.

(2) Chapter 2 treats the gamma and beta functions, and chapters 4, 5, 6, and 7 are devoted to the hypergeometric family: the hypergeometric function, generalized hypergeometric functions, Bessel functions, and the confluent hypergeometric function, respectively.

(3) Chapter 8 is concerned with the generating function concept, as a preparation to chapters 9, 10, 11, and 12, which consider orthogonal, Legendre, Hermite, and Laguerre polynomials, respectively. Chapter 13 contains I. M. Sheffer's classification of polynomial sets; chapter 14 contains Sister M. Celine Fasenmyer's technique for obtaining recurrence relations for polynomials; and chapter 15 contains symbolic relations among classical polynomials. There follow three polynomial chapters: 16, 17, and 18, on Jacobi, ultraspherical and Gegenbauer, and other polynomials, respectively.

(4) The concluding three chapters, 19, 20, and 21, are devoted to elliptic functions, theta functions, and Jacobian elliptic functions, respectively.

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22[L].—L. J. SLATER, *Confluent Hypergeometric Functions*, Cambridge University Press, New York, 1960, x + 247 p., 29 cm. Price \$12.50.

This is a valuable treatise on the subject, and the first of its kind in English. Tricomi [1] and Buchholz [2] have previously written books on the subject in Italian and German, respectively. Tricomi used the notation derived from the theory of hypergeometric functions, and this also is the principal notation employed in the Bateman Manuscript Project [3]. Buchholz uses the notation introduced by Whit-

taker. The notation of the present volume is a fusion of the two, and many results of the same character are given in all of the notations. This should prove most useful to research workers, as the confluent function has many applications, and notation is not uniform.

Chapter I studies the confluent hypergeometric functions as solutions of differential equations. Power series expansions are developed and relations between the various functions are carefully detailed.

Chapter II deals with differential properties including contiguous relations, Wronskians, addition theorems and multiplication theorems.

Chapter III is principally concerned with definite integrals involving the confluent functions. These include integrals of Barnes, Euler and Pochhammer types, and Laplace, Mellin and Hankel transforms.

Chapter IV takes up asymptotic expansions. It is a very useful compendium. In particular, as the author remarks, the theory is not complete, and it is in this area where important new results are anticipated.

Related functions such as Coulomb wave functions, Bessel functions, incomplete gamma functions, etc., are briefly considered in Chapter V.

Descriptive properties are the subject of Chapter VI. Zeros in x of ${}_1F_1(a; b; x)$ and $W_{k,m}(x)$, and zeros in a and b of the former are studied. Formulas and expansions for the zeros are provided. Numerical evaluation of ${}_1F_1$ is discussed and numerous figures are provided to illustrate its behavior when a , b , and x are real.

Three appendices give tables. These are as follows:

Appendix I. Smallest positive zeros of ${}_1F_1(a; b; x)$

$$a = -4.0(0.1) - 0.1, \quad b = 0.1(0.1)2.5, \quad 7D$$

Appendix II. ${}_1F_1(a; b; x)$

$$a = -1.0(0.1)1.0, \quad b = 0.1(0.1)1.0, \quad x = 0.1(0.1)10.0, \\ 7S - 9S$$

Appendix III. ${}_1F_1(a; b; x)$

$$a = -11.0(0.2)2.0, \quad b = -4.0(0.2)1.0, \quad x = 1, \quad 7D$$

Appendix I was calculated to 8D on EDSAC, and the error does not exceed two units in the seventh decimal. Appendix II was calculated on the same machine, but no mention is made of accuracy. Some spot checks indicate that the error does not exceed four units in the last figure recorded. Appendices I and II are extensions of tables previously published by the author. Other short tables of the material in Appendix II have also appeared previously, but the present table is the most complete available. Appendix III was evolved using recurrence formulas and the data of Appendix II. It is quite surprising that nothing is said about interpolation.

A detailed table of contents, general index, and symbolic index of notation enhance the usefulness of the volume. The list of references, though not exhaustive, is fairly complete for work subsequent to that of Buchholz [2], whom the reader should consult for an extensive bibliography of work prior to about 1952. The reviewer notes that two important tables of Whittaker functions reviewed in [4] are not found in Miss Slater's list of references.

1. F. G. TRICOMI, *Lezioni sulle funzioni ipergeometriche confluenti*, Gheroni, Torino, 1952.
2. H. BUCHHOLZ, *Die konfluente hypergeometrische Funktion mit besonderer Berücksichtigung ihrer Anwendungen*, Springer, Berlin, 1953.
3. A. ERDÉLYI, W. MAGNUS, F. OBERHETTINGER, & F. G. TRICOMI, *Higher Transcendental Functions*, v. 1, Chapter VI, McGraw-Hill, New York, 1953.
4. RMT 46, *MTAC*, v. 12, 1958, pp. 86-88.

23[M, X].—G. DOETSCH, *Einführung in Theorie und Anwendung der Laplace-Transformation*, Birkhäuser Verlag, Basel, Switzerland, 1958, 301 p., 24 cm. Price SFr 39.40.

This book forms an excellent introduction to the subject of Laplace transformations in one dimension, written by one of the leading experts in the field. From his wide knowledge of both the theoretical and applied aspects of the subject, the author has written a very readable, rigorous exposition which also indicates relations with appropriate physical concepts.

After a general introduction, the basic properties of the Laplace transform are developed in ten short chapters. Included are discussions of half-planes of convergence, uniqueness of the inverse, analytic properties of the transform, the effect of a linear transformation of the independent variable on the transform, the effect of differentiation and integration, and the transformation of a convolution. Because each chapter is devoted to a separate topic, the book is very useful for reference purposes. Many examples of specific transforms are given.

The next four chapters deal with the application of Laplace transforms to the following problems: the initial-value problem in ordinary differential equations with constant coefficients; the solution of differential equations for special input functions; homogeneous and non-homogeneous systems of differential equations; the initial-value problem for difference equations.

The next group of chapters deals with further properties of the transform, some of which may not be familiar to the reader: the behavior of the transform at infinity; inversion formulas expressed as integrals along vertical lines, as integrals along deformed paths in the complex plane, and as series of residues; conditions for the representation of a function as a transform; functions given as the sums of series of transforms; the analogue of Parseval's formula and transforms of products; and the asymptotic behavior of the transform and of the "original" function.

The book concludes with three chapters on further applications of the Laplace transform to differential equations with variable coefficients, to simple partial differential equations, and to certain integral equations.

A useful feature of the book is the inclusion of necessary background material, particularly in the later chapters.

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24[S, X].—G. I. MARCHUK, *Numerical Methods for Nuclear Reactor Calculations*, (An English translation of a work originally published in Russian as Supplement Nos. 3-4 of the Soviet Journal of Atomic Energy, *Atomnaya Énergiya*. Atomic Press, Moscow, 1958.) Consultants Bureau, Inc., New York, 1959, 293 p. 28 cm. Price \$60.00.

A more accurate title for this book would have been *Numerical Methods for*

Nuclear Reactor Physics Calculations. As stated by the author, "the book is an attempt at a more or less systematic exposition of numerical methods for the calculation of thermal, intermediate, and fast neutron reactors. Particular attention is devoted to the problems of critical mass, the space-energy neutron flux distribution, and the neutron importance." This is the first book to become available in English that is entirely devoted to this area of calculation, and as such is a helpful addition to our literature.

The book is clearly not addressed to mathematicians. A familiarity with nuclear reactor theory is assumed; terms such as "cross section" are used without definition. The mathematician will probably be disturbed by such statements as "the integrands in the resulting two terms are now continuous, and they can therefore be expanded in a Taylor's series about r_k " (p. 106), or, "primarily, this is due to the fact that a second-order difference equation has two eigenvalues, one of which is positive" (p. 122). Emphasis throughout the book is on the derivation of equations and the mechanics of their solution. The stability of a number of the numerical methods against the exponential growth of round-off errors is considered. Problems related to the existence of solutions and the accuracy with which solutions of the various approximate equations represent the desired solutions are mentioned only occasionally.

In the foreword the author mentions that the numerical methods were "first tested on a large quantity of theoretical and experimental data and were later used in operation." It is unfortunate that the results of some of these calculations were not included in the book. Results of specific calculations are not given. For example, in Chapter VIII, various sets of finite-difference equations are set up to approximate the diffusion equation. Some of these difference equations are more accurate than others. However, no discussion of the actual errors incurred when these various approximations are used is presented other than a statement, "with a sufficiently fine network good accuracy can be obtained." This would have been an excellent place to include numerical results showing the kind of accuracy that could be attained when the various approximate equations are used in typical specific problems.

A useful feature of this book is that corresponding to every set of equations derived, an adjoint set of equations is also obtained.

In the first chapter a basic set of transport equations is derived: a "slowing-down" equation in which the assumption is made that the moderator nuclei are at rest while the neutrons are slowed down by elastic collisions, and a "thermal" equation in which the velocities of the moderator nuclei are assumed to have a Maxwellian distribution. In succeeding chapters, various sets of equations approximating this basic set are obtained. In the second chapter the diffusion approximation is derived following the method of R. E. Marshak, H. Brooks, and H. Hurwitz, Jr., published in *Nucleonics*, v. 4, 5 (1949), in which the angular dependence of the neutron flux and the collision function are both approximated by retaining the first two terms in a series expansion involving spherical functions. The corresponding boundary conditions are also derived. In the third chapter the diffusion-age approximation is obtained when the energy dependence of the flux is approximated by retaining the first few terms of a Taylor series expansion. Here again, the corresponding boundary conditions are derived. In the fourth chapter further approximations to the basic transport equation are derived which take more accurate account of energy dependence than the diffusion-age approximation,

and are therefore applicable in cases where there is stronger absorption of slowing-down neutrons. Equations are derived in which resonance absorption is accounted for based on a method suggested by Wigner. Also, the Greuling-Goertzel equations for the slowing-down of neutrons in an inhomogeneous medium are obtained.

In Chapter V exact solutions are given of the basic and adjoint equations of the diffusion-age approximation for a homogeneous, unreflected reactor with an extrapolated boundary which is energy-independent. Also, for this special case the convergence of "the method of successive approximations" for finding the criticality factor is established. "The method of successive approximations" means here that at each step, assuming an estimate of the fission source, the corresponding neutron flux is calculated, and then using this set of flux values, a new estimate of the fission source is obtained. The criticality factor is obtained using ratios of successive source estimates.

In Chapter VI various sets of multigroup equations are derived which are applicable primarily to thermal reactors where weak absorption of slowing-down neutrons is assumed; in Chapter VII various sets of multigroup equations are derived which are applicable primarily to intermediate reactors where strong absorption of slowing-down neutrons must be considered.

Chapter VIII contains the derivation of various sets of three-point difference equations to approximate the one-dimensional diffusion equation in plane, cylindrical, and spherical geometries. In addition, five-point difference equations are obtained for the two-dimensional diffusion equation in plane (x, y) and cylindrical (r, z) geometries.

In Chapter IX various methods are discussed for solving sets of finite-difference diffusion equations of the types set up in the previous chapter. To solve the three-point difference equation a well-known factorization technique is described, and its stability against the growth of round-off errors is considered. For the solution of the five-point difference equations an analogous factorization technique, as well as a simple iteration procedure, and a third method suggested by N. I. Buleev are considered. Error criteria which can be used to estimate the accuracy of results are also mentioned.

Chapter X contains a discussion of the "net" method for obtaining approximate solutions of the differential slowing-down equation in one spatial dimension. Difference equations are set up to approximate the differential equation and solved for the unknown function in one energy interval in terms of values computed for the preceding interval. Both explicit and implicit schemes are discussed. Their stability is investigated by applying the von Neumann stability criterion.

In Chapter XI perturbation theory is applied to obtain approximate formulas giving the change in the criticality factor when small changes are made in values of the various physical parameters. This is done starting with the basic equations of Chapter I as well as with the various equations obtained in later chapters to approximate these basic equations. Some further applications of perturbation theory are discussed such as the calculation of the reactivity equivalent of a control rod in the center of the core of a cylindrical reactor, as well as for a whole system of control rods in such a reactor.

Chapter XII on heterogeneous effects in nuclear reactors contains descriptions of the Gurevich-Pomeranchuk theory of resonance absorption applicable in calculating the effective resonance integral for "thin" U^{238} blocks, the Wigner theory for

calculating the resonance integral for "thick" blocks, and Orlov's extension of Wigner's method giving a formula for the effective resonance integral which can be used for U^{238} blocks of any thickness. Further, a method is given which was developed by Marchuk and Orlov for calculating the resonance capture of neutrons in a plane lattice of uranium blocks. Sedel'nikov's extension of the Gurevich-Pomeranchuk theory to take account of self-shielding in intermediate neutron reactors is described. Effective boundary conditions for "black" and "grey" bodies are considered. Finally, the Wigner-Seitz method is applied to a cell of a heterogeneous reactor to obtain effective constants for an equivalent homogeneous reactor.

Chapter XIII contains a discussion of the calculation of the neutron flux in fast nuclear reactors. The transport equation including terms for inelastic scattering is given. Corresponding multigroup equations are obtained. For solving these equations both the method of spherical harmonics and Carlson's S_n method are described.

A final chapter on calculations for intermediate and thermal reactors with hydrogenous moderators is included. This chapter was added after the original manuscript had been prepared for publication.

It is unfortunate that this book was not more carefully edited. For example, there is confusion in the use of the terms 'flux' and 'current'; on page 104 the following statement appears, "we wish to find a solution of (30.1) which has a continuous flux

$$I = r^a D \frac{d\phi}{dr}."$$

Throughout the text, the equations which are usually referred to as "transport" equations are called "kinetic" equations. Too frequently proper names are spelled phonetically as a result of the transliteration process, rather than with their usual English spelling; for example, R. E. Marshak appears as R. E. Marchak (p. 6), R. Ehrlich appears as R. Erlich (p. 6), Neumann appears as Neiman (p. 142), etc. Also, it is unfortunate that the publishers did not take more care in their representation of symbols. The equations were apparently reproduced photographically. A parameter represented by a script letter in an equation is frequently represented in another form in the text. On pages 132 and 134 a parameter which appears in the equations as a "chi" is represented as a "kappa" in the text.

This book is a very useful addition to our literature. It is hoped that its presence and imperfections will act as a stimulus for the publication of another book in the area of numerical methods for nuclear reactor calculations which will give a more satisfying mathematical treatment of this subject, and which will be made available at a more reasonable price.

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25[W].—E. BURGER, *Einführung in die Theorie der Spiele*, Walter de Gruyter & Co., Berlin, 1959, 169 p., 23 cm. Price DM 28.

This is the first book on the theory of games to appear in the German language. The author, professor of mathematics at the University of Frankfurt, has previously

given several courses on the subject for economists, but the emphasis in this book is to bring out the mathematical structure of the theory. He has succeeded in this in an admirable way. The comparatively few applications are meant as illustrations rather than as developments of the empirical fields for which the theory of games is particularly suited. There is an extensive discussion of zero-sum two-person games, and of equilibrium points for non-cooperative games. The presentation of linear programming is also clear and rather exhaustive. The theory of cooperative games, that is, those in which the formation of coalitions is advantageous and allowable, and in which side-payments by the players are freely admitted, is developed in fair detail, even including a discussion of Shapley's value of an n -person game. At this point the author confesses that he is less sure of the intuitive background against which this theory has been placed, a position that is common to many mathematicians who have studied the problem of n -person games. However, this is a difficult issue and the author is wise not to have taken too definite a stand, rather withdrawing to the strictly mathematical aspects involved. These problems can only be solved by recourse to an improved description of the socio-economic world. If it turns out that the real problem involves a high degree of cooperation—and I have no doubts whatsoever that this will be the case—then the mathematical theory will have to accommodate itself to these facts, even if the mathematical structure is uncommon and cumbersome, until fundamentally new concepts are established.

Dr. Burger possesses a very high didactic skill; the great clarity which pervades his whole book should make it a welcome tool for the novice in game theory who commands the mathematical knowledge expected of first or second year graduate students. A translation of the work should be seriously considered, since there is no similar book in English which accomplishes as much in such small compass as Dr. Burger's does.

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26[X].—R. P. BOAS, JR. & R. C. BUCK, *Polynomial Expansions of Analytic Functions*, Springer-Verlag, Berlin, 1958, viii + 77 p., 23 cm. Price DM 19.80.

A great part of numerical analysis is concerned with polynomial approximation to analytic functions, and so this booklet appears of immediate interest to the numerical analyst. However, with numerical analysis in its present state, it is more relevant for studies in the general theory of functions of a complex variable or in the theory of special functions.

Given a set of polynomials $\{p_n(z)\}$ and a function $f(z)$, it is reasonable to ask whether we can find coefficients $\{c_n\}$ such that

$$(1) \quad f(z) = \sum c_n p_n(z)$$

in some sense. This is the "expansion problem".

It is also reasonable to ask whether, given linear functionals $\{L_n(f)\}$, for example,

$$(2) \quad L_n(f) = f^{(n)}(0)$$

or

$$(3) \quad L_{2n}(f) = f^{(2n)}(0), \quad L_{2n+1}(f) = f^{(2n)}(1),$$

we can obtain a representation of f in the form (1). This is the "interpolation problem". The solution in case (2) is given by the Taylor expansion and that in (3) by the two-point Lidstone expansion

$$f(z) = \sum A_n(1-z)f^{(2n)}(0) + \sum A_n(z)f^{(2n)}(1),$$

where the polynomials $A_n(z)$ are defined by

$$\frac{\sinh zw}{\sinh w} = \sum A_n(z)w^{2n}.$$

This is valid, for instance, when $f(z)$ is an entire function of exponential type less than π .

These two problems are studied by a general method—"kernel expansion"—for wide classes of polynomials (defined by generating functions) and for the cases when $f(z)$ is entire, or regular at the origin.

The material is accessible to those familiar with the classical methods of complex variable theory, and its study by numerical analysts is recommended. It will, for instance, encourage us to get off the real axis, reveal some thought-provoking "bad examples" (e.g., non-uniqueness in (1)), and show us how mathematics should be written.

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27[X].—RUDOLPH E. LANGER, Editor, *Symposium on Numerical Approximation*, The University of Wisconsin Press, Madison, 1959, x + 462 p., 24 cm. Price \$4.50.

The present volume auspiciously launches the publication efforts of Professor Langer's group at the Mathematics Research Center. It contains the proceedings of a symposium conducted at the University of Wisconsin, April 21–23, 1958. The papers (original and expository) present such a well-rounded survey of numerical approximation because of the careful selection of the invited contributors. A listing of the authors and the titles of their papers will serve to indicate the level and scope of the work.

A. M. Ostrowski	On Trends and Problems in Numerical Approximation
R. C. Buck	Linear Spaces and Approximation Theory
Z. Kopal	Operational Methods in Numerical Analysis Based on Rational Approximations
P. J. Davis	On the Numerical Integration of Periodic Analytic Functions
H. E. Salzer	Some New Divided Difference Algorithms for Two Variables
P. C. Hammer	Numerical Evaluation of Multiple Integrals
M. Golomb	Optimal Approximation and Error Bounds
A. Sard	The Rationale of Approximation
J. L. Walsh	On Extremal Approximations
E. L. Stiefel	Numerical Methods of Tchebycheff Approximation
L. Fox	Minimax Methods in Table Construction

T. S. Motzkin	Existence of Essentially Nonlinear Families Suitable for Oscillatory Approximation
I. J. Schoenberg	On Variation Diminishing Approximation Methods
M. Golomb	Approximation by Functions of Fewer Variables
J. C. P. Miller	Extremal Approximations—A Summary
R. C. Buck	Survey of Recent Russian Literature on Approximation
F. L. Bauer	The Quotient-Difference and Epsilon Algorithms
J. B. Rosser	Some Sufficient Conditions for the Existence of an Asymptotic Formula or an Asymptotic Expansion
J. W. Tukey	The Estimation of (Power) Spectra and Related Quantities
L. Collatz	Approximation in Partial Differential Equations
J. Todd	Special Polynomials in Numerical Analysis

E. I.

28[X].—RUDOLPH E. LANGER, Editor, *Frontiers of Numerical Mathematics*, The University of Wisconsin Press, Madison, 1960, xi + 132 p., 24 cm. Price \$3.50.

This book contains eight papers and a discussion of these papers presented at a Symposium held at Madison, Wisconsin on October 30–31, 1959. This symposium was conducted jointly by the Mathematics Research Center, the United States Army, and the National Bureau of Standards. Its purpose “was not intended to be an occasion for the presentation of research results, but one for a survey of the future; for the identification of some mathematical problems that will have to be faced in the lines of scientific advance.”

The authors and the titles of their papers are:

William Prager	Stress Analysis in the Plastic Range
Garrett Birkhoff	Some Mathematical Problems of Nuclear Reactor Theory
Zdenek Kopal	Numerical Problems of Contemporary Celestial Mechanics
Lee Arnold	Aeroelasticity
Phillip M. Morse	Operations Research
Joseph O. Hirschfelder	Mathematical Bottlenecks in Theoretical Chemistry
S. Chandrasekhar	Magnetohydrodynamics
J. Smagorinsky	On the Application of Numerical Methods to the Solution of Systems of Partial Differential Equations Arising in Meteorology

It is unfortunate that the paper by Lee Arnold was made from a tape recording and was not finally reviewed by the author. It is very difficult for a reader to follow this paper, for many statements in it seem to refer to illustrations which are not included.

The reader of this book should not expect to find a detailed discussion of numerically formulated problems arising in various branches of science. He will find, in the main, discussions and reviews of various open mathematical problems in different scientific areas. Some of the papers refer to numerical treatment of these problems. A surprising number indicate a strong preference on the author's part for analytical methods for dealing with their problems.

One can but agree with J. Smagorinsky when he states “that computing ma-

chines cannot be considered a substitute for the ingenious mathematical and laboratory techniques of analysis which have been devised. . . .” However, this reviewer is convinced that such ingenuity when properly coupled with the power of modern computers will provide greater insight than analytical methods alone.

A. H. T.

29[X].—(a) D. S. MITRINOVIĆ, “Sur les nombres de Stirling de première espèce et les polynômes de Stirling,” *Publ. de la Fac. d’Électrotechnique de l’Univ. à Belgrade* (Série: Math. et Phys.), No. 23, 1959, 20 p. (Serbian with French summary. Tables by Miss RUŽICA S. MITRINOVIĆ.)

(b) D. S. MITRINOVIĆ & R. S. MITRINOVIĆ, “Sur les polynômes de Stirling,” *Bull. Soc. Math. Phys. Serbie*, v. 10 (for 1958), p. 43–49, Belgrade. (Summary in Russian.)

(c) D. S. MITRINOVIĆ & R. S. MITRINOVIĆ, “Tableaux qui fournissent des polynômes de Stirling,” *Publ. Fac. Élect. Univ. Belgrade* (Série: Math. et Phys.), No. 34, 1960, 24 p. (Summary in Serbian.)

These three papers are concerned with the Stirling numbers of the first kind, S_n^r , which may be defined for positive integral n by

$$x(x - 1)(x - 2) \cdots (x - n + 1) = \sum_{r=0}^n S_n^r x^r.$$

Altogether the numbers S_n^{n-m} are tabulated for $m = 1(1)32$, $n = m + 1(1)N$, where $N = 200$ for $m = 1(1)5$, $N = 100$ for $m = 6$, and $N = 50$ for $m = 7(1)32$. The values for $m = 1(1)7$ are given in (a), for $m = 8(1)13$ partly in (a) and partly in (c), for $m = 14(1)20$ partly in (b) and partly in (c), and for $m = 21(1)32$ in (c). The authors found no discrepancy as a result of some checking against unpublished tables by F. L. Miksa (see *MTAC*, v. 10, 1956, p. 37).

Algebraic expressions for S_n^{n-m} in the form of binomial coefficients $\binom{n}{m+1}$ multiplied by polynomials (with factors $n(n - 1)$ separated out if m is odd and not less than 3) are given for $m = 1(1)13$ in (a) and for $m = 1(1)9$ in (c).

S_n^{n-m} may also be expressed as a sum of multiples of binomial coefficients in the form

$$S_n^{n-m} = \sum_{k=0}^{m-1} C_m^k \binom{n}{2m-k}.$$

Altogether the values of the coefficients C_m^k are given for $k = 0(1)31$, $m = k + 1(1)32$, the values for $k = 0(1)19$, $m = k + 1(1)20$ being found in (b) and the remaining values in (c).

A. F.

30[Z].—WAYNE C. IRWIN, *Digital Computer Principles*, Van Nostrand Co., Inc., Princeton, 1960, vi + 321 p., 24 cm., \$8.00.

This book contains material presented at a training course in the Electronics Division of the National Cash Register Company. It is an extremely elementary “book for the beginner. No previous acquaintance with computers, electronics or mathematics is necessary.”

The book is divided into nine sections and forty-two chapters. The section titles are: Methods of Computation, Symbolic Logic, Mechanization of Logic, Mechanization of Storage, Timing, Mechanization of Arithmetic, Control, Communication with the Computer, Preparation of Instructions, Reduction of Errors, and Present Trends.

The author has placed more emphasis on symbolic logic than on any other subject. This section of the book begins with Boolean algebra and terminates with a discussion of the Harvard Minimizing Chart. However, even in this section, the best written one in the book, the author does not develop the mathematical ideas involved adequately or prove statements. He describes various facts and illustrates the use of various techniques.

The author does not go very far in the discussion of any topic he selects, and he omits many significant points in his discussion. Some omissions are noted in footnotes but some are never mentioned. For example, "asynchronous machines" are dismissed with a footnote on page 145. The words "round-off" and "rounded multiplication" are never mentioned in the text and neither word can be found in the index. The existence of a computer using a number representation other than one with a signed absolute value is never mentioned.

It is unfortunate that the author did not see fit to include in the bibliography references to the reports by von Neumann and his co-workers at the Institute for Advanced Study. The discussion given in these reports of arithmetic performed in a computer with a two's complement representation of numbers is clear and complete, and would supplement very nicely the author's limited discussion of arithmetic in a binary machine.

A. H. T.

31[Z].—T. E. IVALL, *Electronic Computers, Principles and Applications*, Second Edition, Philosophical Library, New York, 1960, viii + 263 p., 22 cm. Price \$15.00.

This is a revised version of the first edition published in 1956 under the same title [1]. In the first edition the author acts as an editor, publishing in book form the separate contributions of a number of English writers working in the field of computers. In the present edition the author attempts to regroup and rewrite the material so as to present a more coherent picture. He also adds three short chapters, as follows:

(1) In place of one chapter on Analogue Computing Circuits the new edition now contains two chapters, Analogue Computing Circuits—1, and Analogue Computing Circuits—2.

(2) A chapter on Programming Digital Computers is added.

(3) The chapter, Computers of the Future, somewhat rewritten, now becomes Recent Developments, and a new chapter, Computers of the Future, is added.

The book is intended as an introduction to the computer field for the non-technical reader. It can adequately fulfill this purpose. However, the book has a number of deficiencies. It also appears that the author has not really kept abreast with modern developments in this field, nor with the rapid advances which are taking place in this country. In his brief chapter, Computers of the Future, he discusses mainly some future potential applications of computers rather than the exciting developments which are taking place in the field of computer design. He sometimes betrays

a rather limited appreciation of the flexibility and power of modern digital computers. For instance, on page 214 he states: "It now appears that digital computers, too, may have some applications in the field of simulation, particularly when the system to be simulated has a digital nature." Digital computers have for years been used to simulate very complex physical systems, for example, the temperature distribution within the core of a nuclear reactor.

This book can serve as a simple and not too technical introduction to computers. In the opinion of the reviewer, however, the interested reader can find a number of other books recently published that will more adequately serve this purpose, and probably at a lower price.

H. P.

1. T. E. IVALL, Editor, *Electronic Computers*, Philosophical Library, New York, RMT 118, MTAC, v. 12, 1958, p. 255.

32[Z].—WALTER J. KARPLUS, *Analog Simulation, Solution of Field Problems*, McGraw-Hill, New York, 1958, xv + 434 p., 23 cm. Price \$10.00.

Analog Simulation by Karplus is essentially unique. There are neither competitive contemporary treatises nor older works with which to compare it in review. It almost automatically, then, belongs in any complete library of machine computation. The real question, on the other hand, is whether a real need exists for either a text or a reference book on its subject matter. As a research engineer, the reviewer has a strong affirmative answer to the second half of this question. He can now throw away the collection of reprints and notes which he has been carrying around for years. Its merit as a text is more debatable.

The subject matter of the book should be made clear first. The title, *Analog Simulation*, conveys to most American engineers devices like aircraft and missile simulators and electronic amplifier differential analyzers. Even the subtitle, "Solution of Field Problems," evokes a momentary picture of a fire control computer in the "field." The full title is correctly descriptive of the category of physical problems considered, but a title like "Analog Techniques for Partial Differential Equations" would have made the material covered more readily apparent.

The book is divided into three major parts. Part 1 (Chapters 2-4) gives 98 pages of mathematical background for analog study of field problems. In Part 2 (Chapters 5-10), the actual "hardware" is discussed in 165 pages. Part 3 returns to a mathematically organized discussion of the applications of the previously discussed analog techniques to different classes of partial differential equations (Chapters 11-14, 115 pages). It may be seen that most of the book is problem-oriented.

In spite of this problem orientation, the book is highly recommended as a reference on analog techniques. This is in large part a consequence of the superb list of references at the end of each chapter and the excellent 22-page general bibliography. The only serious omission is *Dynamical Analogies* by H. F. Olson, the closest equivalent to Karplus, although almost solely concerned with acoustics and vibration. If for no other reason, it would be recommended for a place on the engineer's bookshelf alongside good source books on digital computers and electronic differential analyzers.

The reviewer feels that information on direct analog simulators should be more readily available to engineers. The practitioners of the computing art tend to develop rather strong prejudices for the techniques with which they are more familiar. In fact, the three major classes of computers (digital data processors, electronic differential analyzers, and direct analog devices) all figure with equal importance in current engineering research work for which the reviewer is responsible. Not only do we have the continual rather pointless argument about the relative merits of digital versus analog computation, but computer engineers working with operational amplifier machines tend to disparage passive-network and other direct analog devices. The Introduction (Chapter 1) and Chapter 9 on "Electronic Analog Computers" do an excellent job of putting this controversy into proper perspective. The analog systems discussed in Part 2 are not diminishing in importance to engineering. These are: Chapter 5, "Conductive-Solid Analogs"; Chapter 6, "Conductive Liquids—The Electrolytic Tank"; Chapter 7, "Resistance Networks"; Chapter 8, "Resistance-Reactance Network Analogs"; Chapter 10, "Nonelectric-analog Simulation Systems." The value of *Analog Simulation* as a unique reference on distributed system analogs is indisputable.

Some consideration must be given to the merits of the book as a student text, since it is intended for such by the author. The first proposed use for the second half of a one-semester upper-undergraduate or first-graduate introduction to analog computation does not appeal to the reviewer. At this level, I have strong prejudices in favor of orientation along the line of "mathematical and physical principles of engineering analysis," with all techniques of computation introduced for illustration and some familiarization, plus some other topics such as scale modeling and dimensional analysis. My principal reason for objecting to a course based on Karplus is that I feel mathematics beyond an engineering course in ordinary differential equations is required; probably at least an introduction to complex variables or the usual survey course in applied mathematics. As a text for a full-semester course for graduate students, however, it appears excellent. Part 1, "The Mathematical Model," would be an excellent "refresher" on the mathematical and physical principles involved. Graduate engineering students working in mechanical engineering areas should benefit by the familiarization with electrical engineering techniques.

The reviewer's major criticisms of the book are very few. The weakest part is Chapter 3, "Transformations." Although it is a reasonably adequate condensed summary with useful reference tables of transformations, as a text for someone with no previous introduction to the subject it would be useless. The caution on page 66 about applicability of conformal transformations in space, although quite correct, would only be mystifying after a four-page discussion of the subject. The only misprint of any consequence noted is also in this chapter, namely $F(t)$ for $f(t)$ in formula (3.23). There are also occasional implications that physical intuition can be substituted for mathematical understanding, as on pages 11, 12, where it is stated that "It is entirely possible to develop analogs without taking a mathematical route—namely, by applying direct physical insight." Again, on page 79, Karplus warns that use of finite-difference expansions requires an understanding of errors and approximations inherent in the process, and conditions for a valid solution then immediately suggest that physical reasoning and insight may be substituted for mathe-

mathematical understanding. To the reviewer, a contradiction is implied, which, it is doubtful, was intended. It is considered dangerous to suggest to students that physical intuition may be used in solving problems, without parallel mathematical analysis of the properties of the analogous systems involved. Otherwise, there is nothing that can be cited to detract from the value of *Analog Simulation* as a unique contribution to its field.

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33[Z].—W. OPPELT, Compiler, *Anwendung von Rechenmaschinen bei der Berechnung von Regelvorgängen (Application of Computing Machines to the Computation of Servomechanisms)*, R. Oldenbourg, Verlag, Munich, 1958, 128 p., 24 cm. Price DM 16.80.

This book consists of ten papers presented at a meeting in Düsseldorf on November 8, 1957. The authors and titles are:

- | | |
|---------------------------------|----------------------------------------------------------------------------------------------------------------------|
| 1. E. Bucovics | On the Use of Small Digital Computers for the Solution of Fundamental Problems of Servomechanics |
| 2. J. B. Reswick | A Simple Graphical Method for Deconvolution |
| 3. A. Ieonhard | A Special Computer for Polynomials |
| 4. R. Herschel | On the Design of Analog Circuits for Problems of Servomechanics |
| 5. O. Foellinger & G. Schneider | Comparison of Computations for Servomechanisms using Computing Machines of Different Types |
| 6. E. Buehler | On the Mechanical System with Friction and its Electronic Equivalent |
| 7. H. Witsenhausen | Application of Analog Computers for Optimizing Discontinuous Control Circuits with Randomly Varying Input Parameters |
| 8. D. Ernst | Practical Work with Analog Computers |
| 9. Th. Stein | On the Usefulness of Analog Experiments in Practical Applications |
| 10. W. Roth | Investigation of the Control Characteristics of Electric Generator Sets by the Use of Analog Computers |

(All papers are in German, except the second.)

Only two papers consider digital computers: No. 1 describes in much detail the use of an IBM 604 calculating punch for some basic problems; No. 5 compares the accuracy and speed of two analog computers (one mechanical, one electronic) with that obtained on the IBM 650 in the BELL interpretative mode. The comparison is unfair, however, since the Runge-Kutta method is used for solving a system of linear differential equations with constant coefficients (time: 27 sec./step for a system of six equations!).

There is no space here for reviewing each paper separately, so that the following statements may do some injustice to individual papers. Most authors report mainly on their practical work and experiences, not on new theoretical results. There are a

fair number of references, but some are geographically biased and/or obsolete. The book gives an interesting picture of the local situation (Germany and Austria) in the mid-fifties; but as far as applications of computing machines, and in particular digital computers, are concerned, it was almost obsolete when it appeared in 1958 and is, of course, even more nearly obsolete today.

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