

the coefficients

$\nu$	$a_\nu$	$b_\nu$
0	+0.8860 7596	+0.8435 00
1	-0.3087 1705	+0.7108 09
2	+0.1463 8520	-3.7124 56
3	-0.0584 3877	+6.7056 28
4	+0.0143 1771	-5.5948 77
5	-0.0015 0176	+1.7777 87

With these approximations, the relative error  $|F_{1/2}(x) - F_{1/2}^*(x)|/F_{1/2}(x)$  is less than  $2 \cdot 10^{-4}$  and  $5 \cdot 10^{-4}$ , respectively.

Another intensive table of  $F_p(x)$  has been given by G. A. Chisnall [5] who also discusses in [6] a method for the interpolation of the existing tables of  $F_{1/2}(x)$ . It is not difficult to obtain analogous Chebyshev approximations to  $F_p(x)$  for any fixed values of  $p$  to a prescribed degree of accuracy if one is able to generate the function with this (or slightly more) accuracy.

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## On the Congruences $(p-1)! \equiv -1$ and $2^{p-1} \equiv 1 \pmod{p^2}$

By Erna H. Pearson

The results of computations to determine primes  $p$  such that one of the relations

- (1)  $(p-1)! \equiv -1 \pmod{p^2}$ ,
- (2)  $2^{p-1} \equiv 1 \pmod{p^2}$

holds have been published previously [1-5]. The known Wilson primes (those satisfying (1)) are 5, 13, and 563, the last having been determined by Goldberg [3] in testing  $p < 10^4$ . Froberg [4] tested  $10^4 < p < 30,000$  without finding additional Wilson primes.

Froberg [4] determined  $p = 1093$  and  $p = 3511$  to be the only primes less than

50,000 satisfying (2). Kravitz [5] extended the range of primes tested in (2) to  $p < 10^5$  and found no additional primes of this type.

The author recently tested primes  $30,000 < p \leq 200,183$  in (1) and  $10^5 < p \leq 200,183$  in (2). No primes satisfying either relation were found in these ranges.

The computations were carried out on the Control Data 1604 Computer at The University of Texas. The formula used as a basis for programming the computations was

$$(3) \quad (p - 1)! \equiv (-1)^{(p-1)/2} 2^{2p-2} [(p - 1)/2!]^2 \pmod{p^2}$$

for an odd prime  $p$ . This formula is given as Theorem 133, Hardy and Wright [6].

The primes not exceeding 200,183 were generated and stored on tape, from which they were called in blocks to be tested individually. The 1604 computer is a binary computer with a 48-bit word length. The residue of  $2^{p-1}$  was determined by successions of left shifts and reductions modulo  $p^2$ , where the left shifts were long enough to multiply each intermediate residue by a reasonably large power of 2, yet short enough to avoid end-around carry. This residue was tested in (2), then squared, and reduced for use as a factor in (3). The residue of  $[(p - 1)/2]!$  was built up by successive multiplication and reduction modulo  $p^2$ , finally being squared and reduced for use in (3). The computation time per prime was roughly proportional to the size of the prime, about 8.5 seconds for a prime of the order of  $10^5$ .

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EDITORIAL NOTES: (1) The 18,000th prime is 200,183; (2) the residues were not saved and are not available for comparison.

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