REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1 [A, Z].—EDGAR KARST, *Six-Digit Fraction Conversion from Decimal to Octal with Unlimited Accuracy*, ms. of six tables (343 numbered sheets) and 15 handwritten sheets, 23 cm., deposited in UMT File.

Construction of these voluminous decimal-to-octal conversion tables was based on the author's earlier four-digit decimal-fraction conversion tables, which are described in [1]. Use of the present tables is illustrated by six numerical examples, one for each of the tables in this manuscript. The IBM 650 program (of 20 lines) used in the underlying calculations is given, and other details of the construction and checking of the tables are supplied.

Professor Karst also gives a list of references, which includes titles of related tables of Wijngaarden [2], Causey [3], and Fröberg [4].

J. W. W.

1. EDGAR KARST, *Tables for converting 4 digit decimal fractions to periodic octal fractions.* [See RMT 6, *MTAC*, v. 10, 1956, p. 37.]
2. A. VAN WIJNGAARDEN, *Decimary-Binary Conversion and Deconversion*, Report R-130 of the Computation Department, Mathematical Center at Amsterdam. [See RMT 83, *MTAC*, v. 11, 1957, p. 208.]
3. ROBERT L. CAUSEY, *Decimal to Octal and Octal to Decimal Conversion Tables*, U.S. Naval Air Missile Test Center, Point Mugu, California, 1952. [See RMT 65, *MTAC*, v. 10, 1956, p. 227.]
4. CARL-ERIK FRÖBERG, *Hexadecimal Conversion Tables*, second edition, C. W. K. Gleerup, Lund, Sweden, 1957. [See RMT 82, *MTAC*, v. 11, 1957, p. 208.]

2 [C].—JOSEPHINE G. BOERNGEN, *Table of Common Logarithms and their Squares*, Report PB 181 100, Office of Technical Services, Department of Commerce, Washington 25, D.C., December 1961, 3 + 22 p., 27 cm. Price $0.75.

This table, which was prepared in connection with a statistical study of chemical analyses of rock samples, contains log N and $(\log N)^2$ to 5D for $N = 1(1)\ 10^3\ (10^2)10^4$. According to the author, 5D values of log N were used to obtain the tabulated values of $(\log N)^2$. The author's statement that the corresponding truncation errors are as large as a unit in the last decimal place printed was evidently not based on an appropriate error analysis. Such analysis indicates the possibility of terminal-digit errors of as much as *four* units in the listed values of $(\log N)^2$. An error of this magnitude is attained, in fact, in the tabulated square of the common logarithm of 9900. Clearly, the last place in that table is generally unreliable.

J. W. W.

3 [F].—F. J. BERRY, *"Table of prime numbers from 12000000 to 12041000,"* Royal Armament Research and Development Establishment, Fort Halstead, Sevenoaks, Kent, England. Ms. of 11 p., 8″ x 13″, deposited in UMT File.

This list of the 2500 consecutive primes directly following 12,000,000 was computed on a new electronic computer, COSMOS, in a single run of 9 hours 20 minutes, using an unsophisticated program.

In his introductory remarks the author expresses his belief that this range has not been hitherto investigated in his country. He is apparently unaware both of the

efforts of his countrymen J. C. P. Miller and D. O. Claydon [1], who have prepared printed lists of primes to 21,000,000 and punched tape lists to 36,000,000, and of the results of D. H. Lehmer [2] and of Baker and Gruenberger [3] in this country.

J. W. W.

1. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, Addison-Wesley Publishing Company, Inc., 1962, v. I, p. 21.
2. D. H. Lehmer, "Tables concerning the distribution of primes up to 37 millions," reviewed in *MTAC*, v. 13, 1959, p. 56–57, RMT 3.
3. C. L. Baker & F. J. Gruenberger, *The First Six Million Prime Numbers*, The Microcard Foundation, Madison, Wisconsin, 1959. (See *Math. Comp.*, v. 15, 1961, p. 82, RMT 4.)

4 [F].—Ivan Niven, *Diophantine Approximations*, Interscience Publishers, New York, 1963, viii + 68 p., 24 cm. Price $5.00.

From the author's preface:

"At the 1960 summer meeting of the Mathematical Association of America it was my privilege to deliver the Earle Raymond Hedrick lectures. This monograph is an extension of those lectures, many details having been added that were omitted or mentioned only briefly in the lectures. The monograph is self-contained. It does not offer a complete survey of the field. In fact the title should perhaps contain some circumscribing words to suggest the restricted nature of the contents, . . .

"The topics covered are: basic results on homogeneous approximation of real numbers in Chapter 1; the analogue for complex numbers in Chapter 4; basic results on non-homogeneous approximation in the real case in Chapter 2; the analogue for complex numbers in Chapter 5; fundamental properties of the multiples of an irrational number, for both the fractional and integral parts, in Chapter 3. . . .

"A unique feature of this monograph is that continued fractions are not used. This is a gain in that no space need be given over to their description, but a loss in that certain refinements appear out of reach without the continued fraction approach. Another feature of this monograph is the inclusion of basic results in the complex case, which are often neglected in favor of the real number discussion. . . ."

"Homogeneous" and "non-homogeneous" above have reference to the following: If θ is irrational, the problem of finding integers k and h such that $k\theta - h$ is small (relative to some inverse power of k) is the homogeneous problem, while if α is real, the corresponding problem for $k\theta - h - \alpha$ is non-homogeneous.

"Not offer a complete survey" above has reference to the omission of important but more difficult topics such as Markoff numbers, Weyl's criterion for equidistribution, and the celebrated Roth theorem on approximations of real algebraic numbers. (See the following review.)

These more advanced topics have been treated in the older, more complete, and more difficult book [1] by Cassels: *An Introduction to Diophantine Approximation.* In fact, it would be appropriate if that book and this could exchange their titles. The present volume is certainly much more readable for a beginner and can be strongly recommended as an introduction to the subject.

Perhaps a mild rebuke is due the publisher. The price seems a little high for so slim a volume.

D. S.

1. J. W. S. Cassels, *An Introduction to Diophantine Approximation*, Cambridge Tracts No. 45, Cambridge University Press, 1957.

5 [F].—K. F. ROTH, *Rational Approximations to Irrational Numbers*, H. K. Lewis & Co., Ltd., London, 1962, 13 p., 26 cm. Price 3s. 6d.

In this brief, readable, and stimulating inaugural lecture, the author discusses his famous theorem:

Let α be algebraic and of degree $n \geqq 2$. If the positive number κ is such that the inequality

$$\left|\alpha - \frac{h}{q}\right| < \frac{1}{q^\kappa}$$

has an infinity of solutions h/q, then $\kappa \leqq 2$.

He traces its history via Liouville, Thue, Siegel, Dyson, and Schneider, and emphasizes the "fundamental weakness" of the proof (which goes all the way back to Thue) in that if κ is chosen greater than 2 it is impossible to put an upper bound on the corresponding largest value of q. This impossibility creates difficulties in applications, and is of immediate concern to investigators of some number-theoretic problems who are utilizing high-speed computers.

He also discusses other drawbacks and some unsolved problems, in particular a conjecture of Littlewood "which can hardly be given too much publicity". He agrees with Mahler's remark that "the whole subject is as yet in a very unsatisfactory state".

D. S.

6 [F].—C. D. OLDS, *Continued Fractions*, Volume 9 of the *New Mathematical Library*, Random House, New York, 1963, viii + 162 p., 23 cm. Price $1.95.

This is an easy-going exposition of simple continued fractions, that is, those of the form $n_1 + \frac{1}{n_2 + \frac{1}{n_3 + \cdots}}$. There are applications to the expansions of irrational numbers into infinite continued fractions and to the solution of the Diophantine equations $Ax \pm By = \pm C$ and $x^2 - Ny^2 = \pm 1$. There are many problems (mostly numerical) together with their solutions.

As with other volumes in this series, the material is mostly quite elementary, but brief mention is given of some more advanced material such as Hurwitz's theorem, Farey sequences, the (unnamed) Markoff "chain of theorems" (page 128), unsymmetrical approximations (page 129), and the logarithm algorithm (section 3.11). An interesting Appendix II lists some historically famous numerical or analytic continued fractions.

D. S.

7 [F, Z].—ROBERT SPIRA & JEAN ATKINS, *Coding of Primes for a Decimal Machine*; a deck of 159 IBM cards deposited in UMT File.

This deck of IBM cards is an efficient coding of the primes $< 10^5$ for use in a decimal machine. There are 159 cards, each containing a card number in the first 10 columns and seven other ten-digit coding words. The last word of the last card is not used, and is set equal to zero. Thus, the identification of the primes $< 10^5$ is capable of being stored in 1,112 ten-digit words.

This information was stored as follows: The odd numbers not divisible by 3 were written down. Below them were written binary bits: 1, if a prime; 0, if not.

The binary numbers were converted to octal and stored as such in a decimal machine. This makes for a reasonable decoding time. The process is illustrated by the table below.

5 7 11	13 17 19	23 25 29	31 35 37	41 43 47	49 53 55
1 1 1	1 1 1	1 0 1	1 0 1	1 1 1	0 1 0
7	7	5	5	7	2

Thus the first number in the deck starts: 775572. . . . The cards were prepared by Miss Jean Atkins at the Duke University Computing Center on an IBM 7072.

AUTHORS' SUMMARY

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8 [G].—B. H. ARNOLD, *Logic and Boolean Algebra*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962, 144 p., 23 cm. Price $9.00.

Logic and Boolean Algebra is an introductory text in which attention is mainly directed to an abstract development of the theory of finite Boolean algebras and Boolean rings. The first two chapters, dealing with propositional logic and Boolean functions, respectively, serve to provide illustrative material for the ordered sets and the general notion of an algebraic system in Chapter 3; the book then progresses through lattices (Chapter 4), Boolean Algebras (Chapter 5), Boolean Rings (Chapter 6) and ends in Chapter 7 with a treatment of normal forms and duality. The last chapter deals briefly with applications of Boolean algebra to the design and analysis of switching circuits and computers.

Chapters 5 and 6 form the core of the book. The main result of Chapter 5 is the theorem: Every finite Boolean algebra is isomorphic to the algebra of the set of all subsets of a finite set. In Chapter 6 the equivalence of Boolean rings with a unit and Boolean algebras is demonstrated, and it is shown that every finite Boolean ring is isomorphic to the ring of all n-tuples of Boolean constants for some n.

With the notable exception, in Chapter 1, of a confused discussion of object language versus meta language—a confusion compounded by a misuse of quotation marks and a failure to distinguish adequately names of linguistic objects from the objects themselves—the book is well written. Theorems are clearly stated, and their proofs are sensibly organized. There are numerous problems, many of which give results later used in the text.

The present volume, in short, constitutes a readable, if somewhat elementary, introduction to the study of abstract Boolean algebra.

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9 [K].—CHURCHILL EISENHART, LOLA S. DEMING & CELIA S. MARTIN, *Tables Describing Small-Sample Properties of the Mean, Median, Standard Deviation, and Other Statistics in Sampling from Various Distributions*, Government Printing Office, Washington 25, D.C., 1963, iv + 14 p., 26 cm. Price $0.20.

This note is a brief collection of ten single-page tables useful for the study of the

sampling distributions of some frequently-used statistics, with brief discussion of their construction and use. Their computation was motivated by the need for information on the behavior of these statistics (for example, their relative approach to normality with increasing sample size) for limited sets of data such as occur in measurement work where small numbers of specimens are used. The items tabulated are grouped as follows: (1) The probability level $P(\epsilon, n)$ of any continuous parent distribution corresponding to the level ϵ of the distribution of the median. This table is basic in the construction of the eight tables following. (2) Probability points of certain sample statistics for samples from six distributions: normal and double-exponential (mean, median), rectangular (mean, median, midrange), and Cauchy, sech, sech^2 (the median only, for these three). In all these nine tables, the sample size $n = 3(2)15(10)95$ and the probability levels are $\epsilon = .001, .005, .01, .025, .05, .10, .20, .25$. For the normal and double-exponential distributions there are also given the values of certain ratios useful for comparing the various statistics. (3) Probability that the standard deviation σ of a normal distribution will be underestimated by the sample standard deviation s and by unbiased estimators of σ based on s, based on the mean deviation, and based on the sample range. These results show striking biases in certain cases for some of the statistics in popular use. The sample sizes used for this table are slightly different from the others, all values to $n = 10$ and eight variously-spaced values to $n = 120$ being shown.

The user of these tables should not overlook the footnote on the first page. This warns that the probability level ϵ corresponds to the left tail for only the first table, and to the usual right tail for all other tables.

The explanations and implications in the brief text accompanying the tables are quite lucid and, unlike the facts of modern life, the price bears no relation to the effort in producing the tables, which anyone interested in detailed study of the distributions (or the results of such study) need, therefore, not hesitate in acquiring.

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10 [K].—D. B. OWEN, *Factors for One-Sided Tolerance Limits and for Variables Sampling Plans*, Sandia Corporation Monograph SCR-607, available from Office of Technical Services, Department of Commerce, Washington 25, D.C., 1963, 412 p., 28 cm. Price $5.00.

Tables are given of a quantity k which is used to define single-sample variables sampling plans and one-sided tolerance limits for a normal distribution. The probability is γ that at least a proportion P of a normal population is below $\bar{x} + ks$, where $\bar{x}$ has a normal distribution with mean μ and variance σ^2/n, and fs^2/σ^2 has a chi-square distribution with f degrees of freedom. The quantity k just described corresponds to a percentage point of the non-central t-distribution, and is extensively tabulated. Tabulations of other functions computed from the non-central t-distribution, and various expected values are also given. Many other applications are discussed and various approximations compared. One section gives the mathematical derivations, and there is an extensive bibliography which has been cross-referenced to several indices of mathematical and statistical literature.

The variables sampling plans given are to be preferred to most other such variables plans (including the MIL STD plans) in cases where the protection of the consumer is of primary interest and the costs of items are high. These plans may also be preferred in other circumstances but an analysis of costs of the alternative plans should precede any decision on which plan to use.

AUTHOR'S SUMMARY

11 [K].—MARVIN ZELEN, editor, *Statistical Theory of Reliability*, The University of Wisconsin Press, Madison, Wisconsin, 1963, xvii + 166 p., 25 cm. Price $5.00.

This volume contains six papers presented at a seminar sponsored by the Mathematics Research Center, U. S. Army, at the University of Wisconsin May 8–10, 1962. The papers survey recent research developments and results in the statistical theory of reliability. They were written mainly for mathematical statisticians doing research in the area rather than for analytical statisticians or engineers wishing to use the latest techniques. However, the volume does contain some techniques immediately useful to the applied statistician with adequate mathematical background.

A paper by Richard E. Barlow, entitled "Maintenance and Replacement Policies," includes two tables that could be used in developing a replacement policy when the distribution of component life is not known but the first, or the first and second, moments of the distribution can be estimated.

The other articles are: "A Survey of some Mathematical Models in the Theory of Reliability," by George H. Weiss; "Redundancy for Reliability Improvement," by Frank Proschan; "Optimum Checking Procedures," by Larry C. Hunter; "Confidence Limits for the Reliability of Complex Systems," by Joan Raup Rosenblatt; and "Problems in System Reliability Analysis," by William Wolman.

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12 [K].—L. E. MOSES & R. V. OAKFORD, *Tables of Random Permutations*, Stanford University Press, Stanford, California, 1963, 233 p., 24 cm. Price $7.00.

This book presents random permutations of integers: specifically, 960 permutations of the integers 1–9; 850 permutations of the integers 1–16; 720 of 1–20; 448 of 1–30; 400 of 1–50; 216 of 1–100; 96 of 1–200; 38 of 1–500; and 20 of 1–1000.

The permutations were created from the RAND deck of a million random digits by an algorithm especially suited to machine computation and for which a flow chart is given. The randomness of the permutations of size 50 or less was studied by means of goodness-of-fit tests on the observed distributions of (a) the longest run up or down, (b) rank correlation with order position, (c) Friedman's analysis-of-variance statistic, and (d) the distribution of the square of a linear function of the deviation from expectation of the number of runs (up or down) of length 1, 2, and 3. The tests show two results significant at the 1 % level: in test (a) for permutations of 1–30, and in test (d) for permutations of 1–16. Such performance would certainly

be acceptable on its own, but, in addition, the randomness of the RAND deck, on which these tables are based, has been checked [1] and thus provides a check on the randomness of the permutations, and vice versa.

In the experimental sciences the most frequent uses of random numbers are in connection with assignments of experimental units, and for this purpose tables of random permutations are required. Cochran and Cox [2] give 1000 permutations of the integers 1–9 and 1000 permutations of the integers 1–16. For small numbers, creating a permutation from tables of random numbers is not difficult; however, in permutations of 20 or more numbers the work involved is not negligible, and one would accordingly predict that the Moses-Oakford tables will be extensively used in the experimental sciences.

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1. RAND CORPORATION, *A Million Random Digits with 100,000 Normal Deviates*, Free Press, Glencoe, Illinois, 1955.
2. W. G. COCHRAN and G. M. COX, *Experimental Design*, second edition, John Wiley and Sons, Inc., New York, 1957.

13 [K].—J. ARTHUR GREENWOOD & H. O. HARTLEY, *Guide to Tables in Mathematical Statistics*, Princeton University Press, New Jersey, 1962, lxii + 1014 p., 26 cm. Price $8.50.

Greenwood and Hartley's *Guide* will be on the desks of all sophisticated users of statistical methods and of all applications-oriented statisticians. It will be on a nearby bookshelf of nearly all theory-oriented statisticians. The rapidly increasing use of statistical methods in science and industry and the rapid expansion of the body of available statistical methodology has caused a corresponding increase in the production of tables to facilitate application of the new methods. While a new high-speed calculating machine may, in many cases, calculate a needed constant more quickly than it can consult a table, statisticians who do not have access to such a machine need tables, and statisticians with high-speed machines use tables anyway rather than to program the calculation of the needed constants. Fortunately, the new machines have made table-making less expensive. The *Guide* will direct one to an appropriate table if it is among the approximately 1500 tables, published mostly before 1961, which the authors have catalogued.

The body of the *Guide* is arranged in chapters, sections, and subsections to facilitate rapid discovery of the catalogue of tables of the type sought. Within each section and subsection the authors have described the functions tabulated, either by exhibiting a functional form or by describing the method of calculation. Authors of guides to mathematical tables have noted great difficulty in producing such descriptions for statistical tables. Happily the authors have done even more and, except for tables of types which have been in wide use for many years, have described the purpose for which each table was intended. The authors have been remarkably successful in giving concise, yet fully adequate explanations. In a few cases where there is no concise explanation possible, the reader is told where a competent explanation can be found. The range of the table and the number of decimals or significant figures tabled are, of course, given and frequently there are

comments on the accuracy of the entries. Methods of approximations beyond the limits of available tables are often presented.

Three chapters are worthy of particular note. Chapter 13 deals with tables of random numbers. By fault of the authors of the tables, rather than of the authors of the *Guide*, the description of the method of computation of some of these tables is less complete than might be wished. Chapter 14 covers acceptance sampling, control charts, and tolerance limits. Chapter 15 (one of the longer chapters) is a collection of tables for the design of experiments.

Throughout the *Guide* the appearance of a listed table in one or more of 16 standard collections of statistical tables is noted, so that the table seeker can avoid a library search for a table he already has handy.

A detailed subject index provides reference to the proper page in case the table seeker uses key words which do not fall under the system of classification used by the authors of the *Guide*. The devices used by the authors to conserve space are simple, and a glance at the introduction is enough to make them clear. The introduction does give considerable detail on the more subtle aspects of the bibliographic art and on the applications of that art in the *Guide*.

Main chapter headings, in abbreviated form, are as follows:

1. The normal distribution
2. The chi-squared and Poisson distributions
3. The beta and binomial distributions
4. The t-, F- and z-distributions
5. Various discrete distributions
6. Likelihood-test statistics
7. Correlation, mostly product-moment
8. Rank correlation; asymptotic theory of extreme values
9. Non-parametric tests
10. Frequency curves; symmetric functions
11. Regression and other curves
12. Variate transformations
13. Random numbers
14. Quality control
15. Design of experiments, etc.
16. Sundry mathematical tables

Appendices are devoted to:

1. Supplement to the descriptive catalogue
2. Contents of some standard collections of tables for statisticians
3. Material treated both in this *Guide* and in Fletcher *et al.* 1946

The *Guide* is equipped with extensive author and subject indices, respectively, comprising the concluding sections.

The authors and the sponsors, the Committee on Statistics of the Division of Mathematics of the National Academy of Sciences—National Research Council, and the Office of Naval Research are to be commended for time and money well spent on behalf of nearly all users of statistical tables.

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14 [L].—F. M. Arscott & I. M. Khabaza, *Tables of Lamé Polynomials*, The Macmillan Company, New York, 1962, xxviii + 526 p., 27 cm. Price $20.00.

Consider the Lamé equation

$$\frac{d^2w}{dz^2} + \{h - n(n+1)k^2 sn^2 z\}w = 0 \tag{1}$$

where $sn\ z = sn(z, k)$ is the Jacobian sine-amplitude (elliptic) function. If $sn\ z = t$, we get

$$(1 - t^2)(1 - k^2t^2)\frac{d^2w}{dt^2} - t(1 + k^2 - 2k^2t^2)\frac{dw}{dt} + \{h - n(n+1)k^2t^2\}w = 0 \tag{2}$$

Equation (1) may be transformed into four other forms like (2) by use of the substitutions $sn^2\ z = \zeta$, $k\ sn\ z = u$, $cn\ z = x$, and $dn\ z = y$. A trigonometric form results from putting $am\ z = v$ in (1). If in (1) we put $sn^2\ z = \zeta$ and then $\zeta = P(\xi)$, where $P(\xi)$ is the Weierstrass elliptic function, we then get the "Weierstrassian" form. Each form possesses finite solutions only when n is a positive integer and h is one of $(2n + 1)$ eigenvalues. For each form, these solutions fall into eight types. For example, for (1), they are of the form

$$w = sn^\rho\ z\ cn^\sigma\ z\ dn^\tau\ z\ F(sn^2\ z) \tag{3}$$

where $\rho, \sigma, \tau = 0$ or 1 and $F(sn^2\ z)$ is a polynomial in $sn^2\ z$ of degree $\frac{1}{2}(n - \rho - \sigma - \tau)$.

Let $N = n/2$ or $(n - 1)/2$, according as n is even or odd. For $N = 1(1)15$ and $C = k^2 = 0.1(0.1)0.9$, this volume gives to 6S the $(2n + 1)$ values of h and the corresponding coefficients of the polynomial F for each type. Similar tables for $N = 16(1)30$ are deposited with the Royal Society, in the Depository of Unpublished Mathematical Tables, as Reference 78. There are a few other scattered tables in the literature, but this appears to be the first systematic tabulation attempted.

An introduction clearly presents the basic properties of the functions, correspondence with other notations, and the method of computation. The computations were done on the Ferranti MERCURY computer, and the computer program is given. The tables were printed by photo offset from the computer output. The entries are legible, but the type is not pleasing to the eye.

Y. L. L.

15 [L].—G. Blanch & Donald S. Clemm, *Tables Relating to the Radial Mathieu Functions, Vol. 1: Functions of the First Kind*, U.S. Government Printing Office, Washington 25, D.C., 1962, xxiv + 383 p., 27 cm. Price $3.50.

This table provides numerical solutions of the differential equation

$$\frac{d^2f}{dz^2} - (a(q) - 2q\cosh 2z)f = 0 \tag{1}$$

where the $a(q)$ are the eigenvalues corresponding to which

$$\frac{d^2f}{dz^2} + (a(q) - 2q\cos 2z)f = 0 \tag{2}$$

has solutions of period π or 2π. The tabulated solutions depend on three parameters; namely q, z, and the order of the eigenvalue r.

The solutions of (2) fall into four categories, namely even or odd, and periodicity π or 2π. Solutions of (1) can be obtained from (2) by replacing z by iz. The even solutions of (1) are denoted by $Mc_r^{(1)}(z, q)$ and the odd ones by $Ms_r^{(1)}(z, q)$. For convenience, these are represented by

$$Mc_r^{(1)}(z, q) = M_r \cosh rz \, Pc_r(z, q)$$

$$Ms_r^{(1)}(z, q) = M_r \sinh rz \, Ps_r(z, q)$$

and

$$\frac{d}{dz} Mc_r^{(1)}(z, q) = rM_r \sinh rz \, Qc_r(z, q)$$

$$\frac{d}{dz} Ms_r^{(1)}(z, q) = rM_r \cosh rz \, Qs_s(z, q)$$

where

$$M_r = q^{1/2r}/(r!)2^{r-1}$$

Actually, the functions tabulated are the P and Q functions. The extraction of the hyperbolic functions leads to data which are readily interpolable in both z and q. The table must, therefore, be used in conjunction with a table of hyperbolic functions.

There are four basic tables. They provide 7D approximations to $Pc_r(x, q)$, $Qc_r(x, q)$, $Ps_r(x, q)$, and $Qs_r(x, q)$ for $q = 0$ (0.05) 1; $r = 0$ (1) 7, $x = 0$ (0.02) 1, and $r = 8$ (1) 15, $x = 0$ (0.01) 1.

In addition, the values of $M_r(q)$ are furnished to 8S, as are those of the functions $C_r(q)$ and $S_r(q)$, which are defined on pages 1 and 197. The latter can be used instead of $M_r(q)$, corresponding to a different normalization. Also, the eigenvalues $a_r(q)$ and $b_r(q)$ are given to 8D. The computations were performed by a stepwise numerical integration of the differential equations for the P and Q functions. Some of the computations were performed on an 1103 ERA computer; the rest, on an IBM 7090.

The superscript appearing in $Mc_r^{(1)}(z, q)$ indicates that these are functions of the first kind (corresponding to Bessel functions for $q = 0$). A table for functions of the second kind is now in preparation.

Preceding the table is a good general discussion. A helpful chart relates the many non-standardized notations in this field.

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16 [L].—L. S. BARK & P. I. KUZNETSOV, *Tablitsy tsilindricheskikh funktsiĭ ot dvukh mnimykh peremennykh* (*Tables of Cylinder Functions of Two Imaginary Variables*), Computing Center, Acad. Sci. USSR, Moscow, 1962, xx + 265 p., 27 cm. Price 2.87 rubles.

On replacing x and y in the Lommel functions of two variables

$$\sum_{m=0}^{\infty} (-1)^m \left(\frac{y}{x}\right)^{n+2m} J_{n+2m}(x)$$

by ix and iy respectively, one is led to the functions

$$\Upsilon_n(y, x) = \sum_{m=0}^{\infty} \left(\frac{y}{x}\right)^{n+2m} I_{n+2m}(x)$$

which are here tabulated. These functions may thus be regarded as Lommel functions of two pure imaginary variables. A collection of formulas in the volume uses also the closely related notation

$$\theta_n(y, x) = \sum_{m=0}^{\infty} \left(\frac{x}{y}\right)^{n+2m} I_{n+2m}(x).$$

The tables give values of $\Upsilon_1(y, x)$ and $\Upsilon_2(y, x)$ to 7S for $y = 0(.01)1(.1)20$, $x = 0(.01)1(.1)y$. There are also second differences in both x and y. Although these are denoted by Δ^2_{xx} and Δ^2_{yy}, they are central differences; the second equation of line 2 on page xiv should accordingly read $\Delta^2_{xx}f(x_0) = f(x_1) - 2f(x_0) + f(x_{-1})$. Ordinary Everett coefficients of second differences are tabulated to 8D without differences at interval 0.001. The scheme of bivariate interpolation recommended is clearly set out, with a diagram on page xiv and worked examples, and will be quite intelligible to anyone who does not read Russian.

These extensive tables (computed on the electronic computing machine STRELA) are a development of part of a small table published by Kuznetsov in 1947; see *MTAC*, v. 3, 1948, p. 186 (*for* Kuznetsev, *read* Kuznetsov), or FMRC *Index*, second edition, 1962, Art. 20.72.

There is mention of several integrals which have been shown by Kuznetsov to be expressible in terms of Lommel functions of two imaginary variables. No fewer than nine fields of application are briefly mentioned; in eight of these cases, the bibliography includes at least one reference in a Western language. The present tables have been made to remedy a lack which has made numerical applications difficult, and are clearly of importance.

A. F.

17 [L].—J. W. McClain, F. C. Schoenig, Jr. & N. J. Palladino, *Table of Bessel Functions to Argument 85*, Engineering Research Bulletin B-85, The Pennsylvania State University, University Park, Pennsylvania, September 1962, v. + 30 p., 28 cm. Price $1.00.

This table consists of 4S values, in floating-point form, of the Bessel functions $J_n(x)$, $Y_n(x)$, $I_n(x)$, and $K_n(x)$ for $n = 0, 1$ and $x = 0(0.1)85$.

An introduction of five pages describes the conventional mathematical procedures used in the underlying calculations, which were performed on an IBM 7074 system, using a Fortran program reproduced in the Appendix.

One infers from the Preface that the authors were apparently unaware of the existence of such fundamental related tables as those of Harvard Computation Laboratory [1] and of the British Association for the Advancement of Science [2].

Moreover, the reliability of the least significant figure appearing in the table under review is uncertain, as revealed by a comparison with the corresponding entries in the fundamental tables cited. Such examination has disclosed 26 terminal-digit errors in the entire range of values tabulated herein for $J_0(x)$ and $J_1(x)$ and

27 errors (ranging up to 25 units) in the tabulated values of the remaining functions for $x \leqq 20$.

Thus we have still another example of the result of insufficient planning to insure complete tabular accuracy, especially in the vicinity of zeros of oscillatory functions such as $J_n(x)$ and $Y_n(x)$, where single-precision computations cannot always be relied upon to yield the desired number of significant figures.

Despite these flaws, the present table performs a valuable service in listing approximations to the values of $Y_n(x)$, $I_n(x)$, and $K_n(x)$ over a range not hitherto tabulated.

J. W. W.

1. Harvard University, Computation Laboratory, *Annals*, v. 3, *Tables of the Bessel Function of the First Kind of Orders Zero and One*, Harvard University Press, Cambridge, Massachusetts, 1947.

2. British Association for the Advancement of Science, Committee on Mathematical Tables, *Mathematical Tables*, v. 10, *Bessel Functions, Part II: Functions of Positive Integer Order*, Cambridge University Press, Cambridge, England, 1952.

18 [I, M].—Ruel V. Churchill, *Fourier Series and Boundary Value Problems*, McGraw-Hill Book Company, Inc., New York, 1963, viii + 248 p., 23 cm. Price $6.75.

Professor Churchill has written several very successful text books at the intermediate level. Each of these books has been marked by careful attention to mathematical detail. They are extremely easy to teach from and contain plenty of exercises for the student.

This second edition of *Fourier Series and Boundary Value Problems* continues this tradition. The chapter topics are: 1. Partial Differential Equations of Physics; 2. Superposition of Solutions; 3. Orthogonal Sets of Functions; 4. Fourier Series; 5. Further Properties of Fourier Series; 6. Fourier Integrals; 7. Boundary Value Problems; 8. Bessel Functions and Applications; 9. Legendre Polynomials and Applications; and 10. Uniqueness of Solutions.

These chapter headings are, in many ways, similar to those in the first edition. A separate chapter on the Fourier integral has been added, and various chapters have been rearranged. The lists of problems have been either added to or, in many cases, completely replaced with new ones. References have now been placed in a bibliography at the end of the book.

These changes appear to the reviewer to be definite improvements over the first edition. The book should supply the needs for an intermediate text for students in mathematics, physics, or engineering. The only criticism that the reviewer can make concerns the lists of problems. There are plenty of problems, but no really difficult ones. For the most part, they are simple applications of material presented in the text. It would seem that the book would appeal to mathematics students more if it contained some additional, difficult problems dealing with applications and with further mathematical theory.

Teachers will welcome Professor Churchill's book as a clear, well written text which makes their job easier for them.

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19 [L, M, S, T].—JAMES A. BIERLEIN, "Gouy diffractometry in thermal diffusion," *The Journal of Chemical Physics*, v. 36, 1962, p. 2793–2802.

This paper contains (on p. 2797–2799) a 4D table of zeros and turning points of the incomplete Airy integral

$$\mathrm{Ai}(A, x) = \frac{1}{\pi} \int_0^A \cos\left(\frac{u^3}{3} + xu\right) du,$$

corresponding to $A = 0(\pm 0.25) \pm 5(\pm 0.5) \pm 6$. For each listed value of A, a total of from 28 to 33 interlacing zeros and turning points are tabulated.

Included also are corresponding data for the complete Airy integral, given by $\lim \mathrm{Ai}(A, x)$ as $A \to \infty$, and for the ratio $\lim \mathrm{Ai}(A, x)/A$, as $A \to 0$. The latter appears erroneously in the column heading as Ai/x.

The associated maximum and minimum values of these integrals are given to 5 or 6D.

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20 [L, P].—GUSTAV DOETSCH, *Guide to the Applications of Laplace Transforms*, D. Van Nostrand Co., Ltd., London, 1961, 255 p., 24 cm. Price $9.75.

This book, by the well-known author of standard books on the Laplace Transformation, is intended to aid engineers in their use of this transformation. Theorems are carefully stated and the reader is referred for the proofs of the more involved of these to the author's book *Theorie und Anwendung der Laplace-Transformation.* Common pitfalls are indicated by a special warning sign printed in the margin. In addition to the usual treatment of linear differential equations with constant coefficients, a problem in automatic control involving a nonlinear differential equation is discussed. A chapter is devoted to difference equations and sampled data systems. A short account of the application of the Laplace Transformation to partial differential equations, treating the heat conduction equation and the equations of a twin conductor line with distributed constants, is given. The book closes with a chapter on the asymptotic behavior of functions and an appendix listing 256 Laplace transforms. The translation into English from the second edition of the original German book is well done and the printing is excellent. Translations of the first edition into Russian, French, and Japanese have previously appeared.

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21 [M, X].—V. I. KRYLOV, V. V. LUGIN & L. A. IANOVICH, *Tables for the Numerical Integration of Functions with Power Singularities*, Izdat. Akad. Nauk BSSR, Minsk, 1963 (Russian), 434 p. Price 1r. 45k.

This book contains tables of Gauss-Jacobi quadrature formulas of the form

$$\int_0^1 x^\beta (1 - x)^\alpha f(x)\, dx \simeq \sum_{i=1}^n A_i f(x_i),$$

which are exact whenever f is a polynomial of degree $\leqq 2n - 1$. Formulas are given for $\alpha, \beta = -0.9(0.1)3.0, \beta \leqq \alpha$, and $n = 1\ (1)8$. The A_i and x_i are given to 8 significant figures. The x_i $(i = 1, \cdots, n)$ are the zeros of the corresponding Jacobi polynomial $P_n^{(\alpha,\beta)}(x)$ for the segment [0, 1]. The tables comprise 410 pages; there is a short introduction of 20 pages.

These are important tables, both for practical calculations and for the information they give concerning Jacobi polynomials; they will be useful, for example, for investigations of the distribution of the zeros of Jacobi polynomials. For use in numerical integration on a high-speed computer, however, it may be more convenient in many cases to compute a formula of this type before it is used. Each of these low-order formulas could be computed on, say, the IBM 7090 in a very few seconds.

For this reason, we believe it is more important to give tables of highly accurate formulas, which are more difficult and time consuming to compute. This reviewer is preparing such a set of tables; among the formulas already computed are the following Gaussian-type formulas:

$$\int_{-1}^{1} f(x)\,dx \simeq \sum_{i=1}^{n} A_i f(x_i), \qquad n = 2(1)64(4)96(8)168,256,384,512$$

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)\,dx \simeq \sum_{i=1}^{n} A_i f(x_i) \qquad n = 2(1)64(4)96(8)136$$

$$\int_{0}^{\infty} e^{-x} f(x)\,dx \simeq \sum_{i=1}^{n} A_i f(x_i) \qquad n = 2(1)32(4)68$$

We have the A_i and x_i in these formulas correct to 30 significant figures.

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22 [P].—Frederick C. Grover, *Inductance Calculations Working Formulas and Tables*, Dover Publications Inc., New York, 1962, xiii + 286 p., 21 cm. Price $1.85.

This book first appeared in 1946. At that time it would have been an excellent tool for the practical engineer or designer of coils. Although the entire work is based upon one conceptually simple general definition, the author has heroically catalogued, in some 280 pages, an impressive number of special cases. He proceeds systematically through all types of coil shape, winding types, relative orientation, and other parameter situations. Formulae for such special cases are provided and adequate approximations and/or tables of special functions supplied. Today, the type of calculation in which this book would be of practical use would be carried out by a relatively small number of general-purpose inductance computing codes on a digital computer.

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EDITOR'S NOTE: See also *MTAC*, v. 3, 1948–1949, p. 521, RMT 674.

23 [P, X].—R. H. MACMILLAN, *Non-Linear Control Systems Analysis*, The Macmillan Company, New York, 1963, xvi + 174 p., 20 cm. Price $3.75.

This is an introductory volume for the engineer and applied mathematician who wishes to make the transition from prewar to postwar control theory. The authors cover in a very lucid fashion, with numerous examples and discussions, nonlinear control in practice, the phase-plane method, the describing function technique, the calculation of transients, and the rudiments of feedback control.

The book suffers, as do many of its kind, from a complete lack of awareness of the effect of the digital computer upon modern mathematics and control theory. There is no discussion of the use of large-scale computers, nor of dynamic programming, quasilinearization, or any other theories which depend upon modern devices for their successful utilization. There is nothing on the calculus of variations and no mention of the Pontryagin maximum principle. Furthermore, there are no references for advanced reading.

This book would have been extremely useful in 1940, or even in 1946.

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24 [X].—JOSEPH BECKER & ROBERT M. HAYES, *Information Storage and Retrieval: Tools, Elements, Theories*, John Wiley & Sons, Inc., New York, 1963, xi + 448 p., 23 cm. Price $11.95.

This is the first volume of the publisher's Information Science Series. It is written primarily from the point of view of the system designer, and is intended for use as either a textbook or a reference work. According to the preface, its purpose is to guide the newcomer through the maze of scattered development that has characterized the early growth of information storage and retrieval. The result is a well-balanced exposition of a field with tantalizing possibilities.

The first seven chapters survey the tools used by documentalists to organize recorded knowledge and make it available. Techniques developed by the library profession include classification schemes, indexes, and card catalogs. Over the years, however, both logical and physical inadequacies have appeared in the traditional approach. To cope with the logical difficulties a number of new techniques have been devised, viz., coordination of subject terms, superimposed coding of terms, generation of formalized abstracts, and analysis of subject relationships. To cope with the physical problems new recording and processing devices are being utilized, e.g., microfilm, fast reproducing equipment, and electronic computers.

In the next five chapters the authors examine how investigators with various technical backgrounds can contribute to the solution of outstanding problems. The areas of responsibility of user, operator, designer, and equipment supplier are defined, and their common areas of interest and conflict are analyzed in terms of requirements and capabilities. Questions of value arise in seeking to optimize system design. Thus, one must weigh the cost of complexity against its value to the user. In recognition of the importance of a logical structure for explaining the contents of a file, the authors observe that "the better the theory, the simpler the question which can be asked and the more complex the possible answer."

The final three chapters attempt to provide a theoretical foundation for design-

ing information systems. It is proposed that such systems be investigated by building mathematical models, so that their performance can be predicted quantitatively. Techniques which could be applied to the determination of similarity or relevance include symbolic logic, matrix algebra, and statistical analysis.

The emphasis, however, is almost exclusively on means for increasing the effectiveness of retrieval systems through improvements in the physical organization of files. Theories of logical organization are largely ignored except those based upon patterns of usage. In particular, the authors regard an *a priori* organization of descriptive terminology from a semantic viewpoint as being too confining and inflexible in any operational situation. Despite this, the book is one of the most informative yet to appear and is a welcome addition to the field.

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25 [X].—H. S. M. COXETER, *Regular Polytopes*, second edition, The Macmillan Company, New York, 1963, xix + 321 p., 23 cm. Price $4.50 (Paperback).

This second edition is essentially the same as the first edition of 1948, but in paperback by another publisher, with slightly larger pages and appreciably larger plates. Corrections and minor additions have been made, and six pages have been revised.

It still remains the most extensive and authoritative summary of the derivations and enumeration of the n-space generalizations of the regular and quasi-regular polyhedra. It includes their metric, topological, and group properties, and the history of their development. Although the subject of polyhedra is quite ancient, new discoveries concerning these polytopes have been made since the first edition, many by the author. Some of the new work is mentioned in the text and in the extensive bibliography.

A review of the first edition by the present reviewer appeared in *Mathematical Reviews*, v. 10, 1949, p. 261.

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26 [X].—OYSTEIN ORE, *Graphs and Their Uses*, Volume 10 of the *New Mathematical Library*, Random House, New York, 1963, viii + 131 p., 23 cm. Price $1.95.

This is an excellent introduction to graph theory. The exposition is elementary, although less so than that of most of the volumes of this series. The intended audience (for the series) of "high school students and laymen" may have some difficulty with a number of the proofs, but a reader with a little more mathematical maturity, who is seeking a simple introduction to the subject, could hardly do better.

There are nine chapters. In the list of these that follows we add in parentheses the problems to which the corresponding concepts are applied. 1. What is a Graph?

(Mostly definitions). 2. Connected Graphs (Euler lines, Paths, Mazes, Hamilton lines, Knight's Tour). 3. Trees (Connector problem, Matching vertices and edges). 4. Matchings (Assignment problems, Marriage Problem, Round-Robins). 5. Directed Graphs (One-Way Traffic, Genetic Graphs). 6. Questions concerning Games and Puzzles (Puzzles, Theory of Games, Sportswriter Paradox). 7. Relations (Use of graphs to illustrate different types of relations). 8. Planar Graphs (Kuratowski Criterion, Euler's Formula, Platonic Bodies, Mosaics). 9. Map Coloring (The Five-Color Theorem).

There are many problems. Their solutions and a glossary are given in the appendix. One of the definitions in the latter has an interesting error: "*Dodecahedron.* A polyhedron of twenty faces." Is it typographical, or did he multiply when he should have added?

D. S.

27 [X].—S. VAJDA, *Readings in Mathematical Programming*, second edition, John Wiley & Sons, New York, 1962, viii + 130 p., 22 cm. Price $4.25.

This book is a revised edition of *Readings in Linear Programming*, which was first published in 1958. The title of the present edition reflects the inclusion of applications dealing with discrete linear, dynamic, and quadratic programming.

Two methods are described for solving the Hitchcock transportation problem; namely, the stepping-stone method and a network flow algorithm developed by Ford and Fulkerson. The formulation and solution of several problems in terms of a transportation model are given. In addition, variants of the transportation problem are reduced to the problem of determining a maximal flow through a network, which is solved by the Ford-Fulkerson maximal flow algorithm. The simplex method and the dual simplex method are introduced for solving the general linear programming problem. The simplex calculations are described in terms of a tableau. Many examples are provided to elucidate the details of the computational procedures involved. The concept of dynamic programming is exemplified by two simple problems involving multi-stage planning. Only linear objective functions are considered. The discussion of the application of linear programming to game theory is addressed to those who are familiar with the concept of a zero-sum two-person game.

Two types of discrete linear programming examples are presented: (1) the condition of integrality is imposed on all variables; and (2) the condition of integrality is imposed only on some specified variables. The treatment is based on the methods of R. E. Gomory.

The subject of quadratic programming is restricted to an exposition of two methods, one due to P. Wolfe, the other to E. M. Beale. These methods are applicable to the case in which the constraints are linear, but the objective function is convex quadratic.

Except for the chapter on game theory, elementary algebra is the only prerequisite for an understanding of the material presented in this book. Emphasis throughout the book has been placed on the details of specific algorithms rather than on the fundamental concepts underlying the various methods. Although the author has presented excellent representative examples of the manifold applications

of mathematical programming, this book would have been more meaningful and interesting to the reader if some of the basic theorems had been included.

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28 [X].—R. L. ACKOFF & P. RIVETT, *A Manager's Guide to Operations Research*, John Wiley & Sons, Inc., New York, 1963, x + 107 p., 22 cm. Price $4.25.

This is a remarkable little book, which this reviewer and many practitioners of the art will heartily recommend to management personnel who ask, "What is operations research? Where can I find out about it in a form I can underatand?" The practical experience of the authors in dealing with management and their knowledge of the field are readily apparent throughout the pages of the book.

The two major chapters expand on the nature of operations research and describe the form and content of typical problems that lend themselves to such an approach. The shorter chapters are concerned with the relationship with other management services and the organization and administration of operations research. Differences in practice between the United Kingdom and the United States are identified, but one is more struck by the essential similarity. In addition to the major textual content there is included a list of consultants, schools and universities offering courses in operations research, a list of firms, arranged by industry, that use operations research, and an annotated bibliography. Ackoff's and Rivett's contribution should receive an enthusiastic response; the enthusiasm is merited.

JACK MOSHMAN

CEIR
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29 [X].—ALAN S. MANNE & H. M. MARKOWITZ (editors), *Studies in Process Analysis*, John Wiley & Sons, Inc., New York, 1963, viii + 427 p., 26 cm. Price $14.00.

As used by the editors, "process analysis" identifies studies "which approach the analysis of industrial capability through models reflecting the structure of productive processes." Process analysis should be differentiated in this sense from a capability analysis based on gross national product or the input-output studies which are founded on inter-industry product flows. In an introductory chapter, the editors state that "input-output analysis fail to account for alternate methods of production," seeing this as a major drawback to this type of approach. This may be true in a narrow sense, but comprehensive linear programming models based on input-output analysis have been formulated and used in which resource substitutability has been incorporated. This point is discussed by Dorfman, Samuelson, and Solow in *Linear Programming and Economic Analysis* (New York, McGraw-Hill, 1958).

Process analysis involves a model-building activity, the development of appropriate computational algorithms, and the application of model and algorithm to

provide insights into a specific problem area. The orientation of the papers is toward the construction and application of models which are usually based on a linear-programming or integer-programming analysis or on simulation. Little discussion centers on algorithms or policy application.

Over 80 percent of the text is devoted to detailed studies of the petroleum and chemical industry, food and agriculture, and metals and metalworking. A short two-chapter section of 41 pages discusses the application of process analysis to investment planning for newly developing countries.

The chapters relating to industrial applications are quite detailed; they provide a comprehensive discussion of the technology involved, its representation by an analytic model, and the sources of data and problems associated with such applications.

This book is composed of the proceedings of a conference held at Yale University in April 1961. The contributions hold up well, but the developments of the past several years in economics and in computing are necessarily absent.

The mathematician can read this monograph with profit for the detailed case studies describing the construction and use of mathematical models of complex phenomena. The economist can read this monograph with profit for the detailed case studies of analyzing a complete industry, separating the wheat from the chaff, and to appreciate the power of analytical descriptions of the technological processes.

JACK MOSHMAN

30 [X].—CHRISTIAN GRAM, EDITOR, *Selected Numerical Methods*, Regnecentralen, Copenhage, 1962, ix + 308 p., 24 cm. Price D.kr. 70,—.

This book contains four survey articles prepared at the Danish Institute of Computing Machinery by a study group for numerical analysis. As stated in the preface, "Only a small part of the present report . . . represents . . . research; the bulk . . . is a description and treatment of papers by other authors with the purpose of estimating and comparing different numerical methods." The four articles, their authors, and their lengths are:

(1) Linear Equations, C. Andersen and T. Krarup, 28 pages.

(2) Partial Differentive Equations, C. Gram, P. Naur, E. T. Poulsen, 85 pages.

(3) Conformal Mapping, C. Andersen, S. E. Christiansen, O. Møller, and H. Thornhave, 148 pages.

(4) Polynomial Equations, T. Busk and B. Svejgaard, 34 pages. Each article contains theoretical background material and a selected number of methods. Scattered through the text are ALGOL codes and numerical examples. All in all, this is a useful book to have around.

P. J. D.

31 [X].—NATHANIEL MACON, *Numerical Analysis*, John Wiley & Sons, Inc., New York, 1963, xiii + 161 p., 24 cm. Price $5.50.

This book is written as a text for an introductory one-semester course in numerical analysis. A good introductory course in calculus will suffice for prerequisite to a course using this book as a text. The book is machine oriented. In several instances,

flow charts adequate for programming are supplied with the discussion of particular numerical methods. With each topic, there is, in general, a good discussion of error. The exercises supplied are aptly chosen and adequate to check the student's understanding of the material. Answers for computational exercises are supplied. Briefly, the topics covered by this text are: an introduction to machine computation, with a discussion of problem formulation and flow charting; iterative methods for solving functional equations, with emphasis on Newton's method for finding real and complex roots of an equation; an introduction to matrices and linear equations (both iterative and elimination methods are presented); the characteristic value problem for matrices, with a discussion of the method of iteration for finding characteristic roots and vectors; interpolation, using the Aitken-Neville, and Lagrange methods; the Weierstrass theorem, and Bernstein polynomials as an approximation device; numerical differentiation and integration; a thorough discussion of Simpson's rule and the trapezoidal rule; solution of ordinary differential equations by Euler's and Heun's methods; and an introduction to difference equations.

The book is remarkably free of typographical errors. However, on page 47 the third component of the vector should be -9, not -20.

The author indicates that he is defining *analytic* on page 20; however, the correct definition is given as a footnote on page 30. The definition of *inner product* on page 44 could be made a bit more explicit. Since the Bernstein polynomials are discussed as an approximating device, their shortcomings in this role should be indicated.

All in all, the reviewer believes this to be a good text for use at the sophomore or junior level. It has many things to recommend it, including lucid presentation and good selection of topics. This text is aimed at an understanding of numerical analysis rather than a proficiency in problem solving. In developing a curriculum for the student who wishes to become a numerical analyst, it seems that two alternatives present themselves. They are either the inauguration at the sophomore or junior level of a course using a text similar to the one being reviewed or the inclusion of topics presented in this text in existing courses such as calculus, matrix theory, and differential equations.

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32 [X].—K. A. REDISH, *An Introduction to Computational Methods*, John Wiley & Sons, Inc., New York, New York, 1962, xii + 211 p., 25 cm. Price $5.75.

The intended readers of this book are, in the author's words, the "occasional" computer and students of science and engineering. The tools assumed available are tables and, preferably, a desk calculator. The knowledge assumed is that of a sophomore at college, although more advanced terms are used occasionally. Matrix notation is used only briefly, and familiarity with it is not necessary.

Someone approaching this subject for the first time could teach himself from this book, for it is admirably clear in style and nearly every method is illustrated by several worked out examples designed to cover various cases which can occur. The procedures chosen for discussion are those which are the simplest mathematically, not necessarily the most economical computationally. Emphasis is laid on always

trying to estimate accuracy and on using a helpful layout. Actually, only a small part of the book would be irrelevant to someone with access to a high-speed automatic computer.

Problems of varying difficulty, taken largely from the London University examinations, follow each chapter, and answers are given where appropriate.

Chapter headings, with the methods chosen for explanation, are: Linear Algebraic Equations (elimination with row interchanges, attainable accuracy when the data are approximate, iteration on the residuals, Gauss-Seidel and relaxation methods); Non-linear Algebraic Equations (graphs, regula falsi, Newton and Graeffe methods); Finite Differences (the basic difference operators, propagation of table errors); Interpolation, Differentiation, and Integration (usual basic formulas, preparation of tables, inverse interpolation); Ordinary Differential Equations (graphs; series; Picard, predictor-corrector and Fox-Goodwin methods; applications to linear, non-linear, first- and second-order, initial-value and boundary-value problems); Functions of Two Variables (basic formulas, partial differential operators, relaxation); Miscellanea (brief notes on: approximating functions, difference equations, Gauss integration formulas, Lagrangian interpolation, eigenvalues and vectors, Runge-Kutta methods, and summation of series).

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33 [X, Z].—BRUCE W. ARDEN, *An Introduction to Digital Computing*, Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1963, ix + 389 p., 23 cm. Price $8.75.

As the author states in his preface, this is a text for an undergraduate introductory course in digital computing techniques. Integral calculus is the only prerequisite.

The first third of the book introduces the student to computer programming, the basic structure of digital computers and number systems. In his treatment of computer programming, the author uses the MAD language as an example and carefully guides the student through the various types of statements which go into a computer program. Use of flow charts is illustrated and machine language is discussed briefly. Arithmetic operations, scaling, and rounding are also presented in introductory fashion.

A little more than a third of the book is devoted to numerical methods. The topics include finite differences, interpolation, numerical integration, the solution of linear algebraic equations, least-squares approximation, and the solution of ordinary differential equations. In most instances the methods are presented with accompanying programs and flow charts, but with little or no analysis.

The final sixty pages deal with non-numerical problems such as sorting, compilers, and formal differentiation.

E. K. BLUM

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34 [Z].—PHILIP M. SHERMAN, *Programming and Coding Digital Computers*, John Wiley and Sons, Inc., New York, 1963, xiv + 444 p., 24 cm. Price $11.00.

This is an elementary text on digital computer programming. The author's intent is that "no prior knowledge of digital computers and no mathematical background beyond that which is ordinarily a part of the high school curriculum" is required of the student. By and large, his presentation achieves this objective. The style is crisp, clear, and direct. The topics are treated in simple terms. As in similar texts, machine language programming is explained by introducing a hypothetical "typical" computer. An assembler language is discussed and "coding fundamentals" such as loops, use of index registers, branching, use of subroutines, and input-output operations are explained. Finally, there is a discussion of algebraic languages (Fortran and Algol are treated briefly), non-numerical problems, macro-instructions, and program debugging.

Each chapter contains a fairly complete set of exercises.

E. K. BLUM

35 [Z].—H. L. COLMAN & C. SMALLWOOD, *Computer Language*, McGraw-Hill Book Co., Inc., New York, 1963, xiv + 196 + 14 p., 23 cm. Price $5.95.

This is an elementary autoinstructional text designed to teach FORTRAN programming to students "of almost any background or professional interest," to quote from the foreword. Considerable effort has evidently gone into the autoinstructional aspect of the text. The layout is quite different from the usual text and, according to the preface, it also differs from "traditional autoinstructional texts." Each page resembles a flow chart and consists of brief statements enclosed in boxes connected by arrows. Presumably, in the theory of autoinstruction it is shown that this method of guiding the reader's eye and mind is superior to the traditional page layout. The reviewer is unable to comment on this.

The table of contents is informative. There are eight parts to this booklet as follows: 1. Introduction, 2. Program Structure, 3. Variables and Constants, 4. Input Statements, 5. Arithmetic Expressions, 6. Arithmetic Statements, 7. Control Statements, and 8. Output Statements. The ordering of these eight parts is somewhat surprising. For example, one wonders at the presentation of the notion of subroutine in part 2, almost at the outset. However, the style throughout is clear and concise and, autoinstructional or not, it is a good brief introductory text.

E. K. BLUM

36 [Z].—JAMES A. SAXON, *Programming the IBM 7090: A Self-Instructional Programmed Manual*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963, xiv + 210 p., 24 cm. Price $6.75.

This book calls itself a "Self-Instructional Programmed Manual". It is a workbook that attempts to instruct the beginner in some of the rudiments of coding, using the question-answer technique of "programmed" instruction. The author tells the reader that by the time he has worked through the book "... you should feel confident in being able to pull your weight as a fledgeling programmer."

The book covers only the easier parts of 7090 coding. Input-output is touched on very briefly in lesson 10, which in essence states that input-output programming is outside the scope of the book. Lesson 4 indicates that floating-point arithmetic, except in barest outline, is also outside the scope of the book. Of the 13 floating-point instructions in the 7090 only 4 are mentioned. The FDP instruction (Floating Divide and Proceed) is omitted. The FDH instruction (Floating Divide and Halt), which should *never* be used by a programmer, is the only Floating Divide instruction presented here.

It is the opinion of the reviewer that a procedure-oriented language like Fortran should be used by the beginner in his first contact with the 7090. By the time he is ready to learn the details of 7090 machine language coding he should be much more than a beginner in the field of computer programming, and this text will not satisfy his needs.

Lesson 15, the last lesson in the book, presents the student with an example of a complete 7090 program. The example is poorly programmed. It is the kind of programming one should expect from the "fledgeling" programmer, but not from his instructor.

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37 [Z].—F. J. GRUENBERGER & D. D. MCCRACKEN, *Introduction to Electronic Computers, Problem Solving with the IBM 1620*, John Wiley & Sons, Inc., New York, 1963, vii + 170 p., 28 cm. Price $2.95.

The expressed purpose of this book is to have it serve as an introduction to computer programming by discussing examples of types of problems to which computers may be applied. The specific equipment used by way of illustration is the IBM 1620 (with paper tape input and output), and the book is designed to be used as an adjunct to the manual for the 1620. It is assumed in various parts of the book that the reader, through access to the computer itself, is able to resolve specific details of coding, although some of the essentials thereof are reviewed as the book progresses.

An introduction to organization and flow charting is first presented. Then, dealing in terms of machine language, the book explains the use of loops, subroutines, scaling of numerical data, and floating-point arithmetic. Illustrative problems are drawn from number theory. In addition, some examples of the use of random numbers and the playing of simple mathematical games are introduced. However, the book is addressed to a quite elementary level of reader—a background of high school mathematics is all that is required—and so the illustrations are necessarily of an elementary character. They do serve to convey some feeling for empirical tasks that computers can perform.

There is a brief discussion of assembly programming and debugging. The treatment of automatic programming, i.e., interpreters, compilers, and generators, is very sketchy.

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38 [Z].—ARTHUR GILL, *Introduction to the Theory of Finite-State Machines*, McGraw-Hill Book Company, Inc., 1962, xi + 207 p., 23 cm. Price $9.95.

As pointed out by the author in his preface, this book was written as class notes for an introductory engineering course on the theory of "synchronous, deterministic, and finite-state machines." The material is that contained in articles by Moore, Mealy, Aufenkamp, Hohn, Ginsberg, Zadeh, Simon, Paull, Unger, and the author himself, and is arranged by chapters as follows: 1. The Basic Model; 2. Transition Tables, Diagrams, and Matrices; 3. Equivalence and Machine Minimization; 4. State Identification Experiments; 5. Machine Identification Experiments; 6. Finite Memory Machines; and 7. Input Restricted Machines.

Chapters 5 and 6 include discussions of "information lossless" and "linear binary" machines, respectively. The machines discussed in Chapter 7 are those incompletely specified machines for which an output entry is undefined if and only if the corresponding next state entry is undefined.

The book is as remarkable for its omissions as for the subject matter it includes. The author specifically excludes treatments of Turing machines, Markov chains, and modular sequential machines. In the same spirit, extensions of the finite-state machine model, such as reported by Rabin and Scott, and by Schützenberger, are not dealt with. By defining a machine with input, output, and state sets as arbitrary finite sets, the author avoids discussion of logical nets and the encoding problem. No account is given of the decomposition results reported by Hartmanis and Stearns. The most outstanding omission, however, is that of the theory of regular sets as available in articles by Kleene, Copi, Elgot and Wright, Rabin and Scott, and Chomsky.

In the reviewer's opinion, the book is not successful in collecting the available facts about finite-state machines and presenting them in a simple and concise manner. As a textbook, it may be objected to on two counts.

First, it is unnecessarily lengthy and repetitious. This is a consequence of including numerous "algorithms" which are essentially restatements of definitions and some of which could have been left to the ingenuity of the student, and of restating explicitly special instances of a given proposition.

Second, the book is not written at the (admittedly low) level of mathematical sophistication suitable to the subject. Thus, on one hand, the definitions and proofs are informal and the names 'lemma' and 'theorem' are used loosely. On the other hand, excessive terminology and symbolism are introduced.

A number of errors are present, which the reviewer has communicated to the author.

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