

## On a Constant in the Theory of Trigonometric Series

By Robert F. Church

The note "A constant in the theory of trigonometric series" in the October 1964 issue of *Mathematics of Computation* provided us with a test for our recently constructed algorithms for the computation of roots of functions, and for numerical quadrature in the presence of singularities. The latter algorithm, utilizing the Gaussian 8-point quadrature formula applied to sub-intervals of variable length, involves a sufficiently small number of ordinates that computational labor and round-off error do not become problems. Use of these algorithms indicated the value  $\alpha_0 = .3084438$ , for the root of the equation  $\int_0^{3\pi/2} u^{-\alpha} \cos u \, du = 0$ , differing from the reported value, .30483, in the third place. To check this result, we made the transformation  $u = x^4$  to weaken the character of the singularity at the origin, and obtained the following table by conventional numerical quadrature, confirming our result:

$\alpha$	$F(\alpha)$
.308441	— .99 ( $10^{-5}$ )
.308442	— .63 ( $10^{-5}$ )
.308443	— .28 ( $10^{-5}$ )
.308444	.08 ( $10^{-5}$ )
.308445	.44 ( $10^{-5}$ )
.308446	.79 ( $10^{-5}$ ).

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In a recent note, Boas and Klema [1] considered

$$(1) \quad F(\alpha) = \int_0^{3\pi/2} u^{-\alpha} \cos u \, du, \quad R(\alpha) < 1,$$

and gave some computations from which they concluded that a zero  $\alpha_0$  of  $F(\alpha)$  lies between 0.30483 and 0.30484. Since their tabulated values of  $F(\alpha)$  in the vicinity of the root are given to 8D and there are eight such entries, it would seem, since  $F(\alpha)$  is analytic for  $R(\alpha) < 1$ , that the zero could be given to more places by differencing and making use of ordinary inverse interpolation techniques. It is found

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