efficients used in the evaluation of g; and the last column is the number of significant places in g unaffected by increasing M.

N		M	
5	4	5	2
10	5	10	2
15	5	15	2
20	6	20	2
25	6	25	2
30	6	30	2
35	6	35	3
40	6	40	3
		45	4
		50	4
		55	4

This table indicates that a better method should be investigated for the evaluation of g; however, in evaluating the kernel, the Fourier series technique seems to be satisfactory, and it is in this part of the problem that the two-fold advantage of the technique is realized. The present program requires less than two minutes on the 7090 for $N = 20$ and $M = 45$.

5. Acknowledgment. This paper covers research initiated by the Hydromechanics Laboratory, David W. Taylor Model Basin, Washington, D. C., under Subproject No. S-R011-0101, Task 0401.

David Taylor Model Basin
Hydromechanics Laboratory
Washington, D. C.

1. L. Collatz, *The Numerical Treatment of Differential Equations*, 3rd ed. of English transl., Die Grundlehren der Mathematischen Wissenschaften, Bd. 60, Springer, Berlin, 1960, pp. 505–506. MR **22** #322.
2. S. G. Mikhlin, *Integral Equations*, Macmillan, New York, 1957, pp. 19–21.
3. W. B. Morgan, "Theory of the annular airfoil and ducted propeller," Fourth Symposium on Naval Hydrodynamics, ACR-92, U. S. Government Printing Office, Washington, D. C., 1962, pp. 151–197.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[A-F, H-M, R, S, X].—MILTON ABRAMOWITZ & IRENE A. STEGUN, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, New York, 1965, xiv + 1046 pp., 27 cm. Price $4.00 (paperbound).

This is a paperback edition of the second, corrected printing of the handbook previously reviewed here in RMT **1**, vol. 19, pp. 147–149. One would not know that this edition is based upon the second printing from the legend on the present page II, since it states there "... unaltered republication of the work first published ... in 1964." The corrections made here comprise most of those listed in this journal in MTE **362** and MTE **365** but do not include those subsequently listed here in MTE **373, 376,** and **379.**

The printing and paper are very good. Whether the saving of $2.50 over the government hard-cover edition is worthwhile is a question whose answer depends upon the individual prospective user.

D. S.

2[A, C].—G. BARRIÈRE, *Tables de logarithmes de logarithmes, logarithmes de cologarithmes, logarithmes à six décimales*, Gauthier-Villars, Paris, 1965, 185 pp., 28 cm. Price $5.25 (paperbound).

The three principal tables in this 6D collection are: Table I: log log N, N = 1.00001 (0.00001) 1.00100 (0.0001) 1.0100 (0.001) 10.000 (0.01) 100 (0.1) 1000 (1) 9999; Table II: log colog N, N = 0.000001 (0.000001) 0.00060 (0.00001) 0.0150 (0.0001) 0.9999; Table III: log N, N = 1 (1) 10000. These tables are supplied with first differences.

Table IV, entitled "Table d'interpolation," is a 6D table of log log N − log $|N - 1|$ for N = 0.950 (0.001) 1.399, with differences. This table is designed for use when interpolation in Tables I and II is not feasible.

This is followed by a table of 10D values of the first 99 multiples of $M(= \log e)$ and of its reciprocal.

In Table VI appear 6D values of 26 mathematical constants involving π and e and their reciprocals, together with 6D logarithms, log log's, log colog's, etc. of these constants. Six physical constants, involving the gravitational constant (at Paris), are also listed to 6D.

Table VII gives a display of the variations of the logarithmic function for both bases e and 10.

Finally, Table VIII is a systematic collection of 26 types of calculation to which the main tables are applicable.

An introduction of eight pages contains definitions of the functions log log and log colog, a discussion of their properties, and examples of the use of the tables involving them.

These tables of logarithms of logarithms appear to be the most extensive pub-

lished to date. Examples of earlier, less extensive tables are those by Chappell [1] and by Boll [2].

J. W. W.

1. E. Chappell, *Five-Figure Mathematical Tables*, Chambers, London, 1915. (See *MTAC*, v. 1, 1943–1945, p. 131, Q 4.)
2. M. Boll, *Tables Numériques Universelles des Laboratoires et Bureaux d'Études*, Dunod, Paris, 1947. (See *MTAC*, v. 2, 1946–1947, pp. 336–338, RMT **428**.)

3[B-E, K, S].—J. C. P. Miller & F. C. Powell, *The Cambridge Elementary Mathematical Tables*, Cambridge University Press, Cambridge, 1965, 47 pp., 25 cm. Price $0.50 (paperbound).

The title page carries the information that these four-figure tables were compiled and arranged for the Cambridge Local Examinations Syndicate.

Herein we find conveniently arranged 4D (or 4S) tables of common logarithms and antilogarithms, natural and logarithmic values of trigonometric functions (at intervals of 0°.1), powers (reciprocals, squares, cubes, square roots, cube roots), factorials, natural logarithms, exponential and hyperbolic functions, trigonometric functions for angles in radians, conversion tables (radians to degrees and conversely), binomial coefficients (exact values to $n = 20$), normal distribution function and related statistical functions. Also included are conversion tables for weights and measures and for electromagnetic quantities, and an extensive list of physical constants.

All the mathematical tables are supplied with first differences, and a separate table of proportional parts to tenths is included.

The user will benefit from a perusal of the introductory notes on the use of these tables, which include a detailed discussion of interpolation therein and other methods of use.

These excellent tables should well serve the purpose for which they are intended, and will be useful to others requiring a compact set of elementary mathematical tables.

J. W. W.

4[D].—C. Attwood, *Six-Figure Trigonometrical Functions of Angles in Degrees and Minutes*, Practical Tables Series No. 1, Pergamon Press, Oxford, 1965, vii + 68 pp., 20 cm. Price 7s 6d (paperback).

This is the fifth edition of a set of trigonometrical tables originally published in 1942 by the Ford Motor Company, Ltd. in Dagenham, England.

The main table consists of natural values of the six standard functions to 6S for every sexagesimal minute, arranged semiquadrantally, with initial and terminal first differences shown at the top and bottom of each column of tabular data. Auxiliary tables of proportional parts for interpolation in tenths of a minute and for subdivisions of 5 seconds are given. A 2-page table gives decimal approximations of the first 100 multiples and submultiples of π and π^{-1} to 6 or 7S, as well as $\pi n^2/4$ for $n = 1(1)100$ to similar precision. A few other, related constants such as $\pi^{\pm 2}$, $\pi^{\pm 3}$, $\pi^{\pm 1/2}$, and $\pi^{1/3}$ are given to 6 or 7D in a footnote to this table.

The customary conversion tables from minutes and seconds to degrees (6D),

degrees, minutes, and seconds to radians (7D), and radians to degrees (5D) are also included.

The book concludes with a section on interpolation, including tables of $x \cot x$ and $x \csc x$ for $x = 0(10')500'$, and values of the coefficient in Bessel's interpolation formula with second differences (for interpolation to hundredths of a minute and to seconds), and finally a selected bibliography of outstanding related tables.

An attractive feature of this book and its successors in this series is the inclusion of facsimiles of pages from pertinent tables of historical interest and importance.

J. W. W.

5[D].—C. ATTWOOD, *Six-Figure Trigonometrical Functions of Angles in Hundredths of a Degree*, Practical Tables Series No. 2, Pergamon Press, Oxford, 1965, viii + 103 pp., 20 cm. Price 12s 6d (paperback).

In his preface to these tables the author discusses the advantages of the subdivision of the quadrant into decimal fractions of a degree and cites the classical example of Henry Briggs' celebrated *Trigonometria Brittanica*, published in 1633.

In the present work the principal table consists of the natural values to 6S of the six standard trigonometric functions at intervals of one-hundredth of a degree in the argument. First differences are shown at the head and foot of each tabular column. A table of proportional parts to expedite linear interpolation directly follows the main table.

The table of multiples of π and related numbers is reproduced from the first book in this series.

Conversion tables are presented to change degrees to radians (7D) and radians to degrees (5D).

The concluding section on interpolation parallels that in the first book, and a similar selected bibliography of outstanding related tables is included.

J. W. W.

6[C, D].—C. ATTWOOD, *Six-Figure Logarithmic Trigonometrical Functions of Angles in Degrees and Minutes*, Practical Tables Series No. 3, Pergamon Press, Oxford, 1965, vi + 75 pp., 20 cm. Price 7s 6d (paperback).

As the author points out, this is a companion to No. 1 in the Practical Tables Series. All entries in the main table of 6D common logarithms of the six trigonometric functions are printed in full, including the characteristics.

The first edition, published by the Ford Motor Company, Ltd. in Dagenham in 1945, included logarithms and antilogarithms; this arrangement was retained until the present (fifth) edition, which now relegates the tables of logarithms of numbers to a separate volume (No. 5).

A useful section of 16 pages is devoted to a recapitulation of trigonometrical formulas, including fundamental identities and the standard formulas for the solution of plane triangles.

The subject of interpolation in these tables is discussed in detail, and a valuable bibliography of related tables is appended.

J. W. W.

7[C, D].—C. ATTWOOD, *Six-Figure Logarithmic Trigonometrical Functions of Angles in Hundredths of a Degree*, Practical Tables Series No. 4, Pergamon Press, Oxford, 1965, vi + 100 pp., 20 cm. Price 7s 6d (paperback).

This is a companion table to No. 2 in this series of mathematical tables. The main tables consist of 6D common logarithms of the six trigonometrical functions at intervals of one-hundredth of a degree, as indicated in the title. Tables of proportional parts and conversion tables have been reproduced from No. 2.

The book concludes with a discussion of interpolation and a selected bibliography, in the format of the preceding books in this series.

J. W. W.

8[C].—C. ATTWOOD, *Six-Figure Logarithms, Cologarithms and Antilogarithms*, Practical Tables Series No. 5, Pergamon Press, Oxford, 1965, vi + 125 pp., 20 cm. Price 12s 6d (paperback).

The author states that he has now published this table separately from the logarithms of the trigonometrical functions in order to provide flexibility in its use. A new feature is the inclusion of a table of cologarithms to 6D.

Specifically we find 6D common logarithms of numbers from 1 to 10 are increments of 0.001; 6D cologarithms for the same range of arguments; and 6D antilogarithms of numbers from 0 to 1 at intervals of 0.0001. All three tables are supplied with mean proportional parts.

A table of $n\pi, \pi/n, \pi n^2/4$ for $n = 1(1)100$ to 6 or 7S and their common logarithms to 6D is included. This is followed by a tabulation of 6D or 10D values of a large number of constants involving π and e, as well as square roots of small integers.

In a supplementary section, entitled Notes on Using the Tables, the author includes a brief discussion of the conversion of common logarithms to natural logarithms, together with a table of the first 99 multiples of ln 10 to 7 or 8S.

A selected bibliography of eight titles concludes this useful and convenient set of tables.

J. W. W.

9[F].—NEAL H. MCCOY, *The Theory of Numbers*, The Macmillan Company, New York, 1965, vi + 150 pp., 25 cm. Price $4.95.

This is a short, minimal, simple textbook designed solely for a one-semester course in number theory. No features of originality are claimed, and none are observed. The author apparently conceives of the coverage here as a greatest common divisor of all existing first-semester textbooks, but while it is indeed a common divisor, it is not clear that it is the greatest. No mention is made, at all, of the Prime Number Theorem or Fermat's Last Theorem. While it is evident that one could not expect proofs here of these propositions, especially of the second, this seems to be unduly reticent whether the book is intended for an introductory course or for a terminal course. Similarly, Gaussian and other complex integers are not mentioned, and one might be tempted to refer to the book as The Real McCoy, were there not some danger of being misunderstood.

There are many exercises, two short tables of primitive roots and indices, and a bibliography of twenty books.

D. S.

10[F].—JOSEPH B. MUSKAT, *On Divisors of Odd Perfect Numbers*, proof of a pertinent theorem printed by a computer on 14 sheets and deposited in the UMT file.

This is the proof that includes and extends Muskat's Table 2 printed elsewhere in this issue of *Mathematics of Computation*. The entire proof, as given here, comprises approximately 650 lines, the first 44 of which are listed in Table 2.

Many questions concerning optimization would occur to interested parties. For example, in line 5 of Table 2 one finds the factors $5827 \cdot 6073$. The program selects 6073, and finds a contradiction in four lines. But suppose it chose 5827 instead? Presumably, a whole tree could be traced out, and the minimal branches then be selected. More subtle problems, involving the ordering of the primes, are also apparent. On the other hand, a quite simple redundancy is noted in the given proof. Thus, that proof examines the cases $\sigma(19^8)$, $\sigma(3^8)$, $\sigma(3^{14})$, $\sigma(3^{20})$, etc. These are not needed; the only non-redundant powers $\sigma(p^b)$ being those where $b + 1$ is a prime. The author does eliminate the cases $b = 2m + 1$, as he indicates in his text, and no doubt he knew of the existing redundancies also, but preferred not to complicate the program for the small savings possible.

D. S.

11[F, X].—A. YA. KHINTCHINE, *Continued Fractions*, Noordhoff, Groningen, 1963, Translated by P. WYNN, 101 pp., 23 cm. Price Dfl. 16.25.

12[F, X].—A. YA. KHINCHIN, *Continued Fractions*, University of Chicago, Chicago, Illinois, 1964. Translation edited by H. EAGLE, xii + 95 pp., 21 cm. Price $5.00.

Khintchine's classic is concerned with regular continued fractions, that is:

$$x = a_0 + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots \tag{1}$$

with the a_i positive integers ($i \geqq 1$), with their use in representing real numbers, and with their "metric theory". These two translations into English appeared almost simultaneously. To compare the two, let us start with a paragraph from the first preface (1935).

Wynn writes:

"The main aim of this book is to close the gap in our literature described above; thus it is of necessity elementary and as compact as possible. By this reason its *style* is determined. Against this, its *content* slightly exceeds that which is absolutely necessary for the various applications. This is true above all with regard to the last chapter which is concerned with the fundamentals of the metric (or probability-theoretic) application of continued fractions. We are concerned here with an important new chapter which is virtually the exclusive creation of Soviet scientists. Many parts of Chapter II also go beyond the mentioned minimum, since I wished to discuss the fundamental rôle which continued fractions play in the investigation of irrational numbers, in as much detail as is possible within the framework of such an elementary introduction. If the theory of continued fractions is to be the subject of a special book, then it would seem to me futile to omit the very topics and interconnections of this theory which at the present time most occupy scientific thinkers."

Eagle writes:

"Since the basic purpose of this monograph is to fill the gap in our textbook literature, it necessarily had to be elementary and, to as great a degree as possible, accessible. Its *style* is in large measure determined by this fact. Its *content*, however, goes somewhat beyond the limits of that minimum absolutely necessary for any application. This remark applies chiefly to the entire last chapter, which contains the fundamentals of the measure (or probability) theory of continued fractions—an important new field developed almost entirely by Soviet mathematicians; it also applies to quite a number of items in the second chapter, where I attempted, to the extent possible in such an elementary framework, to emphasize the basic role of the apparatus of continued fractions in the study of the arithmetic nature of irrational numbers. I felt that if the fundamentals of the theory of continued fractions were going to be published in the form of a separate monograph, it would be a shame to leave unmentioned those highlights of the theory which are the subject of the greatest amount of contemporary study."

Aside from questions of style, and the reviewer is reluctant to indicate his preference here, one notes some differences in meaning. In all three cases: "compact" vs. "accessible", "slightly exceeds" vs. "goes somewhat beyond", "metric . . . application" vs. "measure . . . theory", the Eagle translation is surely more accurate. There are other, more serious translational errors in Wynn also, but the comparison is by no means one-sided.

For instance, consider "systematic fractions"—a term not (generally) used in English. Wynn explains it as follows (p. 25):

. . . "For this reason the calculus of continued fractions may, at least in principle, make the same claims for consideration as those which may be made on behalf of decimal fractions, or quite generally for systematic fractions (i.e. those which use any fixed radix system).

"Now what are the essential advantages and disadvantages of continued fractions as a means of representing real numbers when compared with the far more widely established systematic fractions?"

But Eagle writes (p. 19):

. . . "Therefore, the apparatus of continued fractions can, at least in principle, claim a role in the representation of real numbers similar to that, for example, of decimal or of systematic fractions (that is, fractions constructed according to some system of calculation).

"What are the basic advantages and shortcomings of continued fractions as a means of representing the real numbers in comparison with the much more widely used systematic representation?"

Here Wynn's experience in computation reveals itself, while Eagle's "according to some system" is vague, even meaningless. Thus, neither translation is perfect.

The metric theory, which is the main point of the book, is largely Khintchine's invention. It culminates in his famous theorem that for almost all x in (1) with a_i some integer, the geometric mean of the first n elements, that is, $(a_1a_2 \cdots a_n)^{1/n}$, tends to an absolute constant $K = 2.6 \cdots$, as $n \to \infty$. Neither translator mentions that Khintchine's value of K as given here is incorrectly rounded. This was noted by D. H. Lehmer long ago [1]. Subsequently, more accurate values of K have been given in [2], [3], and [4].

"Almost all" means, of course, except for a set of measure zero. This latter set contains, however, *all* rational numbers, *all* quadratic surds, the number e, and presumably much more. An unsolved problem with a delightfully ironic flavor is whether K itself is in the "almost all".

D. S.

1. D. H. LEHMER, "Note on an absolute constant of Khintchine," *Amer. Math. Monthly*, v. 46, 1939, pp. 148–152.
2. D. SHANKS, MTE **164**, *MTAC*, v. 4, 1950, p. 28.
3. D. SHANKS & J. W. WRENCH, JR., "Khintchine's constant," *Amer. Math. Monthly*, v. 66, 1959, pp. 276–279.
4. J. W. WRENCH, JR., "Further evaluation of Khintchine's constant," *Math. Comp.*, v. 14, 1960, pp. 370–371.

13[G, X].—T. L. SAATY, Editor, *Lectures on Modern Mathematics*, Volume II, John Wiley & Sons, Inc., New York, 1964, ix + 183 pp., 22 cm. Price $5.75.

14[F, X].—T. L. SAATY, Editor, *Lectures on Modern Mathematics*, Volume III, John Wiley & Sons, Inc., New York, 1965, ix + 321 pp., 22 cm. Price $11.75.

These two volumes complete the series containing the 18 lectures given under O. N. R.—George Washington University sponsorship. For detailed, general remarks concerning this series see the review of Volume I [1]. Volume II contains the six chapters:

"Partial Differential Equations with Applications in Geometry," by L. Nirenberg,
"Generators and Relations in Groups—The Burnside Problem," by Marshall Hall, Jr.,
"Some Aspects of the Topology of 3-Manifolds Related to the Poincaré Conjecture," by R. H. Bing,
"Partial Differential Equations: Problems and Uniformization in Cauchy's Problem," by Lars Gårding,
"Quasiconformal Mappings and Their Applications," by L. Ahlfors,
"Differential Topology," by J. Milnor,

and the last volume has

"Topics in Classical Analysis," by Einar Hille,
"Geometry," by H. S. M. Coxeter,
"Mathematical Logic," by Georg Kreisel,
"Some Recent Advances and Current Problems in Number Theory," by Paul Erdös,
"On Stochastic Processes," by Michel Loève,
"Random Integrals of Differential Equations," by J. Kampé de Fériet.

Detailed individual reviews are not needed or appropriate here. The chapters vary in length from the extensive, 101-page contribution of Kreisel which attempts "to make out a case that real, if modest, progress has been made on foundational problems" to the brief, 14-page lecture by Ahlfors. That latter is the only one that does not include a list of references, even though it is particularly missed there since its subject began in "a small paper" of H. Grötzsch which "was buried in a small journal." The lectures are all difficult and highly-specialized. (If there exists even a single mathematician in the entire world who can read all 18 chapters with

ease and profit the first time around, the reviewer would wish that he would step forward, and so declare.) Perhaps the least difficult here for nonspecialists would be the (nonetheless very meaty) papers of Erdös, Coxeter, and Hille, since the concepts there are relatively simple.

Marshall Hall's subject, like those of Erdös and Coxeter, can utilize some combinatorial and other computer calculations. Hall implies that since Novikov's other shoe may never drop, the Burnside Problem (not Conjecture) must still be considered open for all $n > 6$. Bing's paper is enriched with many spooky diagrams. It contains a number of theorems that begin: "A fake cube is real if . . . ", and an outsider must be forgiven for doubting that the best possible terminology has been chosen. Milnor's subject has a similar occupational disease, e.g.—"How to Recognize an Honest Sphere," and makes more direct contact with Bing's paper through work of Smale, Stallings, and Zeeman pertinent to both topics.

Problem for the reader: Compare Milnor's diagram on p. 175, Vol. II with Coxeter's identical diagram on p. 64, Vol. III. Is their identity accidental, or of significance?

D. S.

1. T. L. Saaty, Editor, *Lectures on Modern Mathematics, Volume* I. Reviewed in *Math. Comp.*, v. 18, 1964, RMT **45**, pp. 329–331.

15[G].—Philip J. Davis, *The Mathematics of Matrices*, Blaisdell Publishing Company, New York, 1965, xiii + 348 pp., 24 cm. Price $7.50.

The most suitable adjective to use in describing this book is the word "appropriate."

Designed for high school seniors or college freshmen, its level is appropriate, there being a lot of motivation for each definition and result.

The style is appropriate. It is written in an informal manner, which can be expected to attract inquisitive young minds to learn about seemingly mystical arrays of numbers. There is a minimum of formalism and abstraction, and there are interesting diversions which hold one's attention.

The title is appropriate, for the material in the book is selected from wide areas of matrix mathematics, with the algebra of matrices being only one of the topics (and even here, with matrices as rectangular arrays being inviolate).

Finally, it is appropriate that one of our profession's best mathematics expositors should apply his energies so generously to the problem of providing really good books suitable for school mathematics. He deserves our thanks.

A list of the chapter titles here cannot possibly impart much about the book. It proceeds from notation and arithmetic; through some transformation theory and associated geometry; on to operators, characteristic values, and applications; and ends with "Pippins and Cheese," a tasty conglomeration of diverse problems, topics for further study, and historical notes. There is only one way to appreciate this book: read it.

This reviewer has no critical remarks to make.

Robert J. Wisner

New Mexico State University
University Park, New Mexico

16[H].—CHIH-BING LING, *Values of the Roots of Eight Equations of Algebraic-Transcendental Type*, Virginia Polytechnic Institute, Blacksburg, Virginia, June 1965, ms of 11 typewritten sheets deposited in UMT File.

Professor Ling considers the eight equations $\sinh z \pm z = 0$, $\sin z \pm z = 0$, $\cosh z \pm z = 0$, and $\cos z \pm z = 0$, giving a detailed historical account of their solution as well as the mathematical procedure used in preparing his tables.

The real and imaginary parts of the first 100 roots appearing in the first quadrant of the complex plane are tabulated to 11D, based on computations performed on an IBM 1401 computer. Rules for deducing the corresponding roots in the remaining quadrants are also given. The single real root of the equation $\cos z - z = 0$ appears to higher precision (20D) in the text.

A valuable feature of this manuscript report is the bibliography of 16 titles covering earlier calculations and applications of such tables.

J. W. W.

17[H, X].—B. E. MARGULIS, *Systems of Linear Equations*, translated and adapted from the Russian by Jerome Kristian and Daniel A. Levine, Pergamon Press, New York, 1965, ix + 88 pp., 22 cm. Price $2.75.

This is a quite literate treatment at the high-school level, that includes elimination, determinants, successive approximation, least squares, and graphical solution, in that order. One is pained to see parentheses missing on page 1 from polynomials to be divided, but elsewhere they seem to be present where needed.

A. S. H.

18[I, X].—PHILIP J. DAVIS, *Interpolation and Approximation*, Blaisdell Publishing Company, New York, 1963, xiv + 393 pp., 24 cm. Price $12.50.

This is an excellent textbook on approximation theory, emphasizing its intrinsic relations with other areas of analysis. For example, a student who pursues this text can learn from it the fundamentals of functional analysis "on the job" while studying approximation theory. The interplay between approximation theory, functional analysis, and numerical analysis is displayed in a highly attractive fashion.

The first chapter serves as an introduction, summarizing mathematical material to be used subsequently in the text.

This is followed by a chapter on (finite) interpolation, the treatment being carried out in a very general setting but including many concrete and important examples.

The third chapter, entitled "Remainder Theory," is concerned with formulas for the difference between a function and its Lagrange interpolation polynomials.

Chapter IV ("Convergence Theorems for Interpolatory Processes") centers about the problem of convergence of the Lagrange interpolation polynomials of a given function in the complex domain.

The following chapter, entitled "Some Problems of Infinite Interpolation," is concerned, too, with interpolation in the complex domain.

Chapter VI ("Uniform Approximation") begins with the Weierstrass approximation theorem. In this connection the author develops the theory of the Bernstein

polynomials, following which he discusses the Hermite-Fejér polynomials, a theorem of J. L. Walsh on simultaneous approximation and interpolation by polynomials, and the Stone-Weierstrass theorem.

Chapter VII ("Best Approximation") deals with the existence, uniqueness, and other properties of best approximations. Its central topic is the Tschebyscheff best approximation (in the real and in the complex domains).

The next chapter, entitled "Least Square Approximation," is concerned with this classical subject, developed via the theory of inner product spaces.

Chapter IX is an introduction to the theory of Hilbert space, while Chapter X is concerned with the subject of orthogonal polynomials. Such polynomials are considered again in an appendix, entitled "Short Guide to the Orthogonal Polynomials," appearing at the end of the book.

The eleventh chapter is entitled "The Theory of Closure and Completeness"; it deals with pertinent topics from both classical and functional analysis. Included are classical results of Runge and Walsh on approximation in the complex domain and the Müntz closure theorems.

In the next chapter ("Expansion Theorems for Orthogonal Functions") we find a study of Fourier series, convergence of the Legendre series for analytic functions, complex orthogonal expansions, and reproducing kernel functions.

Chapter XIII ("Degree of Approximation") deals with measures of best approximation for different norms. Various estimates of such measures are given.

The concluding chapter, entitled "Approximation of Linear Functionals," contains such topics as the Gauss-Jacobi theory of approximate integration, weak* convergence and its applications, and equidistributed sequences of points.

A bibliography, listing 141 books and papers, is appended.

One of the attractice features of the book is the inclusion of a large number of illustrative examples and problems.

One is impressed by the success of the author in attractively presenting many chapters of classical and functional analysis via approximation theory. The author imparts to the reader his own enthusiasm and appreciation for the beauty of mathematical analysis as well as for its practical applications.

O. Shisha

Aerospace Research Laboratories
Wright-Patterson Air Force Base, Ohio

19[K].—V. D. Barnett, *Random Negative Exponential Deviates*, Cambridge University Press, New York, 1965, xxii + 89 pp., 23 cm. Price 10s 6d (paperbound).

This tract presents three tables. The first is a table of 10,000 four-place pseudorandom numbers x belonging to the probability density exp $(-x)$. These numbers were generated in the form $x = -\ln y$, where the numbers y were generated by the congruential method for simulating uniformly distributed random variates on the interval $0 < y < 1$. The second table consists of the 10,000 numbers $x' = -\ln(1 - y)$. These numbers are negatively correlated with the corresponding numbers x. In Monte Carlo applications, corresponding pairs x, x' are used in the method of antithetic variates. The last table consists of 1,000 samples of a pseudo-

random variate z belonging to the chi-square distribution with one degree of freedom. These numbers were formed as $z = g^2$, where g is a Gaussian variate formed by the Box-Muller method from a uniform variate. The numbers x and z can be used to simulate a variate chi-square with any positive-integer degree of freedom. The tables are subjected to various tests of randomness, and an example is given of their use in a Monte Carlo application.

J. N. FRANKLIN

California Institute of Technology
Pasadena, California

20[K, P].—HERBERT FREEMAN, *Discrete-Time Systems: An Introduction to the Theory*, John Wiley & Sons, Inc., New York, 1965, xiii + 241 pp., 24 cm. Price $10.00.

This handsomely-made book attempts to fill an important role in the analysis of discrete-time systems, particularly of systems that contain digital computers. The book is well written, in the sense that the prose is lucid, but it suffers from a lack of organization and unity that reduce it from a self-contained text to a useful reference for a number of somewhat disjointed topics.

The major fault of the book is the absence of an attempt to inform the reader of the reasons for the choice and order of topics. As a result, the underlying unity of the subject matter is never brought out. The material is not elegant enough to form a satisfying mathematical treatise, so the choice of topics has to be based on utility in engineering applications. Unfortunately, the use of this material is brought in only on an ad hoc basis, rather than as an integral part of the fabric of the text.

The preface of the book is excellent, eloquently discussing the requirement for a good treatment of the subject. The introductory chapter is quite good in introducing the basic concepts of the field and in relating the models studied to the actual phenomena that they try to describe. Some of the definitions themselves depend on undefined terms, but this is a minor failing, since the meaning is usually clear, and since the following treatment is descriptive rather than deductive. The chapters that present analytical tools (Chapters 2–5) contain much material that will be useful to the working engineer who knows what his problem is and needs a method of solution. For the reasons given above, these chapters do not of themselves form the basis for a satisfying text. Chapters 6 and 7, on continuous-time systems with discrete-time inputs, and on sampled-data control systems, respectively, are interesting surveys of applications of the preceding theory, but they suffer somewhat from the attempt to handle difficult problems (especially stability theory) on an elementary level.

The final chapter, on discrete stochastic processes, is especially unsatisfactory, since the mathematical level sufficient for most of the rest of the book is wholly insufficient even to transmit an appreciation of the nature of the stochastic problems, let alone of their solutions. (In fact, the book suffers throughout from the absence of an early treatment of noise in this type of system.)

In sum, for the reader who has entered this field without the proper mathematical background, but who understands what problems exist, this will be a readable and useful reference. The individual paragraphs are well-written and reasonably

accurate. For the reader from an allied technical or scientific discipline who is interested in the activity of his colleagues working on discrete-time systems, and for the graduate student who is first being introduced to discrete-time systems, the book will be something of a disappointment in its attempts to fulfill the need recognized in the preface.

IVAN SELIN

Office of Assistant Secretary of Defense (Comptroller)
Systems Analysis
Department of Defense
Washington, D. C.

21[K, P].—THORNTON C. FRY, *Probability and Its Engineering Uses*, Second Edition, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1965, xv + 462 pp., 24 cm. Price $12.00.

Thirty-seven years have elapsed since the famous first edition appeared. Such an exceptional delay is fitting, however, since the book has often been called a classic, and one should not tamper (too much) with classics. Nonetheless, considerable changes have now been made. Three chapters have been added: Chapter II, *The Language of Probability*; Chapter X, *Matrix Methods and Markov Processes*; and Chapter XI, *The Foundations of Statistics*; and two chapters have been deleted: the old Chapter IX, *Curve Fitting*; and the old Chapter XI, *Fluctuation Phenomena in Physics*. The book is now slightly shorter than before. No explanation is given for the deletions, and the reviewer wishes that the theory of fluctuation phenomena had been kept; the physical phenomena there are of interest, and the theory may even be applicable to some number-theoretic situations (cf. *MTAC*, v. 13, 1959, p. 279).

The remaining nine chapters (those with their previous names) have been thoroughly rewritten, even to the extent of including minor stylistic changes here and there. There are nine tables in the Appendix (factorials, binomial coefficients, normal error and Poisson functions, etc.). Of these, the table of Student's Test of Significance is new, while three other short tables have been deleted.

A publisher's blurb on the jacket neatly characterizes the book as follows:

> "The point of view of the first edition has been retained in the revision. Thus, it is less pragmatic and more postulational than was fashionable in 1928, but less abstract and more attuned to the realities of the physical world than is usual today."

The new edition will therefore be read eagerly by those who continue to have some interest in the real world.

D. S.

22[K, P].—LEONARD KLEINROCK, *Communication Nets: Stochastic Message Flow and Delay*, McGraw-Hill Book Company, New York, 1964, ix + 209 pp., 24 cm. Price $12.50.

Elementary queueing theory and linear programming form the basis for the author's paradigm of communication nets presented in this much revised doctoral thesis on electrical engineering carried out at MIT. Mathematical prerequisites

for the "easy" reading of the essentially self-contained treatise include a year of calculus, and, say, the first-half of either Feller's *Introduction to Probability Theory and its Applications* (Wiley, New York, 1957) or else Riordan's *Introduction to Combinatorial Analysis* (Wiley, New York, 1958).

The research monograph (it is *not* a textbook) commences with numerous preliminary examples; e.g., the well-known cases of static maximal flow, minimal cost flow, multiterminal network flow, and multicommodity network flow. That discussion leads the author into the principal concern of his investigation, namely, the behavior of connected networks subjected to stochastic flow capable of accommodating queues. His most crucial measure of performance for such nets is the average time for a message to arrive at its destination. The remainder of the text is primarily concerned with optimization of performance for various nets. For example, supposing a fixed-cost constraint, he optimizes network performance relative to a prescribed assignment of channel capacities to the branches.

The author's format is, for the most part, the concise theorem-proof style supplemented by occasional concrete examples. Further, the book is replete with figures and diagrams as heuristic aids for the reader. For those readers unfamiliar with elementary queueing theory there is an appendix which includes the results needed in the text-proper. The author even provides a discussion on the simulation of communication nets on Lincoln Laboratory's TX-2 digital computer as a means of experimentally checking his theoretical results. Finally, he includes a brief section on suggestions for further research.

Investigators and students interested in communication theory and operations research should find this well-indexed and well-documented tract of considerable use.

ALBERT A. MULLIN

Lawrence Radiation Laboratory
University of California
Livermore, California

23[L].—G. BLANCH & DONALD S. CLEMM, *Tables Relating to the Radial Mathieu Functions*, Vol. 2: *Functions of the First Kind*, U. S. Government Printing Office, Washington, D. C., 20402, 1965, xxiii + 481 pp., 27 cm. Price $4.00.

The volume under review represents the second of a two-volume set. Volume 1 was reviewed earlier in this journal (Volume 18, 1964, pp. 159–160). The functions tabulated are solutions of the equation

$$(1) \qquad \frac{d^2f}{dx^2} - (a(q) - 2q \cos 2x)f = 0, \quad \text{where } a(q) \text{ is an eigenvalue}$$

corresponding to which the related equation

$$(2) \qquad \frac{d^2g}{dx^2} + (a(q) + 2q \cos 2x)g = 0$$

has solutions of period π or 2π. The tabulated solutions depend on three parameters; namely, q, x, and the order of the eigenvalue r.

The solutions of (2) fall into four categories; namely, even or odd, and periodicity π or 2π. When the variable x is replaced by ix in these solutions they be-

come solutions of (1). Then even solutions are denoted by $Mc_r^{(1)}(x, q)$, and the odd ones by $Ms_r^{(1)}(x, q)$. For $q = 0$ the eigenvalues of (2) are double eigenvalues. Then $Mc_r^{(1)}(x, o)$ and $Ms_r^{(1)}(x, o)$ become linearly independent solutions of the same differential equation. They are in this case hyperbolic cosines and sines respectively.

Volume 1 was devoted to the functions mentioned above and their derivatives. Volume 2 extends the range of values tabulated for these functions, and also tabulates a second linearly independent solution and its derivatives. As in Volume 1, the functions are not tabulated directly. Thus, we have, for example, $Mc_r^{(2)}(x, q) = e^{-rx}Tc_r(x, q)/rM_r(q)$.

The extraction of the factor e^{-rx} leads to a function $Tc_r(x, q)$. These functions are readily interpolable in both x and q. Necessarily, therefore, this table must be used in conjunction with a table of exponential functions. Tabulated data for these functions are provided for $x = 0(0.02)1$; $q = 0(0.05)1$; $r = 0(1)7$; 7D, and $x = 0(0.01)1$; $q = 0(0.05)1$; $r = 8(1)15$; 7D.

An auxiliary set of tables is provided for a second set of linearly independent solutions and their derivatives denoted by $Dc_r(x, q)$, $Ds_r(x, q)$, $Ec_r(x, q)$, $Es_r(x, q)$.

Tables V–XII provide data by means of which all solutions can be computed for $x = 1(0.02)2$, $\sqrt{q} = 0.5(0.02)1$, to 7D. These are tabulated in terms of a class of functions denoted by $Fc_r^{(j)}(x, q)$, $Fs_r^{(j)}(x, q)$, $Gc_r^{(j)}(x, q)$, and $Gs_r^{(j)}(x, q)$, where $j = 1, 2$. These again are so defined as to lead to smooth data but must be used in conjunction with a table of Bessel functions.

These tables in conjunction with the asymptotic formulas provided in the introduction to Volume 2, provide a thorough numerical knowledge of the solutions of equation (1).

HARRY HOCHSTADT

Polytechnic Institute of Brooklyn
Brooklyn, New York

24 [L].—ROBERT SPIRA, *Check Values, Zeros and FORTRAN Programs for the Riemann Zeta Function and its First Three Derivatives*, University Computing Center, Report No. 1, University of Tennessee, Knoxville, Tennessee. Copy deposited in the UMT file.

This report contains documented FORTRAN programs for calculating $\zeta^{(k)}(s)$, $k = 0(1)3$, and also the following tables:

(a) $\zeta(.5 + it)$, $|Z(t)|$: $t = 500, 1000, 2000$; 18D.

(b) $\zeta^{(k)}(\sigma + it)$: $k = 1(1)3$: $\sigma = -2(1)0(.5)1(1)3$,
$t = 0, 10, 50, 100, 200$ (except for pole);
also $\sigma = 0(.5)1$, $t = 500, 1000$;
Accuracy: Nearly all 10D or 10S.

(c) Zeros of $\zeta(s)$: First 30, 13D.
Zeros of $\zeta'(s)$, $\zeta''(s)$: $-1 \leqq \sigma, 0 < t \leqq 100$, 10D.

(d) $-b_\mu = B_{2\mu+2}/B_{2\mu}$: $\mu = 1(1)50$, 30D.

Cross checking was accomplished by also computing the derivatives by a differences program that is included. The zeros of $\zeta'(s)$ and $\zeta''(s)$ were obtained from previously known approximations by a Newton's method program (also included).

AUTHOR'S SUMMARY

25[L, M].—H. E. SYRET & M. W. WILSON, *Computation of Fresnel Integrals to 28 Figures: Approximations to 8 and 20 Figures*, Computer Science Department, University of Western Ontario, London, Ontario, ms of 65 pp. + 96 pp. (unnumbered) of tables, deposited in UMT file.

The main tables in this unpublished report are those of the Fresnel integrals $S(x)$ and $C(x)$ to 28 S in floating-point form for $x = 0(0.001)2(0.01)10$.

The prefatory textual material is presented in ten chapters devoted, respectively, to definitions of the Fresnel integrals, expansions in Taylor series and in asymptotic series (with 12S and subsequently 28S tables of the first 24 coefficients of the latter), generating functions as introduced by Boersma [1], approximation by finite series of Chebyshev polynomials (with tables of coefficients to 10S and 20S), description of the main tables (including a discussion of the use of Lagrange interpolation), and a list of six references.

On p. 28 we find a brief list of pertinent constants, mostly to 28S; included are π, $\pi/2$, $(2/\pi)^{1/2}$, and $2^{1/2}/\pi$. Rather surprisingly, terminal-digit errors of a unit occur in the first and third of these.

The present tables of Fresnel tables are by far the most precise of their kind, inasmuch as previously published tables such as those by Watson [2], Pearcey [3], and Corrington [4] extend to at most 8D.

The user will observe the occasional omission of leading figures in the third group of six digits in the tabular entries. This is due to the unfortunate suppression of zeros in these places in the course of the computer editing of these data.

J. W. W.

1. J. BOERSMA, "Computation of Fresnel integrals," *Math. Comp.*, v. 14, 1960, p. 380.
2. G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, second edition, Cambridge University Press, Cambridge, 1944, pp. 744–745.
3. T. PEARCEY, *Table of the Fresnel Integral to Six Decimal Places*, Cambridge University Press, Cambridge, 1957. (See *MTAC*, v. 11, 1957, pp. 210–211, RMT **87**.)
4. M. S. CORRINGTON, *Tables of Fresnel Integrals, Modified Fresnel Integrals, the Probability Integral, and Dawson's Integral*, Radio Corporation of America, R.C.A. Victor Division. (See *MTAC*, v. 7, 1953, p. 189, UMT **166**.)

26[L, X].—N. N. LEBEDEV, *Special Functions and their Applications*, translated from the Russian by RICHARD A. SILVERMAN, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1965, xii + 308 pp., 24 cm. Price $12.00.

A subject basic to applied mathematics and the mathematics of computation is that of special functions. The subject, unfortunately, has little academic status in the mathematical curriculum, though some topics are often covered as a by-product of courses in theoretical and mathematical physics. Thus, by far and large, it is necessary to learn the subject by self-study. The present volume is ideal for this purpose, and indeed should prove suitable for an academic text.

The book presupposes that the reader is familiar with the elements of real and complex variable theory, although the author attempts to keep the required background material to a minimum. Of course, a better understanding of the subject is also facilitated by some knowledge of differential equations and asymptotics. This material is not introduced in any systematic fashion, but is presented to the reader as the need arises in connection with certain special functions. The arrangement of the material in the various chapters is dictated by the desire to make the chapters

independent of one another as much as possible, so that one can study the simpler functions without becoming involved in more general functions. In illustration, the subject of hypergeometric functions is deferred to Chapter 9, though the transcendents of Chapters 2–8 are for the most part special cases of the generalized hypergeometric function. To reinforce the theoretical development, considerable attention is devoted to applications.

The gamma function is treated in Chapter 1. Chapters 2 and 3 take up the important special cases of the incomplete gamma function. Orthogonal polynomials and expansions in series of these functions are studied in Chapter 4. Chapters 5 and 6 are devoted to Bessel functions and their applications, while Chapters 7 and 8 take up spherical harmonics and applications. An introduction to the Gaussian hypergeometric function is given at the start of Chapter 7. As already implied, a more systematic development is presented in Chapter 9, where the confluent hypergeometric function is also taken up in some detail. Here the connection between the functions of the previous chapters and hypergeometric functions is noted, and a short introduction to generalized hypergeometric functions is given.

The exposition is clear and rigorous, and careful attention is paid to conditions of validity. This is an excellent volume. Each chapter contains a list of problems, which should facilitate use of the book as a text or for self study,

Y. L. L.

27[M, X].—E. T. Copson, *Asymptotic Expansions*, Cambridge University Press, New York, 1965, 120 pp., 23 cm. Price $6.00.

Certain important functions may often be represented by asymptotic series which are usually divergent. Nevertheless, the functions may be calculated to some level of accuracy by taking the sum of a suitable number of terms. In some situations, the sequence obtained by a certain weighting of the sequence of partial sums of an asymptotic series converges. Solutions of ordinary differential equations can often be expressed in the form of a definite integral or a contour integral. Thus, the subject of asymptotics is very important to both pure and applied mathematicians.

This volume gives an excellent treatment of asymptotic expansions of transcendents defined by integrals. After an introductory account of the properties of asymptotic expansions (Chapters 1 and 2), the standard methods of deriving asymptotic expansions are explained in detail and illustrated with special functions. These techniques include integration by parts (Chapter 3), the method of stationary phase (Chapter 4), Laplace's approximation (Chapter 5), Laplace's integral and Watson's lemma (Chapter 6), the method of steepest descent (Chapter 7) and the saddle-point method (Chapter 8). Chapter 9 treats Airy's integral by various methods. For the most part, the expansions discussed are not uniform. Uniform asymptotic expansions is the subject of Chapter 10.

Professor Copson's volume presupposes only a knowledge of the more elementary notions of real and complex variable theory. The subject matter is within the capabilities of undergraduate students.

The volume is very readable and suitable for self-study or as an academic textbook. In this connection, the utility of the text would have been considerably enhanced by the inclusion of exercises.

Y. L. L

28[M, X].—PHILIP HARTMAN, *Ordinary differential Equations*, John Wiley & Sons, Inc., New York, 1964, xiv + 612 pp., 24 cm. Price $20.00.

This book treats both the classical theory of differential equations (linear theory, Sturm-Liouville boundary value problems) and the modern nonlinear theory (Poincaré-Bendixson theory, Liapounov's methods, invariant manifolds, stability in the large, applications of fixed-point methods). In addition, there is material found only in the periodical literature (A. J. Schwartz's theory of flows on 2-manifolds, H. Weyl's theory of the generalized Blasius equation of boundary-layer theory).

The writing is clear and meets the high standards one has come to expect from the author. The notes on the literature are especially valuable. To round this all off, this book has one of the best collections of exercises (together with hints) ever seen by this reviewer.

HARRY POLLARD

Purdue University
Lafayette, Indiana

29[M, X].—PETER HENRICI, *Discrete Variable Methods in Ordinary Differential Equations*, John Wiley & Sons, Inc., New York, 1962, xi + 407 pp., 24 cm. Price $11.50.

How does one solve an ordinary differential equation numerically, and if a method is chosen how accurate will the answer be? These are the questions which this book aims to answer for the method of finite differences. In short, it presents an elaborate analysis of methods that are used on today's computers.

The book is divided into three parts: one-step methods, multi-step methods, and two-point boundary value problems. The last part makes up only about one-seventh of the book, while the first occupies about one-half. The second part gives us, for the first time in a text, a presentation of some of the beautiful results of Dahlquist on multi-step methods.

The mode of presentation in each part is to present a method for solving a differential equation problem, followed by an existence and convergence proof. The author then proceeds to give an elaborate discussion of error propagation and rate of convergence. Thus, in the first part, we find a detailed account of Euler's method (for one equation of first order), which includes not only a constructive existence theorem but also a complete analysis of error propagation from the usual and from a probabilistic point of view. This is expanded to general one-step methods with applications to the methods of Runge, Kutta, Heun, and others. This analysis is subsequently repeated for systems of first- and higher-order equations. For this first half of the book only a knowledge of calculus and a modest knowledge of matrix calculus and probability is called for.

The multi-step methods are introduced by examining the methods of Adams, Bashforth, Moulton, Nyström, and Milne. This is followed by the more general case with a presentation of some of the theorems of Dahlquist. Here we are given necessary and sufficient conditions for convergence of such schemes. Included is a discussion of the properties of stable operators and the startling results of Dahlquist on the maximal order achievable with such operators. Only a minimal knowledge of complex variable theory is required.

The last part discusses Newton's method in n-dimensional space, and this is then applied to a second-order two-point boundary-value problem. In all cases we find very detailed discussions of the discretization errors and other numerical errors.

Each chapter has a generous number of exercises of varying degrees of complexity plus a useful bibliographical summary. Most of the methods are illustrated by examples and flow charts.

Apart from a few minor misprints, which do not mar the excellence of the presentation, we have only the following notes of criticism: The uninitiated reader should be prepared for a large variety of "errors" to be encountered. Apart from the usual discretization and round-off errors, of both the "local" and "genuine" variety, we find induced, adduced, inherent, starting, accumulated (both primary and secondary), and magnified errors. There is a tendency to couple and uncouple words such as stepnumber, which appears uncoupled in the index. The row sum given on p. 371 is evaluated on p. 375. It is not true, in general, that if algorithm (7–21) breaks down, then A is singular. Is there an extra hypothesis to be made in (7–74) that L_2 exists?

The text is otherwise carefully written and is a welcome addition to the growing body of literature on the analysis of finite-difference methods. This work will, I am sure, enlighten those interested in both discrete problems as well as non-discrete (or is it in-discrete) problems.

SAMUEL SCHECHTER

AEC Computing Center
New York University
New York, New York

30[M, X].—ALEKSANDR SEMENOVICH KRONROD, *Nodes and Weights of Quadrature Formulas*, Consultants Bureau, New York, 1965, vii + 143 pp., 28 cm. Price $12.50.

This book is a translation of a Russian book published in 1964. It contains tables of two types of quadrature formulas: Gauss-Legendre and so-called "improved" quadrature formulas.

The Gauss-Legendre formulas are the well-known approximations of the form

$$\int_0^1 f(x)\,dx \simeq \sum_{i=1}^{N} A_i f(\nu_i) \tag{1}$$

which are exact for all polynomials of degree $\leqq 2N - 1$. The A_i and ν_i in (1) are tabulated for $N = 1(1)40$.

Also tabulated are formulas of the form

$$\int_0^1 f(x)\,dx \simeq \sum_{i=1}^{N} A_i^* f(\nu_i) + \sum_{j=1}^{N+1} B_j f(\mu_j) \tag{2}$$

which are called improved formulas. In (2) the ν_i are the points in (1); the A_i^*, B_j and μ_j are chosen so that (2) is exact for all polynomials of the highest possible degree k; for N even, $k = 3N + 1$, and for N odd, $k = 3N + 2$. The constants in (2) are also given for $N = 1(1)40$. From the tables it is seen that the μ_j separate the ν_i, but no proof is given.

The constants in (1) and (2) are tabulated in two ways: to 16D and also in octal floating point to 16 significant octal places. Also given are auxiliary tables pertaining to the errors in (1) and (2). The tables comprise 133 pages.

To the reviewer's knowledge, this is the first place in which formulas (2) have been studied. They are proposed for the following reason: Suppose one has approximated an integral using (1) for a fixed N. Then by computing $N + 1$ additional values of the integrand one obtains a formula (2) of degree $3N + 1$ (or $3N + 2$) which serves as a check on (1). If one were to use an $(N + 1)$-point formula (1) as a check, this would only be a formula of degree $2N + 1$.

The reviewer, however, is not convinced of the value of checking by this method. What is gained by checking a formula of degree $2N - 1$ by one of degree $3N + 1$ instead of by one of degree $2n + 1$?

The introduction to the tables reproduces at least one error of the original Russian and has several added typographical errors. The displayed equations in the text and the tables are reproduced photographically from the original. The price seems about twice what would be necessary in a book of this nature.

A. H. STROUD

Computation Center
University of Kansas
Lawrence, Kansas

31[M, X].—FLOYD E. NIXON, *Handbook of Laplace Transformation: Fundamentals, Applications, Tables, and Examples*, Second Edition, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1965, xii + 260 pp., 24 cm. Price $7.50.

This handbook is intended as a guide for those already familiar with the subject or as a text, albeit a short one, for the uninitiated. On both counts, it leaves much to be desired, and there are already available many references which are better by far. In the applications, knowledge of methods for finding the roots of polynomials are essential. This subject is taken up in Chapter 2. For the solution of equations or order higher than four, the author's only suggestion is an iterative method due to S. Lin. The discussion is woefully inadequate, as there is no discussion of convergence. In fact, the procedure does not always converge; and if it converges, the convergence is usually linear. The Newton-Raphson, Bairstow and other useful processes are ignored.

In general, the material is directed to the solution of ordinary differential equations with constant coefficients, with applications mostly to mechanical and electromechanical systems. For readers of this journal, the only useful feature of the book is a table of Laplace transform pairs [$F(s)$ is the Laplace transform of $f(t)$], where $F(s) = p(s)/q(s)$, $p(s)$ is at most a cubic in s, and $q(s)$ is most often a quartic in s, though there are some cases where $q(s)$ is a quintic or a sextic. The table comprises about 75 pages. In each case, $q(s)$ is represented in factored form as a product of linear and/or quadratic factors. A certain coding is used to facilitate location of $f(t)$ corresponding to a given $F(s)$.

Y. L. L.

32[P, X].—R. E. D. BISHOP, G. M. L. GLADWELL & S. MICHAELSON, *The Matrix Analysis of Vibration*, Cambridge University Press, New York, 1965, x + 404 pp., 26 cm. Price $19.50.

It is stated in the preface that this book "can be regarded as a sequel to an earlier volume" [R. E. D. Bishop & D. C. Johnson, *The Mechanics of Vibration*, Cambridge University Press, 1960], but that it can be read independently. Four of the chapters deal mainly with the mathematical formulation, in matrix form, of vibration. These are Chapter 2, The Vibration of Conservative Systems having a Finite Number of Degrees of Freedom; Chapter 4, Further Development of the Theory of Conservative Systems; Chapter 5, Damped Forced Vibration; and Chapter 6, Continuous Systems. The remaining five chapters can be read independently of these, and two provide a very elementary introduction to the theory of matrices, while the remaining three provide equally elementary descriptions of computational techniques.

For solving linear systems and inverting matrices, only methods of triangular factorization are discussed (Gaussian elimination with pivoting). Chapter 8 deals mainly with the power method for finding the root, with Aitken's acceleration, deflation, and some techniques for finding error bounds based mainly on the use of the Rayleigh quotient. The use of a Rayleigh-quotient iteration is implied, and could be so easily developed, that one wonders why it was not. The final chapter bears the slightly misleading (but often used) title "Direct Methods for Characteristic Values." The direct methods, of course, produce only a reduced matrix or the characteristic equation, to which it is still necessary to apply some iterative method. For symmetric matrices they choose the method that goes by the name of the reviewer; for nonsymmetric matrices they describe the Lanczos method. For the triple-diagonal matrices, they discuss Newton's method and Muller's method.

There are many exercises, and about 30 pages of detailed solutions at the end. In addition, a number of illustrative examples are worked out in the text. The complete novice to matrix theory, even if also a novice to vibration theory, should have little difficulty in reading the matrix chapters on his own and getting a limited but good introduction to the theory and to the computational techniques.

A. S. H.

33[P, X].—C. T. LEONDES, Editor, *Advances in Control Systems, Theory and Applications*, Academic Press, Inc., New York, 1964, x + 365 p., 24 cm. Price $13.00.

This is the first of a series the purpose of which is "to disseminate current information from leading researchers in the ever broadening field of automatic control." It is to consist of a collection of "critical and definitive reviews" at a level between a journal and a monograph covering both theory and applications.

Volume I contains six contributions:

1. "On optimal and suboptimal policies in control systems," by Masano Aoki (pp. 1–53). The control system is disturbed by random noise. If the distribution function of the noise is known, it is called a "stochastic" control problem. If less information is given, it is called an "adaptive" control problem. A review is given of the linear theory of such systems.

2. "The Pontryagin maximum principle and some of its applications," by James A. Meditch (pp. 55–74). The maximum principle and its application to the design of systems is described and illustrated.

3. "Control of distributed parameter systems," by P. K. C. Wang (pp. 75–172). The extension of the theory for lumped parameter systems (ordinary differential equations) to distributed parameter systems (partial differential equations or integral equations) and the difficulties are discussed.

4. "Optimal control for systems described by difference equations," by Hubert Halkin (pp. 173–196). This contribution provides in the simpler context of difference equations an introduction to the geometrical and topological method in the theory of optimal control and includes a proof of the maximum principle for difference equations.

5. "An optimal control problem with state vector measurement errors," by Peter R. Schultz (pp. 197–243). The problem discussed is a linear stochastic one with a quadratic performance criterion, and its study is based on the method of dynamic programming.

6. "On line computer control techniques and their application to re-entry aerospace vehicle control," by Francis H. Kishi (pp. 245–557). Methods available "to perform adaption in a control process" are based on observation of input-output data, the estimation of parameter values and state-variables, and thence the computation of optimal control. Methods available for the linear problem with quadratic performance criteria are summarized and then extended to the same problem with control constraints. Results of computer simulation are given. An application to a phase of the re-entry problem is presented in outline.

This first volume indicates that the series will prove to be of considerable value to all concerned with the theory and design of control systems and with the education of system engineers. The chapters are written independently and this results in much repetition, particularly in the statement and review of the control problem. This has both advantages and disadvantages, but it is to be hoped that the contributors to Volume II will, with the appearance of Volume I, avoid needless repetition and the introduction of new terminology, and that they will relate their contributions to those previously appearing in the series.

J. P. LaSalle

Brown University
Providence, Rhode Island

34[P, Z].—M. Nadler, *Topics in Engineering Logic*, Pergamon Press, New York, 1962, ix + 231 pp., 21 cm. Price $9.50.

Too often simplified notations and methods of procedure are not sufficiently emphasized in the literature. Indeed, as Nadler states ". . . the history of mathematics shows that progress is not indifferent to notation." This remark is particularly apropos when applying mathematical results to modern technological problems. There are many instances in the history of modern technology in which mathematical results that could be applied with great benefit are not so applied, simply because of the lack of proper notations and procedures.

The need for such notation arises from three reasons. First, the engineer or

technologist who is to apply the mathematical results will, in general, not be familiar with the mathematical theory or its mathematical background. In order to bring the mathematical results to his immediate attention, without his extensively reviewing mathematical background and theory, the engineer requires a specialized notation designed especially to display the required results. Second, the engineer's primary attention will be on his technological problem rather than on the mathematical methods. Therefore, a systematic mathematical notation that can enable the mathematical results to be applied by rote, with as little analytical thought as possible, will more likely be utilized by the engineer, unless analytical ingenuity must be concentrated on the engineering pattern at hand. Third and finally, applying a mathematical theory to a specific technological situation is frequently far from trivial. This transition, from theoretical mathematical results to specific applications, can be substantially aided by means of an application-oriented notation, by means of which the relationships between the mathematical entities and their properties can be more readily identified with the hardware components and "real world" phenomena being investigated.

Nadler's book can therefore be praised as a step in the right direction, since its main purpose is to present one such application-oriented notation. The notation involves "logical matrices," and the application is the logical design of digital electronic circuits. The logical matrix is attributed to Marquand, Veitch, and Svoboda, and consists in the more extensive use of the "Veitch Chart" than is normally considered in most texts. The main advantage of the method rests upon the convenience that results from writing the truth values of a Boolean function of n variables as a two-dimensional array, where the columns correspond to all combinations of truth values of r of the variables, and the rows correspond to all combinations of the remaining $n - r$ variables. In particular, the combinations of truth values are so chosen that they form the binary numbers from 0 to 2^r from left to right, and from 0 to 2^{n-r} from top to bottom. Various observations about patterns made by such arrays are applied to the problems of minimizing Boolean functions, of forming self-correcting codes, of synthesizing circuit designs, etc. Other subjects associated with logical design of digital circuits are considered, including redundancy.

Nadler's presentation is best read by one already familiar with the problems of logical circuit design, and presents one approach to computational methods for various problems. Manual computational feasibility is stressed. The book could be a useful addition to the library of the engineer who is interested in various notations devised to aid such logical design.

ROBERT S. LEDLEY

National Biomedical Research Foundation
Silver Spring, Maryland

35[S, X].—J. S. R. CHISHOLM & ROSA M. MORRIS, *Mathematical Methods in Physics*, W. B. Saunders Company, Philadelphia, Pennsylvania, 1965, xviii + 719 pp., 23 cm. Price $10.00.

In spite of its title, and the correct number of authors, one should not expect to find a competitor here for Courant-Hilbert, Morse-Feshbach, Jeffreys-Jeffreys,

Margenau-Murphy, Frank-von Mises, and the like. The present textbook is quite elementary. Written for students who have studied algebra, trigonometry, and analytic geometry, it includes calculus, some ordinary differential equations, vector algebra and analysis, complex variables, matrices, Fourier series, special functions, and probability. There is little more than mention of partial differential equations and tensor analysis, and no treatment whatsoever of integral equations, calculus of variations, group theory, graph theory, numerical analysis, or other modern tools. In a word, the title is misleading; an adequate treatment of that field would assume knowledge of calculus, and some allied material, and not attempt to teach it.

Mathematically, the book is not always adequate, and sometimes is incorrect. On page 609 it purports to prove a proposition known to be false, namely, that the Fourier series of a continuous $f(x)$ converges to $f(x)$. This is accomplished by generous interchange of limiting processes and sets the theory back 130 years.

D. S.

36[S, X].—JON MATHEWS & R. L. WALKER, *Mathematical Methods of Physics*, W. A. Benjamin, Inc., New York, 1964, x + 475 pp., 24 cm. Price $12.50.

As the authors point out in the preface, this book evolved from the notes of a course which they have taught at the California Institute of Technology for the last fourteen years. That course was intended primarily for first-year physics graduate students. The book thus assumes, as far as physics is concerned, that the reader has been exposed to the standard undergraduate physics curriculum: mechanics, electricity and magnetism, introductory quantum mechanics, etc. To quote further from the preface: "It is assumed that the student has become acquainted with the following mathematical subjects:

1. Simultaneous linear equations and determinants
2. Vector analysis, including differential operations in curvilinear coordinates
3. Elementary differential equations
4. Complex variables, through Cauchy's theorem."

The book has an Appendix for a review of some topics in the theory of a complex variable. But even after studying this Appendix, the reader who studied complex variable theory only through Cauchy's theorem will find that Section 3 of Chapter 3 (Contour Integration) and Chapter 5 (Further Application of Complex Variables) call for more intensive preparation.

The stated prerequisites make it clear that the book is not another "Mathematics for Engineers and Physicists." It assumes that the reader either completed a course of such or similar title, or, even better, that he has taken individually the several courses which are often telescoped into one course of Mathematics for Engineers and Physicists.

In the presentation of the various subjects elementary topics are, therefore, only briefly summarized. For instance, the first chapter (Ordinary Differential Equations) gives only a cursory description of solutions in closed form, with the understanding that the reader is familiar with the subject and needs only to be reminded of it briefly. The rest of the first chapter is then devoted to a section on Power-Series Solutions, introducing the concept of regular singular point, a section on Miscellaneous Approximate Methods, and one on the WKB Method.

There are altogether sixteen chapters and one Appendix, each followed by a section of problems, usually of a widely varying level of difficulty. Each section is followed by references to the quite extensive bibliography.

Since the book is intended to be as the preface says, "a book about mathematics, for physicists," the level of rigor is governed by this consideration. At frequent occasions the reader is referred to the bibliography for a more rigorous treatment of the subject. There are occasions when the deemphasis of rigor is carried too far, for example, when Problem 3-2 calls for evaluation of the integral $\int_0^\infty \sin bx\, dx$, followed by the hint: "apply a convergence factor; do integral; remove the convergence factor."

Numerous cross references from one chapter to another appear, and the relatedness of subjects treated in different chapters is frequently brought out. An example for such a frequent cross reference is the subject of eigenvalues, which is part of Chapter 6 (Vectors and Matrices), Chapter 9 (Eigenfunctions, Eigenvalues, and Green's Functions) and Chapter 11 (Integral Equations). The authors point out in the preface, "there is deliberate nonuniformity in the depth of presentation. Some subjects are skimmed, while very detailed applications are worked out in other areas." The reader may well be surprised that in Chapter 16 (Introduction to Groups and Group Representation) the presentation is quite elaborate on the subject of group representations, while Chapter 14 (Probability and Statistics) does not mention at all the modern approach to probability.

In addition to the topics mentioned, the book deals with Infinite Series, Evaluation of Integrals, Integral Transforms (Chapters 2, 3, 4), Special Functions, Partial Differential Equations (Chapters 7, 8), Perturbation Theory (Chapter 10), Calculus of Variations, Numerical Methods (Chapters 12, 13), and Tensor Analysis and Differential Geometry (Chapter 15).

FEODOR THEILHEIMER

Applied Mathematics Laboratory
David Taylor Model Basin
Washington, D. C.

37[V, X, Z].—BERNI ALDER, SIDNEY FERNBACH & MANUEL ROTENBERG, Editors, *Methods in Computational Physics*, Volume 3: *Fundamental Methods in Hydrodynamics*, Academic Press, New York, 1964, xii + 386 pp., 24 cm. Price $13.50.

A large number of very complex hydrodynamic computer codes have been in existence for several years, particularly at the various A.E.C. Laboratories. These codes were designed to solve unsteady-flow problems in one and two space dimensions and included provisions for handling multiple shocks. Some unclassified reports have been written describing the numerical techniques used, but these reports usually had limited circulation. Relatively little has appeared in journal or book form which describes in detail how these numerical procedures are carried out.

This volume contains ten papers describing either general difference methods for unsteady hydrodynamics or giving details of particular codes. A list of the contributions is as follows:

"Two-Dimensional Lagrangian Hydrodynamic Difference Equations," by William D. Schulz.

"Mixed Eulerian-Lagrangian Method," by R. M. Frank and R. B. Lazarus.

"The Strip Code and the Jetting of Gas Between Plates," by John G. Trulio.

"Cel: A Time-Dependent, Two-Space-Dimensional, Coupled Eulerian-Lagrange Code," by W. F. Noh.

"The Tensor Code," by G. Maenchen and S. Sack.

"Calculation of Elastic-Plastic Flow," by Mark L. Wilkins.

"Solution by Characteristics of the Equations of One-Dimensional Unsteady Flow," by N. E. Hoskin.

"The Solution of Two-Dimensional Hydrodynamic Equations by the Method of Characteristics," by D. J. Richardson.

"The Particle-In-Cell Computing Method for Fluid Dynamics," by Francis H. Harlow.

"The Time Dependent Flow of an Incompressible Viscous Fluid," by Jacob Fromm.

The editors have gathered together a variety of different methods and are to be congratulated for making them readily available in this volume. Some of the papers go into more detail than others but the general impression of all of them is a good one. Some examples are given, and the editors state that the next volume of the series will be devoted to hydrodynamics from an applied point of view.

RICHARD C. ROBERTS

U. S. Naval Ordnance Laboratory
White Oak, Silver Spring, Maryland

38[X].—N. K. BARY, *A Treatise on Trigonometric Series*, Pergamon Press, New York, 1965, Vol. I, xxiii + 553 pp., price $12.00; Vol. II, xix + 508 pp., price $15.00.

This is an interesting and valuable treatise. It contains a lot of material not otherwise available in book form. Moreover, following an old and well established Russian pedagogical tradition, the author pays much attention to the matter of presentation. The proofs are laid out carefully and in great detail, both mathematically and graphically, so that the difficulties of reading and following the arguments are at a minimum. Another important aspect of value to the beginner: the basic facts from various parts of the theory are assembled together in Chapter I of the book, which, preceded by an Introduction containing auxiliary material, may be treated as a textbook within the textbook, giving fundamentals without overwhelming the reader with excessive details. The book contains a considerable number of examples and exercises, often with hints of solutions.

The theory of trigonometric series has a long history. It has had a strong impact on other branches of mathematical analysis and has, in turn, been affected by developments in other fields. The modern theory emerged roughly some sixty years ago, primarily in connection with the appearance of Lebesgue's integral, but now we see a number of trends in the theory. These trends are not pure and are constantly influencing one another, but still they are easily discernible.

The oldest of them goes back to Lebesgue himself and treats the subject primarily as a branch of Real Variable. Problems are stated in terms of properties of real functions, and progress is achieved through a refined analysis of sets and functions. Of considerable importance here was the work of the Russian school. It

could be traced back to Egorov, but *spiritus movens* here was actually Lusin. His M.A. thesis ("The integral and trigonometric series" in Russian, Moscow, 1915) as well as his personal influence made a very strong impact on Russian Mathematics and left trends which remain in existence even now, fifty years later. Lusin had very outstanding students, among them Khintchin, Menshov, Kolmogorov, and the author of this book—Nina Bary (she dedicates the book to "my teacher," Lusin). Of comparable significance for trigonometric series in the context of Real Variables was the work of Denjoy in France, though he was concerned with a somewhat different problematics and, unfortunately, did not have pupils to give further impetus to his ideas.

The second direction in trigonometric series recognizes the affinity of the subject with analytic functions and does not hesitate to borrow ideas and methods from the latter. If one wanted to label this direction, the names that come most naturally to one's mind would be those of Hardy and Littlewood, whose systematic work over the period of more than 30 years was probably the strongest individual influence on the field. If one wanted to mention others, the long list should begin with the names of Frederick and Marcel Riesz and those of Lusin and Privalov. The highly original work of the two Russians was an application of delicate methods of Real Variable to the study of boundary behavior of analytic functions and thence the behavior of trigonometric series; it remains a prototype of quite a lot of research done currently in similar but more general contexts.

In recent years a new trend is appearing in trigonometric series and is rapidly gaining strength—in the direction of Functional Analysis. The initial attempts to apply Functional Analysis to trigonometric series may be associated with the appearance of Banach's "Opérations Linéaires" (or even with some earlier work of F. Riesz), but the first major successes here are due to Beurling. It seems to be beyond doubt that this direction offers great possibilities, but only if amalgamated with methods of Real and Complex Variables. In spite of beliefs of many enthusiasts, it seems unlikely that Functional Analysis can tackle single-handedly the really difficult and significant problems of trigonometric series.

In addition to the three trends just described, one might mention another possibility, which lies almost totally in the future: tying trigonometric series with the theory of numbers. The very form of a trigonometric series—a superposition of harmonic oscillations in a fixed order—would indicate that arithmetic properties of real numbers could become a decisive tool, but the difficulty is that at present we cannot even ask intelligent questions here. The few results we know are very interesting, some of them even exciting, but they are too disconnected to offer any general hints.

But let us return to the book. Though the author tries to give a very complete presentation of the theory, it is clear that the methods of Real Variable are closest to her heart and, in view of what has been said above, this is not surprising. These methods are presented with mastery and very exhaustively. A specialist will be particularly appreciative of the chapters giving the results of Menshov concerning "adjustments" of functions in sets of small measure and representation by trigonometric series of general measurable functions. The original papers of Menshov were written in Russian and are not as well known as they deserve to be.

In what follows we give a brief description of the material covered by the book, warning the reader that this description cannot give a completely adequate idea of the contents.

The Introduction has the following sections: I. Analytical theorems (summation by parts, Second Mean-Value Theorem, Convex functions and sequences); II. Numerical series (in particular the methods (C, 1) and Abel's); III. Inequalities (Hölder, Minkowski, etc.); IV. Theory of sets and functions (relevant results from Lebesgue and Riemann-Stieltjes integral); V. Functional Analysis (linear functionals, spaces L^p); VI. Approximation of functions by trigonometric polynomials (best approximation, modulus of continuity, smoothness, Bernstein's inequality).

Chapter I is quite long (pp. 43–204) and gives a self-contained presentation of elements of the theory. It is roughly comparable with the Hardy-Rogosinski Cambridge Tract on Fourier series and contains both Fourier and Riemann theory. The main points are as follows: generalities about trigonometric series and Fourier series; the Riesz-Fischer theorem; series with monotonically decreasing coefficients; elementary tests for convergence of Fourier series; theorems of Fejér and Fejér-Lebesgue; Poisson's integral; factors of convergence; divergence and non-uniform convergence of Fourier series of continuous functions; the theorem of Lusin-Denjoy on absolute convergence; elements of Riemann's theory of trigonometric series (Cantor's lemma, Riemann's method of summation, formal multiplication, elementary theorems on uniqueness).

Chapter II gives a detailed discussion of Fourier coefficients: order of magnitude of coefficients of functions of bounded variation, of Lip k, of L^p (Hausdorff-Young); applications of Rademacher functions; Helson's theorem (trigonometric series with positive partial sums have coefficients tending to 0).

Chapter III deals with the convergence of a Fourier series at a point and gives all the classical tests not presented in Chapter I. Chapter IV is devoted to Fourier series of continuous functions: for uniform convergence we have the classical Dini-Lipschitz test and its generalization to an integral form (Salem); for divergence equally classical constructions (divergence in a prescribed denumerable set, in a dense set of the second category, uniform boundedness together with divergence in a set everywhere of the power of the continuum).

Chapter V deals with convergence and divergence of a Fourier series in sets. We have here the theorem of Kolmogorov-Seliverstov-Plessner for f in L^2; its extension by Marcinkiewicz (to f in L^p, $1 \leqq p \leqq 2$); tests of Salem and Marcinkiewicz and the proof that the latter cannot be strengthened; convergence of Fourier series and capacities of sets; Kolmogorov's construction of an everywhere divergent Fourier series; Ulyanov's theorem that if f is not in L^2, then its Fourier series can be so rearranged that the resulting series diverges almost everywhere.

Chapter VI on "Adjustments" of functions in sets of small measure is devoted almost exclusively to results of Menshov. Two main results are, first, that any integrable function can be so modified in a set of arbitrarily small measure that the resulting function has a uniformly convergent Fourier series and, second, that, given any closed non-dense set P, we can modify f outside P so that the new Fourier series converges almost everywhere.

This is the final chapter of Vol. I of the book, and is followed by a number of

addenda dealing with a number of auxiliary but advanced topics from Functional Analysis and Complex Variables (the Banach-Steinhaus theorem, the Phragmén-Lindelöf Principle, etc.).

Chapter VII, Vol. II deals with various kinds of summabilities of numerical series: $(C^*, 0)$, Lebesgue, Rogosinski (usually called Bernstein-Rogosinski in Russian literature) and $(C^*, 0)$ (the latter bears the same relation to ordinary convergence as non-tangential approach to radial approach in Abel's summability). The main results of the chapter are proofs of the Hardy-Littlewood theorem about strong summability of Fourier series and its extension from L^p, $p > 1$, to L^1 (due to Marcinkiewicz).

Chapter VIII, "Conjugate trigonometric series," is an introduction to complex methods. It mainly centers around M. Riesz's theorem (that a function conjugate to a function in L^p, $p > 1$, is also in L^p) and its various extensions and complements, but also gives the proof of the fundamental theorem of Plessner asserting that if a trigonometric series converges in some set, the conjugate series converges almost everywhere in the set. A section is also devoted to a description of a new kind of integral (the A-integral) generalizing that of Lebesgue, and gives a few applications of the notion to Fourier series.

Chapter IX discusses the absolute convergence of Fourier series, and in particular proves the Wiener-Lévy theorem. To the specialist, however, this chapter will be of interest not primarily because of the fundamental results, which are very well known, but because it brings to light a number of connections between absolute convergence and the various notions of best approximation. These theorems are not as well known as they deserve to be, and it is perhaps a pity that in some interesting cases the results are stated without proof.

In Chapter X we have a study of properties of sine and cosine series with monotone coefficients and, in particular, the asymptotic behavior (due to Salem) of such series near the origin.

The whole of Chapter XI is devoted to lacunary series. Such series are quite special but interesting for several reasons. In the first place, their behavior resembles that of series of independent random variables in the calculus of probabilities; second, they have a number of sharp properties as regards, say, convergence almost everywhere, uniform convergence, uniqueness, best approximation, etc. Finally, and this is the most important, they are a rich source of examples illuminating various points of the general theory.

The next two chapters deal with convergence, ordinary and absolute, of general trigonometric series. They contain very many results which are rather disconnected, but this is just the picture of the situation as it exists at present, and which is likely to persist until some major breakthrough is achieved. In any case, this is a good source of information.

The remaining two chapters are probably the most interesting in this volume. The first deals with the theory of uniqueness in which the author herself was very much interested and obtained some fundamental results. It begins with the classical theorems of du Bois Reymond, W. H. Young, and de la Vallée-Poussin, goes through theorems of Menshov, Rajchman and her own (the first examples of perfect sets of uniqueness and multiplicity of measure zero) and discusses the most modern developments, the most important of which are due to Salem (and indicate unex-

pected links between trigonometric series and algebraic numbers). The chapter gives a very exhaustive presentation of the situation as it exists now.

The last chapter is interesting for a different reason: it gives detailed proofs of a number of theorems little known outside Russia concerning the representation of an arbitrary measurable function by an almost everywhere convergent trigonometric series. That this is possible was first shown by Menshov some twenty years ago. Nina Bary completed the result by showing that the series can be obtained by termwise differentiation of the Fourier series of a continuous function. Of course, Menshov's series is not unique, and the problem whether the function can also take the value $+\infty$ in a set of positive measure still remains open (it is conjectured that no trigonometric series can diverge to $+\infty$ in a set of positive measure).

Volume II also terminates with a long list of appendices.

The English translation is graphically attractive but, unfortunately, it has a number of defects. Obvious mathematical misprints of the Russian original have been retained in the translation, and a number of serious distortions introduced. (To give examples from Chapter I: the expression "in the metric of the space L^2" of the original is systematically translated "in the metric space L^2"; in Steinhaus' theorem on p. 104 the word "equiconvergent" is replaced by "convergent" making the formulation incomprehensible; at the bottom of p. 110 we find the following passus: ". . . in fact, it can be proved that for a bounded function the partial sums of a Fourier series should be bounded. However, this is untrue even for continuous functions.") Such distortions are no real obstacle to a specialist, but may prove serious stumbling blocks to the beginner. Finally, one more point which, however, applies to many translations from Russian: references to Russian textbooks give pages of the original, even if the textbook has meanwhile been translated into English. Such references are useless for readers not familiar with the Russian language.

A. ZYGMUND

University of Chicago
Chicago, Illinois

39[X].—MARTIN BECKER, *The Principles and Applications of Variational Methods*, The M.I.T. Press, Cambridge, Massachusetts, 1964, vii + 120 pp., 24 cm. Price $5.00.

The present book is concerned with variational methods for solving problems in science and engineering. Various methods are described in detail, giving the advantages and disadvantages of each method. The author restricts himself mainly to "trial function methods" for solving an equation of the form $H\varphi = f$, where H is a given operator and f is a known function. In these methods a set of trial functions $\varphi_1, \cdots, \varphi_N$ are given. The problem is to determine a linear combination $a_1\varphi_1 + \cdots + a_N\varphi_N$ which is a "best" approximation to the solution φ of $H\varphi = f$. Particular attention is given to the method of least squares.

The book begins with the study of self-sufficient equations, that is, equations that are the Euler equations of a functional J. The method of adjoint functions is developed for non-self-sufficient problems. The method of weighted residuals is discussed as well as various methods for solving equations subject to constraints.

The author sets up a set of desirable criteria for variational methods and shows

that the method of least squares satisfies these criteria. Here constraints are handled by the method of penalty functions. Various special techniques for handling eigenvalue problems are given. A comparison of the least-squares method with others is made. The least-squares method is applied to the problem of fuel depletion in a nuclear reactor and the results are compared with those obtained by standard methods. Excellent results are obtained.

The book contains numerous examples which illustrate the effectiveness of the various methods employed. The basic theory upon which the method is based is summarized in appendices. No attempt is made to give a priori error estimates.

The book should prove to be useful to one who is interested in solving problems and to be instructive to one who is interested in theory. Many useful ideas are set forth. The examples are well chosen and illustrate difficulties as well as advantages of a particular method.

M. R. HESTENES

University of California
Los Angeles, California

40[X].—FRANCIS B. HILDEBRAND, *Methods of Applied Mathematics*, Second Edition, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1965, ix + 362 pp., 24 cm. Price $10.00.

This volume is the second edition of a book published originally in 1952. It consists of three long chapters entitled "Matrices and Linear Equations," "Calculus of Variations and Applications," and "Integral Equations." The first edition contained an additional chapter entitled "Difference Equations." This chapter has been removed and is to be expanded and published as a separate volume.

The material in each chapter is essentially independent of the other chapters. Each chapter is a brief but reasonably comprehensive treatment of the topic from an applied point of view. There are a large number of problems and a list of answers is given at the back of the book.

In order to fairly appraise the book it is necessary to consider it in conjunction with the author's text *Advanced Calculus for Applications*, together with his forthcoming book on difference equations and finite-difference methods. The present volume is an obvious extension of *Advanced Calculus for Applications*, containing additional topics which could not be included there. The two volumes can be nicely used in a three- or four-semester course on methods of applied mathematics at an intermediate level. The book can also be used for reference or self study. It is well written, and considerable care has been taken in introducing the topics in each chapter. It can be highly recommended if used as noted above.

RICHARD C. ROBERTS

41[X].—L. B. RALL, Editor, *Error in Digital Computation*, Volume I, John Wiley & Sons, Inc., New York, 1965, ix + 324 pp., 24 cm. Price $6.75.

This book is the proceedings of an advanced seminar conducted by the Mathematics Research Center, United States Army at the University of Wisconsin, October 5–7, 1964. As such it contains five papers based on the addresses of the invited speakers. The latter part of the book consists of a bibliography of books and papers on error analysis taken from the Mathematical Reviews.

The speakers and their topics were:

The Problem of Error in Digital Computation, by John Todd

Techniques for Automatic Error Monitoring and Control, by Robert L. Ashenhurst

The Automatic Analysis and Control of Error in Digital Computing Based on the Use of Interval Numbers, by Ramon E. Moore

Error in Digital Solution of Linear Problems, by Ernest L. Albasiny

The Propagation of Error in the Digital Integration of Ordinary Differential Equations, by Peter Henrici.

John Todd's paper describes some of the recent efforts in analyzing the error in digital computation, illustrated with several examples which have arisen recently at Caltech. He has many suggestions for the future directed at various segments of the computer field from mathematician to design engineer.

Robert L. Ashenhurst discusses the effect on error propagation of various types of computer arithmetic. Starting with a precise definition of terms, the exposition leads to significance adjustment rules and the question of normalized versus unnormalized arithmetic.

In his paper Ramon E. Moore defines an arithmetic system of closed intervals which contains the real numbers as a subsystem. A topology is introduced which leads to the concept of continuity of set functions defined for these intervals. An integral calculus is then developed together with an interval form of Gaussian quadrature. The principal application is to the approximate solution of the initial-value problem for ordinary differential equations. By use of interval arithmetic and appropriate rounding procedures, numerical methods are described which give a numerical solution together with rigorous bounds on the error. The bounds themselves behave like the mesh size to an appropriate power. This method has been programmed for an IBM 7094, and the results of certain sample cases are given.

Ernest L. Albasiny discusses recent studies (mainly those of J. H. Wilkinson) of the effect of round-off errors in the numerical solution of various problems in linear algebra. Topics included are the solution of linear equations, matrix inversion, determinant evaluation, and the determination of eigenvalues.

Peter Henrici surveys recent developments in the numerical solution of the initial-value problem for ordinary differential equations by finite-difference methods. The problem of defining the term "stability" is discussed, and some consideration is given to the value of error estimates and error bounds.

As can be seen from the above discussion, this book brings together many of the recent developments of significance in the fields covered.

BERT HUBBARD

University of Maryland
College Park, Maryland

42[X].—*The Universal Encyclopedia of Mathematics* (with a foreword by JAMES R. NEWMAN), Mentor Book, New York, 1965, 715 pp., 18 cm. Price $1.50 (paperbound).

This is an inexpensive paperback edition of the $8.95 hard-cover volume previously reviewed here in *Mathematics of Computation*, v. 19, 1965, p. 164, RMT **22**. See that review for further details. We might repeat the previously made point that the coverage is not as broad as that suggested by the title, since a publisher's press

release states flatly: "Every subject of mathematics is covered in the encyclopedia, from absolute value (the first listing) to zero (the last)." While that makes it evident that *abacus* and *Zorn's Lemma* are not covered, the reader should know that other terms, more in the mainstream, such as *derivative*, are also missing.

D. S.

43[X, Z].—RALPH H. PENNINGTON, *Introductory Computer Methods and Numerical Analysis*, The Macmillan Company, New York, 1965, xi + 452 pp., 24 cm. Price $9.00.

This book is written for students of engineering or science with a mathematical background through integral calculus, but who have little or no acquaintance with digital computers. In the author's words, the book attempts "to give usable computer methods for solving the more elementary problems in applied mathematics and to give some perspective as to how easy or difficult these problems are to solve on a computer."

In line with these aims, about one half of the book is devoted to an elementary introduction to digital computers and to programming (primarily in FORTRAN), and the remaining half contains a slightly more advanced discussion of standard introductory topics of numerical analysis, with a view toward the application of computers.

The discussion on computers and programming begins with a very elementary presentation of number systems, followed by a description of the basic structure of digital computers and a discussion of the elements of machine-language coding for a simple hypothetical computer. Then the principal features of standard FORTRAN II are introduced, including FORTRAN functions and subroutines. An additional chapter presents some comments on computer running times and on de-bugging. The role of errors in numerical calculations is emphasized already very early in the book, this is followed later by a chapter on problems of error accumulation and loss of significance.

The second half of the book on numerical analysis covers the traditional topics expected in any text of this type: Simple quadrature methods, the iterative solution of algebraic and transcendental equations, the application of these methods to polynomial equations (including nice sections on the manipulation of polynomials and on Sturm sequences), the evaluation of determinants and the solution of linear systems of equations by elimination techniques, the Gauss-Seidel iteration, an introduction to matrices (including the solvability of m equations in n unknowns, eigenvalues and their determination using the Leverrier-Faddeev method), least squares, difference tables, polynomial interpolation and numerical differentiation, and finally a brief chapter on ordinary differential equations presenting some material on explicit linear, first-order systems as well as the Runge-Kutta and Milne method. Throughout these chapters a general discussion is followed by flow charts and frequently by complete FORTRAN programs. The text is interspersed with detailed numerical examples, and each chapter ends in a list of exercises, most of which ask the student to write particular FORTRAN programs or to apply some method to specific cases.

The mathematical presentation of the books stays on the indicated level, except in the first half of the book where it is decidedly lower. The exposition is clear and the style lucid, and, all in all, the book appears to satisfy the particular aims set for it by the author.

WERNER C. RHEINBOLDT

University of Maryland
College Park, Maryland

44[Z].—N. G. BRUYEVICH, Editor, *Problems of the Design and Accuracy of Complex Continuous Action Devices and Computer Mechanisms*, Pergamon Press, Inc., New York, 1964, xi + 264 pp., 24 cm. Price $10.00.

This collection of papers by Russian experts in the design of precision mechanisms is probably the most thorough treatment to be found in textbook form. The art has received its greatest development in military analogue computers and, therefore, the knowledge of the techniques and methods has been confined, until recently, to classified documents. The most complete exposition of these techniques appeared in the *Proceedings of the Seminar on the Theory of Machines and Mechanisms* sponsored by the USSR Academy of Sciences (1950–1955). This book is, essentially, a selection of material from these *Proceedings*.

Several of the papers deal with the mathematical analysis of the reduction of errors in making the computation from the continuous inputs to the continuous outputs. This includes the design of automatic-control devices (servos). Other papers deal with detailed studies of errors in special mechanisms like universal joints, toothed gearing, three-dimensional cams, variable-speed mechanisms and friction drives (like mechanical integrators). Many of these studies are based on experimental testing of actual mechanisms.

There are extensive references, mostly to Russian papers and books.

MICHAEL GOLDBERG

5823 Potomac Avenue, N. W.
Washington 16, D. C.

45[Z].—MARTIN DAVIS, Editor, *The Undecidable: Basic Papers on Undecidable Propositions, Unsolvable Problems and Computable Functions*, Raven Press, Hewlett, New York, 1965, 440 pp., 24 cm. Price $8.95.

This book is an anthology of fundamental papers in the area of undecidability, unsolvability and computability. It is much more than a mere collection of papers, however. By means of well-chosen editorial remarks which precede most papers, the editor facilitates the correlation of these early papers with modern work in the area. For example, he calls attention to changes in terminology, points of view, and errors in technical detail (without pinpointing them). Some important papers of Gödel originally published in German have been rendered into English for this volume. In addition, Gödel has made available to the editor, a Postscriptum of his 1934 lecture notes on "Undecidable Propositions . . ." as well as corrections and emendations. These notes are published here for the first time. Another interesting paper appearing here for the first time is one by E. L. Post, "Absolutely Unsolvable

Problems. . .". The reviewer believes that this volume is a valuable addition to the literature. The contents follow:

KURT GÖDEL

On Formally Undecidable Propositions of the Principia Mathematica and Related Systems. I (1931)
On Undecidable Propositions of Formal Mathematical Systems (1934)
On Intuitionistic Arithmetic and Number Theory (1933)
On the Length of Proofs (1936)
Remarks Before the Princeton Bicentennial Conference on Problems on Mathematics (1946)

ALONZO CHURCH

An Unsolvable Problem of Elementary Number Theory (1936)
A Note on the Entscheidungsproblem (1936)

ALAN M. TURING

On Computable Numbers, with an Application to the Entscheidungsproblem (1936–1937)
Systems of Logic Based on Ordinals (1939)

J. B. ROSSER

An Informal Exposition of Proofs of Gödel's Theorem and Church's Theorem (1939)
Extensions of Some Theorems of Gödel and Church (1936)

STEPHEN C. KLEENE

General Recursive Functions of Natural Numbers (1936)
Recursive Predicates and Quantifiers (1943)

EMIL POST

Finite Combinatory Processes, Formulation I (1936)
Recursive Unsolvability of a Problem of Thue (1947)
Recursively Enumerable Sets of Positive Integers and Their Decision Problems (1944)
Absolutely Unsolvable Problems and Relatively Undecidable Propositions—Account of an Anticipation
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CALVIN C. ELGOT

Thomas J. Watson Research Center
International Business Machines Corp.
Yorktown Heights, New York

46[Z].—S. C. PLUMB, *Introduction to FORTRAN*, McGraw-Hill Book Company, New York, 1964, vii + 203 pp., 23 cm. Price $3.50 (paperback).

This is a self-instructional manual for FORTRAN II as designed for the IBM 700 series of computers. The material is presented at a very elementary level (high-

school algebra is the prerequisite) in a format designed for self-teaching. Information is presented to the student in very small units (usually a short paragraph of six to a dozen lines) and each unit is followed by a short question. The brevity and simplicity of some of these units may tax the patience of the better student who would probably prefer to digest a larger piece of information before being interrupted by questions.

E. K. Blum

Wesleyan University
Middletown, Connecticut