An Approximation to the Fermi Integral $F_{1/2}(x)$

By H. Werner and G. Raymann

The Fermi Integral as defined, for instance, in the *Handbuch der Physik*, Bd. XX, S. 58 [1], is given by

$$F_p(x) = \int_0^\infty \frac{t^p}{e^{t-x} + 1} \, dt.$$  

The function $F_{1/2}(x)$ has for negative values of $x$ an expansion of the form

$$F_{1/2}(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{e^{-x}}{x^n},$$

and for large positive $x$ the asymptotic expansion

$$F_{1/2}(x) \sim x^{3/2} \left[ \frac{2}{3} + \frac{\pi^2}{12 \cdot x^2} + \left( \frac{1}{3} \right) \frac{7}{60} \cdot \frac{\pi^4}{x^4} + \cdots \right]$$

$$+ \left( \frac{\pi}{2n - 1} \right) \frac{2^{2n-1} - 1}{n} \cdot \frac{\pi^{2n}}{x^{2n}} + \cdots,$$

compare [2], formulas (10) and (12);

$B_{2n}$ are the Bernoulli numbers, given for example in [3], page 298. We obtained Chebyshev approximations to $F_{1/2}(x)$, based upon the table by McDougall and Stoner [4]. This table was subtabulated by interpolation with a fifth-degree polynomial. The approximations are

$$F_{1/2}^*(x) = e^x \sum_{r=0}^{5} a_r e^{rx} \quad \text{for} \quad -\infty < x \leq +1,$$

$$F_{1/2}^*(x) = x^{3/2} \left[ \frac{2}{3} + \sum_{r=0}^{5} b_r x^{2r+2} \right] \quad \text{for} \quad +1 < x < +\infty.$$
the coefficients

\[
\begin{array}{ccc}
\nu & a_\nu & b_\nu \\
0 & +0.8860 & 7596 & +0.8435 & 00 \\
1 & -0.3087 & 1705 & +0.7108 & 09 \\
2 & +0.1463 & 8520 & -3.7124 & 56 \\
3 & -0.0584 & 3877 & +6.7056 & 28 \\
4 & +0.0143 & 1771 & -5.5948 & 77 \\
5 & -0.0015 & 0176 & +1.7777 & 87 \\
\end{array}
\]

With these approximations, the relative error \(|F_{1/2}(x) - F_{1/2}^*(x)|/F_{1/2}(x)| \) is less than \(2 \cdot 10^{-4}\) and \(5 \cdot 10^{-4}\), respectively.

Another intensive table of \(F_p(x)\) has been given by G. A. Chisnall [5] who also discusses in [6] a method for the interpolation of the existing tables of \(F_{1/2}(x)\). It is not difficult to obtain analogous Chebyshev approximations to \(F_p(x)\) for any fixed values of \(p\) to a prescribed degree of accuracy if one is able to generate the function with this (or slightly more) accuracy.

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On the Congruences \((p - 1)! \equiv -1\) and \(2^{p-1} \equiv 1 \pmod{p^2}\)

By Erna H. Pearson

The results of computations to determine primes \(p\) such that one of the relations

\[(p - 1)! \equiv -1 \pmod{p^2},\]
\[2^{p-1} \equiv 1 \pmod{p^2}\]

holds have been published previously [1-5]. The known Wilson primes (those satisfying (1)) are 5, 13, and 563, the last having been determined by Goldberg [3] in testing \(p < 10^4\). Froberg [4] tested \(10^4 < p < 30,000\) without finding additional Wilson primes.

Froberg [4] determined \(p = 1093\) and \(p = 3511\) to be the only primes less than \(10^9\), but

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