bounds is in order. While the lower bound would be particularly important, the improved upper bound would also be useful.

Applied Mathematics Laboratory
David Taylor Model Basin
Washington 7, D. C.

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An Approximation to the Fermi Integral $F_{1/2}(x)$

By H. Werner and G. Raymann

The Fermi Integral as defined, for instance, in the *Handbuch der Physik*, Bd. XX, S. 58 [1], is given by

$$F_p(x) = \int_0^\infty \frac{t^p}{e^{t-x} + 1}\, dt. \tag{1}$$

The function $F_{1/2}(x)$ has for negative values of x an expansion of the form

$$F_{1/2}(x) = \frac{\sqrt{\pi}}{2} \sum_{\nu=1}^{\infty} (-1)^{\nu-1} \cdot \frac{e^{\nu x}}{\nu^{3/2}}, \tag{2}$$

and for large positive x the asymptotic expansion

$$\begin{aligned} F_{1/2}(x) \sim x^{3/2} \Bigg[\frac{2}{3} + \frac{\pi^2}{12 \cdot x^2} + \binom{\frac{1}{2}}{3} \cdot \frac{7}{60} \cdot \frac{\pi^4}{x^4} + \cdots \\ + \binom{\frac{1}{2}}{2n-1} \frac{2^{2n-1} - 1}{n} \mid B_{2n} \mid \cdot \frac{\pi^{2n}}{x^{2n}} + \cdots \Bigg]; \end{aligned} \tag{3}$$

compare [2], formulas (10) and (12);

B_{2n} are the Bernoulli numbers, given for example in [3], page 298. We obtained Chebyshev approximations to $F_{1/2}(x)$, based upon the table by McDougall and Stoner [4]. This table was subtabulated by interpolation with a fifth-degree polynomial. The approximations are

$$\begin{aligned} F^*_{1/2}(x) &= e^x \sum_{\nu=0}^{5} a_\nu e^{\nu x} && \text{for} \quad -\infty < x \leqq +1, \\ F^*_{1/2}(x) &= x^{3/2} \left[\frac{2}{3} + \sum_{\nu=0}^{5} \frac{b_\nu}{x^{2\nu+2}} \right] && \text{for} \quad +1 < x < +\infty, \end{aligned} \tag{4}$$

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the coefficients

ν	a_ν	b_ν
0	+0.8860 7596	+0.8435 00
1	−0.3087 1705	+0.7108 09
2	+0.1463 8520	−3.7124 56
3	−0.0584 3877	+6.7056 28
4	+0.0143 1771	−5.5948 77
5	−0.0015 0176	+1.7777 87

With these approximations, the relative error $|F_{1/2}(x) - F^*_{1/2}(x)|/F_{1/2}(x)$ is less than $2\cdot10^{-4}$ and $5\cdot10^{-4}$, respectively.

Another intensive table of $F_p(x)$ has been given by G. A. Chisnall [5] who also discusses in [6] a method for the interpolation of the existing tables of $F_{1/2}(x)$. It is not difficult to obtain analogous Chebyshev approximations to $F_p(x)$ for any fixed values of p to a prescribed degree of accuracy if one is able to generate the function with this (or slighty more) accuracy.

Institut für Angewandte Mathematik
der Universität Hamburg, Hamburg, and
Deutsches Rechenzentrum
Darmstadt, Germany

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On the Congruences $(p-1)! \equiv -1$ and $2^{p-1} \equiv 1 \pmod{p^2}$

By Erna H. Pearson

The results of computations to determine primes p such that one of the relations

$$(p-1)! \equiv -1 \pmod{p^2}, \tag{1}$$

$$2^{p-1} \equiv 1 \pmod{p^2} \tag{2}$$

holds have been published previously [1–5]. The known Wilson primes (those satisfying (1)) are 5, 13, and 563, the last having been determined by Goldberg [3] in testing $p < 10^4$. Froberg [4] tested $10^4 < p < 30{,}000$ without finding additional Wilson primes.

Froberg [4] determined $p = 1093$ and $p = 3511$ to be the only primes less than

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