Some Additional Factorizations of $2^n \pm 1$

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Herein are set forth some details of three new factorizations of integers of the form $2^n \pm 1$.

The first of these is the complete factorization of $2^{119} - 1$, which possesses as algebraic factors the Mersenne primes $2^7 - 1 = 127$ and $2^{17} - 1 = 131071$. The quotient is known to be divisible by 239 and 20231. There then remains the factorization of the integer

$$N = 82 \; 57410 \; 95583 \; 43357 \; 90279,$$

which was proved composite by E. Gabard of Poitiers, France.

The method of factorization employed was that described by Kraitchik [1] and used by the writer in a previous factorization [2]. Briefly, the procedure consists of exhibiting the integer \( N \) as the difference of two squares \( a^2 - b^2 \), where the integer \( a \) is suitably restricted by a process of exclusion based on a knowledge of several quadratic residues of \( N \).

In this manner the representation \( a = 1019592x + 90874619060 \) was obtained, and this was found to yield a square value for \( a^2 - N \) when \( x = 6051 \); whence we obtain the factorization

\[
N = 62983048367 \cdot 131105292137,
\]

which completes the factorization of \( 2^{119} - 1 \).

The second factorization is that of \( 2^{129} + 1 \), which has the algebraic factor \( 2^{48} + 1 = 3 \cdot 2932031007403 \), and the quotient is divisible by 3 and 1033. The remaining factor

\[
N = 249 \, 66522 \, 25083 \, 17105 \, 80243
\]

was also proved composite by E. Gabard. In this case the same method of exclusion leads to the representation

\[
a = 133128x + 158008074298,
\]

which corresponds to a square value of \( a^2 - N \) when \( x = 57734583 \). Accordingly, we obtain the following decomposition into prime factors:

\[
N = 1591582393 \cdot 15686603697451,
\]

and the factorization of \( 2^{129} + 1 \) is thus complete.

The last factorization considered here is that of \( 2^{141} + 1 \), which is divisible by \( 2^{47} + 1 = 3 \cdot 283 \cdot 165768537521 \). The quotient, \( 2^{94} - 2^{47} + 1 \), is divisible by \( 3 \cdot 1681003 \), and the resulting integer is

\[
N = 39 \, 27623 \, 49394 \, 29899 \, 21473.
\]

Here the representation \( a = 636192x + 62673972097 \) was found, in which the value \( x = 16720 \) was discovered to result in a square value for \( a^2 - N \). Hence,

\[
N = 35273039401 \cdot 111349165273,
\]

which completes the factorization of \( 2^{141} + 1 \).

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