REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

55[F].—KENNETH I. APPEL & J. BARKLEY ROSSER, *Table for Estimating Functions of Primes*, Communications Research Division Technical Report No. 4, Institute for Defense Analyses, Princeton, N. J., 1961, xxxii + 125 p., 22 cm.

This interesting table is quite complicated in its design, and partly so in its subject matter. For the sake of brevity a complete description will not be given here. The functions of the primes referred to in the title are primarily $\pi(x)$, the number of primes $\leq x$, the three sums over the primes p:

$$\sum_{p \leq x} \log p, \qquad \sum_{p \leq x} \frac{1}{p}, \qquad \sum_{p \leq x} \frac{\log p}{p},$$

and the product

$$\prod_{p \leq x} \frac{p}{p-1}.$$

The range of x is from 2 to 10^8.

The approach used here may be illustrated with the first sum:

$$\theta(x) = \sum_{p \leq x} \log p.$$

It is well-known that $\theta(x) \sim x$, (this is equivalent to the Prime Number Theorem), and if one defines the coefficient $TH(x)$ by

$$\theta(x) = x - TH(x) \cdot \sqrt{x},$$

it is found *empirically* that $TH(x)$ has a rather small variation over the range x for which it has been computed. For example, between $x = 85,881,353$ and $x = 87,679,913$, $TH(x)$ has a maximum of 1.337 and a minimum of 1.015. Knowing these extremes, one could therefore obtain bounds for, say, $\theta(86,692,297)$, as follows:

$$86,679,848 \leq \theta \leq 86,682,847.$$

The exact value for this argument is $\theta = 86,681,759.3$.

Similarly, if one knows bounds on

$$PI(x) = [li(x) - \pi(x)] \log x \cdot x^{-1/2},$$

$$SR(x) = x^{1/2} \log x \left[\sum_{p \leq x} \frac{1}{p} - \log \log x - 0.261497 \right],$$

and

$$SL(x) = x^{1/2} \left[\sum_{p \leq x} \frac{\log p}{p} - \log x + 1.332582 \right],$$

one can obtain good estimates for $\pi(x)$ and for the second and third sums above.

Further, the authors also give more complicated, third-order correction coefficients which enable one to obtain even more accurate estimates.

The corresponding coefficient for the product:

$$PR(x) = x^{1/2} \left[e^{-\gamma} \prod_{p \leq x} \frac{p}{p-1} - \log x \right],$$

is not listed as such, since the authors, by study of their preliminary computations, discovered the approximate formula:

$$PR(x) \approx SR(x) + \frac{(SR(x))^2}{2x^{1/2} \log x} - \frac{\log x}{2x^{1/2} (1 + \log x)} .$$

Professor Lowell Schoenfeld later obtained a rigorous justification. His long proof is given in full in the Introduction.

The five primary functions are listed, first, for each of the first 64 primes; $\pi(x)$ exactly, $\theta(x)$ to 5D, and the other three functions to 10D. The primes from 313 to 99,999,989 are divided into 173 rather irregularly spaced intervals, and in each interval the five functions are listed at every x for which one of the auxiliary functions, $TH(x)$, $PI(x)$, etc., takes on a maximum or a minimum value. These extrema of $TH(x)$, etc., are also given.

In addition, the largest gap between successive primes in each interval is listed. The largest gap up to 10^8 occurs between the primes 47,326,693 and 47,326,913.

The orientation here is that of estimating the primary functions in terms of the bounded coefficients. But from a theoretical point of view the opposite orientation is probably one of greater interest. One would like to know the order of the true bounds upon these coefficients. The $x^{1/2}$ that enters into all of their defining equations is related to the Riemann Hypothesis, and, as is well-known, the state of the theory here leaves much to be desired. It has been suggested, in an off-hand manner, in $MTAC$, v. 13, 1959, p. 282, that $PI(x)$ has a mean value equal to 1. The range of its values given here, for $313 \leq x \leq 10^8$, is

$$0.526 \leq PI(x) \leq 2.742.$$

Finally, a word concerning the table *per se*. Since the subject matter is so fundamental, an improved and more elegant edition is probably called for. While the table, as it stands, is quite workable, a less erratic selection of intervals and a somewhat clearer format would be desirable.

<div align="right">D. S.</div>

56[G].—G. E. Shilov, *An Introduction to the Theory of Linear Spaces*, Prentice-Hall, Inc. New Jersey, 1961, ix + 310 p., 23 cm. Price $10.00.

This book is the first in Prentice-Hall's series of translations from the Russian. A bibliography has been added by the translator, R. A. Silverman.

The contents include the usual topics in linear algebra such as determinants, linear spaces, systems of linear equations, coordinate transformations, invariant subspaces and eigenvalues of linear transformations, and quadratic forms. The degree of abstraction is shown by the fact that sections on ideals and tensors are included, but marked with asterisks to indicate that they may be omitted if desired. The final chapter deals with infinite-dimensional Euclidean spaces.

The book is not concerned with computing methods directly, so that its value

to one interested in numerical work lies in its development of needed topics in the theory of matrices. Its lucid treatment of the topics covered makes it a fine addition to the literature. The inclusion of suitable problems also makes it useful for classroom use.

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57[J].—L. B. W. JOLLEY, *Summation of Series*, Dover Publications, Inc., New York, 1961, xii + 251 p., 20 cm. Price $2.00 (Paperbound).

This is a "revised and enlarged version of the work first published by Chapman & Hall, Ltd. in 1925". The 700-odd series in the former edition have now been increased in number to 1146. For most of these series a specific reference is listed. While there is much of use and interest here, there are also, in the opinion of the undersigned, numerous defects.

The notation used is often disturbingly original. Thus:

$$(71) \qquad 1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \tfrac{1}{5} \cdots \infty = \operatorname{logh} 2$$

$$(97) \qquad 1 + ax + \frac{a^2 x^2}{2!} + \frac{a^3 x^3}{3!} + \cdots \infty = \epsilon^{ax}$$

$$(361) \qquad \sum_1^\infty \frac{n^r}{n!} = \mathcal{S}_r$$

$$(373) \qquad \sum_1^\infty \frac{1}{(4n^2 - 1)^r} = \mathcal{S}_r$$

(1133)   Sum of Power Series

$$\mathcal{S}_n = \frac{1}{1^n} + \frac{1}{2^n} + \frac{1}{3^n} + \cdots \infty$$

There are misprints and resulting obscurities:

$$(94) \qquad \mathcal{S}_{2n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{2n} = \frac{\pi \operatorname{logh} 2}{8}$$

$$(335) \qquad \text{If } \sum_1^\infty \frac{1}{n^s} = \zeta(s)\ 2, 3, 5 \cdots p\text{—are prime numbers in order}$$

$$\tfrac{1}{1}s + \tfrac{1}{3}s + \tfrac{1}{5}s + \cdots \infty = \zeta(s)(1 - 2^{-s})$$

It is not clear what is being "summed" in:

$$(68) \qquad 2 + 5 + 13 + 35 + \cdots n \text{ terms} = \tfrac{1}{2}(3^n - 1) + 2^n - 1$$

$$(793) \qquad \theta^2 - \tfrac{2}{3}\theta^4 + \cdots \infty = \operatorname{logh}(1 + \theta \sin \theta)$$

$$(794) \qquad -\frac{\theta^2}{3} - \frac{7}{96}\theta^4 - \cdots \infty = \operatorname{logh} \theta \cot \theta$$

since the continuations of the series on the left are not at all obvious.

The vigorously divergent

$$(976) \qquad \frac{1}{\epsilon}(1 - 2! + 3! - \cdots \infty) = \frac{0.4036526}{\epsilon}$$

is given without any tiresome commentary concerning convergence.

While a good handbook of infinite series is something much to be desired, it is doubtful whether the present book fully meets this need.

D. S.

58[K].—B. M. BENNETT & P. HSU, *Significance Tests in a* 2 × 2 *Contingency Table: Additional Extension of Finney-Latscha Tables*, March 1962. Deposited in UMT File. [See Review 9, *Math. Comp*, v. 15, 1961, p. 88–89; Review 20, *ibid.*, v. 16, 1962, p. 252–253.]

The authors present in these manuscript tables still another addition to the tables of Finney, first extended by Latscha. This latest extension covers the range $A = 31(1)45$, giving the exact test probabilities to four decimal places, as previously, and the significant values of b at the four levels presented in the Finney-Latscha tables and retained in the previous extensions thereto by the present authors.

J. W. W.

59[V].—PHILIP D. THOMPSON, *Numerical Weather Analysis and Prediction*, June 19, 1961. The Macmillan Co., New York, xiv + 170 p., 24 cm. Price $6.50.

In a rapidly developing field it never seems quite appropriate to freeze the state of knowledge in the form of a book. However, enough has evolved since the beginnings of numerical weather prediction to warrant a knowledgeable appraisal of the course of its development. Not only should such a book have a didactic objective but one would hope that the perspective be equally useful as a reference for active workers in the field. This would require the text to assess the road of experience well enough to define the problems and to indicate the avenues which are likely to yield a fruitful expansion of knowledge. Colonel Thompson's book represents a first such attempt. The fact that he has contributed materially to the evolution he sets out to document, taken together with his characteristically smooth expository style, amply qualify him for the task.

In Chapter 1, after a brief discussion of the inherent difficulties of observing and forecasting the atmosphere's evolutions, one finds a description of instrumental, aerological, and analysis techniques, and of the atmosphere's kinematical characteristics. The author then indicates the role of hydrodynamic laboratory models as a research tool and gives an historical development of numerical weather prediction: the antecedents heralding the Norwegian school and the contributions of Richardson, Rossby and the Princeton group.

Chapters 2 and 3 are given over to a summary of the hydrothermodynamic equations of meteorology, first in height coordinates and then in terms of pressure and of potential temperature. The transformation and interpretation of upper and lower boundary conditions are not given. Methods of central differencing and practical problems of numerical weather prediction, especially those engendered by the quasi-non-divergent character of the atmosphere, are then discussed. Chapters 4

and 5 first go into the properties of pure sound, gravitational, and Rossby waves by means of a perturbation analysis of the linear hydrodynamic equations, followed by an analysis of the corresponding finite difference equations with some discussion of their computational stability properties. As an introduction to the use of filtering approximations, the author analyzes in Chapter 6 the properties of mixed wave type solutions and their interaction in linear systems.

In Chapter 7 the author discusses physical systems in which the total kinetic energy is preserved by means of the equivalent barotropic approximation, giving some emphasis to the numerical solution of the corresponding non-linear equations. Chapter 8 lightly covers the question of mapping, examines the finite Fourier series method of Charney, Fjortoft, and von Neumann, and then goes on to discuss the application of relaxation techniques to elliptic difference equations.

Systems in which potential-kinetic energy conversions are admissible and the necessary vertical differencing structure are considered within a geostrophic frame-work in Chapter 9. Here only superficial mention is made of the approximations energetically consistent with the geostrophic constraint. Furthermore, the question of the consistent lateral boundary conditions for the vertical velocity and thermo-dynamic equations is not covered at all. In Chapter 10 Dr. Thompson analyzes the process of baroclinic instability, and the concomitant energy conversions and pole-ward heat transfer, and their role in the index cycle. However he does not take the opportunity to indicate how the barotropic kinetic energy of vortices is transformed to maintain the westerlies—the final link in the energy cycle.

The subject of Chapter 11 is the balance approximation as a filtering device. It could have been useful here to show its connection with the primitive barotropic equations. Also absent is mention of attempts to adapt balance techniques to baro-clinic models. In Chapter 12 he takes up the question of establishing initial condi-tions for the primitive equations. He apparently holds the opinion that the balanc-ing constraint must be applied periodically; however, this has not been borne out by experience. He considers the question of formulating boundary conditions for open boundaries as an unsolved problem.

Getting down to practical problems, the author discusses in some detail in Chapter 13 the question of operational utilization. In particular, he describes the organization of the Joint Numerical Weather Prediction Unit, methods of data processing and objective analysis. After drawing comparisons with routine conven-tional forecasting methods and skills, he then boldly makes an attempt to calculate the economic worth of a forecaster. Finally, he examines the impact of mechaniza-tion on the data processing chain.

Chapter 14 is given over to the question of unsolved problems. Here he points out the systematic errors in the zonal angular momentum which result from ignor-ing surface stresses. He then goes on to emphasize the importance of sources of kinetic energy through baroclinic instability but makes no mention of how the available zonal potential energy is maintained through radiative processes, which are probably of some significance within 48 hours. He makes some mention of how the quasi-stationary long waves may be excited geographically and of the uncer-tainties of the initial state on a forecast. Finally, he says something about the effects of truncation and round-off error. Chapter 15, entitled "The Outlook for the Future," is essentially a recapitulation of the 14 preceding chapters.

One's impression upon reading the book is that it is m re an essay than a text or reference. A good many of its 170 pages are given over to t.ie discussion of peripheral subjects which are treated much more exhaustively elsewhere and could more appropriately have been referred to. This reviewer would rather that the author had taken this space and perhaps more for greater thoroughness in discussing prob- lems intimately germane to the development of dynamical prediction by numerical methods. A more complete discussion of computational instability of the various types that have already been encountered would have been extremely useful. A comprehensive account of mapping techniques for finite differences would be useful if found in one place. Very little attention is given to Green's function techniques and Fourier space, to the process of barotropic stability, to Lagrangian methods, to "staggered" finite difference methods yielding non-redundant solutions (such as that of Eliassen). The powerful methods and useful results of scale analysis are prominent in their omission. One would hope that in the absence of thoroughness Dr. Thompson would have given an exhaustive bibliography; however, his refer- ences are sometimes vague if not scanty (Phillips' significant contributions are virtually ignored) and those which are included often are given a superficial critique. Being the first of its kind, this book does fill a gap. However, this reviewer feels it to fall short of the needs, if not of the author's objectives.

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60[X].—W. F. FREIBERGER, Editor-in-chief, *International Dictionary of Applied Mathematics*, D. Van Nostrand Co., Inc., Princeton, N. J., 1960, 1173 p., 26 cm. Price $25.00.

At the outset it is perhaps appropriate to say a word concerning the title of this useful reference book. One might think that a book so named would confine itself to descriptions of those branches of *mathematics* which are applied to physics, engineering, etc., that is, to numerical analysis, vector analysis, statistics, etc. Instead a large number, or even most, of the entries here, e.g., Binary Stars, Polymer, Isotopes, Pfund Series, etc., are descriptions of those *phenomena* to which such mathematics may be applied. Of the 32 fields covered in this volume only 6 are applied mathematics in the strict sense, while Acoustical Engineering, Acoustics, Aerodynamics and Hydrodynamics, Astronomy, Atomic Structure, Automatic Control, Chemistry, Elasticity, Electromagnetic Theory, and 17 other fields are, rather, physical sciences to which mathematics is applied.

Each of the 32 fields had one or more authorities as a contributing editor. For example, that for Numerical Analysis was A. S. Householder. The 8000-plus entries are all listed alphabetically and not by field.

The many entries differ widely in their length and character. Those on modern physics, e.g., Relativity, Quantum Mechanics, S-Matrix, etc. are often fairly long and informative, but are weakened by a complete lack of references. The reader who wishes to learn more about Positronium or the Zeroth Law of Thermodynamics is given no assistance here. A number of the mathematical entries, on the other hand, are so brief that important qualifications and clarifications are omitted. Thus, in

"Continuous Function" we should have $\delta > 0$; in "Contour Integration" the quantities η_j should be defined; in "Analytic Function" it does not suffice for the derivative to be single-valued at the point itself; in "Gibbs Phenomenon," $x = \pi/(2n + 1)$ is not the discontinuity, but is merely near it; and in "Asymptotic Series" the expansion may be different in different sectors in the complex plane.

The typography is good and there appear to be relatively few errors such as Fronde Number in "Hydraulic Jump" and Hamiltonsion Theory in "Eikonal."

There are a few eccentricities. "Bigit" is advocated for what is now called "bit" in the binary system of numeration, and the number π is defined as the smallest positive *time t* at which the *oscillator* given by

$$\ddot{x} = -x \quad \text{and} \quad x(0) = 1, \quad \dot{x}(0) = 0$$

again attains $\dot{x}(t) = 0$.

In line with the current trend there are appropriate foreign language dictionaries in the appendix. The languages here are French, German, Spanish, and Russian.

Without question, this volume will be a standard reference in many technical libraries.

<div align="right">D. S.</div>

61[Z].—J. F. Davison, *Programming for Digital Computers*, Gordon and Breach Science Publishers, New York, 1962, xi + 175 p., 22 cm. Price $6.00.

The aim of this book is to provide an introduction to the general subject of writing programs, and it is written so as to be intelligible to the non-mathematician. It begins with a general discussion of the role and task of the programmer, assuming that he starts with the statement of a problem that needs to be programmed, and progresses to the point where routine computer operation has been achieved.

The essential vehicle for discussing the techniques of programming is a theoretical machine—TRIDEC—a 3-address decimal machine. With the aid of this machine and the limited set of orders, the author develops the basic concepts of programming up to the point where the reader has a feel for writing a simple routine using index register techniques and loops. There is then a brief discussion of a simple type of console to convey some notion of how the machine is controlled.

Under the heading of more sophisticated techniques there is a look at symbolic programming, subroutines, and floating-point computation.

Interpretive schemes and some aspects of automatic coding are then briefly mentioned. For such a broad subject the treatment is necessarily sketchy and it attempts merely to give general impressions.

Finally there is a discussion of differences among some different types of machines.

Some of the operating concepts will seem odd to American programmers, particularly the idea of using an endless loop as an equivalent to a halt.

For its small size, the book gives a general appreciation of programming. In particular, the details of TRIDEC coding are effectively presented.

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62[Z].—D. S. Evans, *Digital Data, Their Derivation and Reduction for Analysis and Process Control*, Interscience Publishers, Inc., New York, 1961, ii + 82 p., 19 cm. Price $2.95.

This amazingly small monograph presents, with typical English economy of words, a detailed introduction to the considerations involved in producing digital data from mechanical position analogs, such as shaft rotation or linear displacement.

Chapter I, Incremental Measurements, is introductory in nature, and states the prime reasons and principles for automatic digitizing, namely, to conserve manpower and to avoid human failure in applications where both accuracy and speed become increasingly important. The relations among physical, graphical, and digital representations of quantity are discussed, as are the limitations of scaling with respect to ultimate accuracy and precision. The chief advantages, methods, and system considerations are even summarized.

Chapters II and III, Digital Counting Devices and Direct Reading from Coded Scales, give brief, clear presentations of the several counting, direct reading, mechanical, and optical devices for analog-to-digital data conversion in the author's experience. The codes employed, methods to avoid ambiguities in read-outs, and detailed characteristics of each device are presented. A valuable feature here is the listing of performance figures and system requirements for seven specific digitizers.

Chapter IV, Ancillary Equipment, introduces some of the additional hardware and techniques required in incorporating mechanical analog-to-digital converters into data systems. The final chapter, Chapter V, System Arrangement and Application, approaches the analog-to-digital data flow from an over-all system view, indicating some of the end uses to which decoders are often put, and which type of device is then selected.

The author has succeeded in proving, that much valuable and easily applied information can be supplied in a very small volume, which includes 63 excellent figures and photographs within its 78 pages of text.

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63[Z].—Lejaren Hiller, Jr. & Leonard M. Isaacson, *Experimental Music Composition with an Electronic Computer*, McGraw-Hill Book Co., Inc., New York, 1959, vii + 197 p., 24 cm. Price $6.00.

This book is an exposition of the use of programming techniques, mathematics, and musical theory in the composition of music on the ILLIAC computer at the University of Illinois. It is based on the results of a set of experiments designed to determine whether high-speed computers could be used effectively to "compose" music and to analyze musical structures. No attempt was made to generate electronic (synthetic) music, so the performance of the music composed was reserved for conventional musical instruments. Within the limited scope of their aims, the authors were successful not only in carrying out their experiments, but also in producing this neat and scholarly description of their work.

After presenting a chapter on the nature of the aesthetic problem in music, a brief account is given of recent technical and artistic developments in both experimentally composed music and electronically performed music. Here the authors mention the use of mathematical formulations to supply foundations for constructing "supposedly aesthetically significant art structures"; for example, G. D. Birkhoff's theory of aesthetic measure and J. Schillinger's theory of mathematical aesthetics, but none of these formulations are employed in their experiments. This background is followed by an analysis of the technical problems involved and a brief description of the programming operations of a digital computer; then a detailed account of each of the experiments is given.

In developing their computer codes, the authors employed basic concepts from information theory and applied the Monte Carlo method. Random sequences of integers were equated to notes in the (Western) musical scale and also to rhythmic patterns, dynamics, and playing techniques. These random integers were then screened by applying tests expressing various rules of composition (depending on the experiment) and accepted or rejected, depending on the rules in effect. For example, the first experiments began with the simplest rules for writing polyphonic music in C-major in cantus firmus settings, using only the 15 notes from low C to C above middle C. The second experiment produced four-part first-species counterpoint in the same note range. Later experiments were based on the chromatic scale, and provided for the generation of rhythmic patterns, dynamic markings, and playing instructions for stringed instruments (legato, staccato, etc.). The last set of experiments produced what the authors call "Markoff chain music" of zero order and of higher orders. (The music is defined to be of zero order, if the generation of the interval I_n between the notes $n - 1$ and n is independent of the choice of the interval I_{n-1}; it is of first order, if the choice of I_n is dependent on I_{n-1}, etc.)

The programming details are given for each experiment along with explanatory flow-charts, lists of composition rules employed, and tables summarizing the calculations performed. The results of the experiments, that is, the computer-constructed music, were combined by the authors into a composition for string quartet, entitled "The ILLIAC Suite," which was transcribed by hand into conventional musical notation, and is given in the Appendix of the book.

It should be emphasized that the authors do not advocate that computers should replace composers. They only show that computers can serve as useful aids to composers in experimenting with new methods and rules in the field of composition, in studying the classical forms of music (sonata, fugue, etc.), in analyzing the styles of specific composers, and in throwing some light on the aesthetic nature of music.

Numerous references are given in the form of footnotes throughout the book; it would have been helpful if these references had been listed in the form of a bibliography at the end of the book.

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64[Z].—Philip Morrison & Emily Morrison, Editors, *Charles Babbage and His Calculating Engines*, Dover Publications, Inc., New York, 1961, xxxviii + 400 p., 21 cm. Price $2.00 (Paperbound).

The editors have prefaced this convenient compilation of selected writings of Charles Babbage and of several relevant papers by his contemporaries with an excellent biographical sketch, followed by a brief history of punch cards and a selected bibliography of pertinent literature.

The body of this entertaining book is made up of three parts, consisting, respectively, of unabridged chapters from Babbage's autobiographical *Passages from the Life of a Philosopher*, originally published in London in 1864; essays extracted verbatim from *Babbage's Calculating Engines*, published in London in 1889 under the editorship of his son Henry P. Babbage; and miscellaneous papers. These last include a complete listing of Babbage's published papers, a general plan of his Analytical Engine, several plates relating to an eight-day clock and a hydraulic ram as exemplifying his notation for expressing the action of machinery, and a reproduction of the table of contents of *Passages from the Life of a Philosopher*, which is revelatory of certain interesting aspects of his personal life and of his multifarious scientific interests. His versatility is revealed by these fields of interest, which include mathematics, railway engineering, cryptanalysis, and submarine navigation, in addition to his lifelong preoccupation with calculating machines. Furthermore, his only significant *completed* work, namely, the book, *Economy of Manufactures and Machinery*, published in 1832, foreshadowed the field now known as operations research.

The essays from *Babbage's Calculating Engines* include Dr. Lardner's detailed description of Babbage's Difference Engine, originally published in 1834; a memoir on the Analytical Engine published by an Italian engineering officer, L. F. Menabrea, following a visit of Babbage to Turin in 1840, and subsequently translated into English, with extensive annotations, by the Countess of Lovelace, only child of Lord Byron. The Countess, we are told, thoroughly understood and appreciated Babbage's elaborate plans for a universal automatic digital computer, and is reputed to have written the most lucid contemporary accounts of that machine as designed by its inventor. One of her contributions, which was appended to the translation of Menabrea's paper, is a detailed program for evaluating the Bernoulli numbers on the proposed Analytical Engine.

In this book one reads of Babbage's ambitious plans for an increasingly complex series of calculating machines, of his subsequent difficulty in securing continued financial support of the British Government in this research, and of his ultimate failure to bring his farsighted plans to fruition because of their magnitude and the resulting extraordinary demands on the technology of his time.

As the editors aptly remark of Babbage in concluding their Introduction, "The wide range of his practical and scientific interests and his clear commitment to the notion that careful analysis, mathematical procedures, and statistical calculations could be reliable guides in almost all facets of practical and productive life give him still a wonderful modernity. . . . His monument, not wholly beautiful, but very grand, is the kind of coupled research and development that is epitomized today, as it was foreshadowed in his time, by the big digital computers."

<div align="right">J. W. W.</div>

65[Z].—Paul Siegel, *Understanding Digital Computers*, John Wiley & Sons, Inc., New York, 1961, ix + 403 p., 23 cm. Price $8.50.

This book is intended as a comprehensive introduction to digital computers and data processors for a reader with no previous knowledge of the field. The author does expect the reader to have a basic understanding of electronics such as that acquired by a "ham."

The material is presented in an introductory chapter and three sections: Section I is devoted to the basic logical elements; Section II summarizes the circuits which can be used as computer building blocks; Section III describes the functional parts of a computer, and culminates with the detailed description of a specimen computer and its use.

The author states that the introductory chapter, which describes the digital computer and its functions, "provides the reader with an appreciation—a feel—for digital computers and a craving to learn more about them." This reviewer is not as certain as the author seems to be of the reader's reaction, for he finds the style of the book condescending and the material presented in a superficial fashion.

The first example of a superficial discussion is the following: In the introduction addition is described in terms of counting, and multiplication is described in terms of repeated addition; however, in the second chapter addition tables and multiplication tables are given for binary numbers. The relation between the two descriptions of addition and multiplication is never given.

Another example has to do with complements of numbers which are discussed in the first section, where subtraction is done by addition of complements. However, the multiplication of negative numbers by means of the products of their complements is never treated or even mentioned. Thus, the use of signed absolute values for the representation of numbers in a computer is never motivated, and the discussion of multiplication in the specimen computer described in the final chapter is very unclear. Therein computer subtraction is carried out by use of complements and multiplication is done by repeated addition, but the numbers are represented in absolute-value form.

The final example is furnished by the author's discussion of instruction modification on pages 338 and 339. In this discussion the author states that an add-address instruction may be used "together with *one* add instruction to cause the computer to accumulate a long list of numbers." The reader is never warned about the confusion that may result when the same accumulator is used for address modification and the accumulation of the sum, nor is he told of the price that must be paid in red-tape orders. Thus, he has no notion of the price that may have to be paid in orders and speed of computation when address modification is used.

These examples convince the reviewer that the book under discussion does not answer the great need for a good introductory book on digital computers.

<div align="right">A. H. T.</div>