

operation E may be required twice for a singly degenerate matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

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1. H. GARNER, "The residue number system," *IRE Trans. on Electronic Computers*, Vol. EC-8, pp 140-147, June 1959.

2. FRAZER, DUNCAN & COLLAR, *Elementary Matrices*, Cambridge University Press, 1960.

A Remark on a Paper of Bateman and Horn

By A. Schinzel

Let f_1, f_2, \dots, f_k be distinct irreducible polynomials with integral coefficients and the highest coefficient positive, such that $f(x) = f_1(x)f_2(x) \cdots f_k(x)$ has no fixed divisor > 1 . Denote by $P(N)$ the number of positive integers $x \leq N$ such that all numbers $f_1(x), f_2(x), \dots, f_k(x)$ are primes.

P. T. Bateman and R. A. Horn [1] recently gave the heuristic asymptotic formula for $P(N)$:

$$(1) \quad P(N) \sim \frac{N}{\log^k N} (h_1 h_2 \cdots h_k)^{-1} \prod_p \left(1 - \frac{\omega(p)}{p}\right) \left(1 - \frac{1}{p}\right)^{-k},$$

where h_i is the degree of f_i and $\omega(p)$ is the number of solutions of the congruence $f(x) \equiv 0 \pmod{p}$.

Formula (1) contains as particular cases six conjectures from a well-known paper of Hardy and Littlewood [3] called by the latter Conjectures B, D, E, F, K, P, as well as their conditional theorem X 1. This is evident except for Conjecture D which concerns the number of solutions of the equation

$$(2) \quad ap - bp' = k \quad (a > 0, b > 0, (a, b) = 1)$$

in primes p, p' with $p \leq n$. In order to apply formula (1) here one should put $f_1(x) = u_0 + bx, f_2(x) = v_0 + ax, N = \frac{n - u_0}{b}$, where u_0, v_0 are fixed integers such that $au_0 - bv_0 = k$.

Conjectures denoted by Hardy and Littlewood by J, M, N are of distinctly different character; besides the first has been proved by S. Chowla [2] and Ju. V. Linnik [4]. Conjecture A, (a strong form of Goldbach's Conjecture), is a particular case of C, Conjectures H and I are particular cases of G. It remains therefore to consider Conjectures C, G, L, which are, according to Hardy and Littlewood, conjugate to Conjectures D, F, K respectively. We quote them below for the convenience of a reader, with slight changes in the notation (e.g. p, p' denote primes).

Conjecture C. *If a, b are fixed positive integers and $(a, b) = 1$ and $P(k)$ is the number of representations of k in the form*

$$k = ap + bp'$$

then

$$P(k) = o\left(\frac{k}{(\log k)^2}\right)$$

unless $(k, a) = 1, (k, b) = 1$, and one and only one of k, a, b is even. But if these conditions are satisfied then

$$P(k) \sim \frac{2C_2}{ab} \frac{k}{(\log k)^2} \prod \left(\frac{p-1}{p-2}\right),$$

where

$$C_2 = \prod_{\substack{p=3 \\ p \nmid k, a, b}}^{\infty} \left(1 - \frac{1}{(p-1)^2}\right)$$

and the first product extends over all odd primes p which divide k, a or b .

Conjecture G. *Suppose that a and b are integers, and $a > 0$, and let $P(n)$ be the number of representations of n in the form $am^2 + bm + p$. Then if n, a, b have a common factor, or if n and $a + b$ are both even, or if $b^2 + 4an$ is a square then*

$$P(n) = o\left(\frac{\sqrt{n}}{\log n}\right).$$

In all other cases

$$P(n) \sim \frac{\epsilon}{\sqrt{a}} \frac{\sqrt{n}}{\log n} \prod \left(\frac{p}{p-1}\right) \prod_{\substack{p \equiv 3 \\ p \nmid a}} \left(1 - \frac{1}{p-1} \left(\frac{b^2 + 4an}{p}\right)\right),$$

where p is a common odd prime divisor of a and b , and ϵ is 1 if $a + b$ is odd and 2 if $a + b$ is even.

Conjecture L. *Every large number n is either a cube or the sum of a prime and a (positive) cube. The number $P(n)$ of representations is given asymptotically by*

$$P(n) \sim \frac{n^{\frac{1}{3}}}{\log n} \prod_p \left(1 - \frac{1}{p-1} (n)_p\right),$$

where $p \equiv 1 \pmod{3}$, $p \nmid n$, and $(n)_p$ is equal to 1 or to $-\frac{1}{2}$ according as n is or is not a cubic residue of p .

A comparison of formula (1) with the above formulas of paper [3] suggests forcibly the following conjecture.

Let polynomials $f_1, f_2, \dots, f_k (k \geq 0)$, $f = f_1 f_2 \dots f_k$ satisfy the same conditions as above. Let g be a polynomial with integral coefficients and the highest coefficient positive. Let n be a positive integer such that $n - g(x)$ is irreducible and $f(x)(n - g(x))$ has no fixed divisor > 1 . Denote by $N(n) = N$ the number of

positive integers x such that $n - g(x) > 0$ and by $P(n)$ the number of x 's such that all numbers $f_1(x), f_2(x), \dots, f_k(x)$ and $n - g(x)$ are primes. Then for large n we have

$$(3) \quad P(n) \sim \frac{N}{\log^{k+1} N} (h_0 h_1 \cdots h_k)^{-1} \prod_p \left(1 - \frac{\omega(p)}{p}\right) \left(1 - \frac{1}{p}\right)^{-k-1}$$

where h_0 is the degree of g and $\omega(p)$ is the number of solutions of the congruence $f(x)(n - g(x)) \equiv 0 \pmod{p}$.

Conjectures C, G, L and therefore also A, H, I are particular cases of formula (3). To see this, as far as C is concerned, one should put

$$f_1(x) = bx + l, \quad g(x) = ax, \quad n = \frac{k - al}{b},$$

where l is an integer such that $al \equiv k \pmod{b}$, $-b < l \leq 0$. Conjecture A has been extensively verified [3, p. 37]. I have had no possibility to verify by computation the agreement of formula (3) with reality in other cases. For such comparisons one should replace $N(\log N)^{-k-1}$ by $\int_2^N (\log u)^{-k-1} du$, as is pointed out in [3].

I conclude with expressing my thanks to the referee for his valuable suggestions.

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1. P. T. BATEMAN & R. A. HORN, "A heuristic asymptotic formula concerning the distribution of prime numbers," *Math. Comp.*, v. 16, 1962, p. 363-367.

2. S. CHOWLA, "The representation of a number as a sum of four squares and a prime," *Acta Arith.*, v. 1, 1935, p. 115-122.

3. G. H. HARDY & J. E. LITTLEWOOD, "Some problems of 'Partitio numerorum'; III: On the expression of a number as a sum of primes," *Acta Math.*, v. 44, 1923, p. 1-70.

4. JU. V. LINNIK, "An asymptotic formula in an additive problem of Hardy-Littlewood," (Russian), *Izv. Akad. Nauk SSSR. Ser. Mat.*, v. 24, 1960, p. 629-706.

Some Miscellaneous Factorizations

By John Brillhart

1. Introduction. The factorizations presented here have accumulated in the author's files for several years and have not heretofore appeared in print. Twenty-five contain new prime factors (designated by an asterisk), while twenty-one, listed as complete, possess a cofactor that is prime. Included are the complete factorizations of four Mersenne numbers and twelve Fibonacci numbers. Further results include a second factor of the Mersenne number M_{191} and of the Fermat number F_{10} , as well as the least prime factor of M_{271} . These new results relating to Mersenne numbers supplement an earlier tabulation by G. D. Johnson and the author [1].

The factorizations, with the exception of those numbered (6)-(9), inclusive, were obtained by the author on various IBM computers at the University of California at Berkeley and at Los Angeles. The factorizations (6)-(9) of certain alge-