## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

**47[A, S].**—A. F. NIKIFOROV, V. B. UVAROV & YU. L. LEVITAN, *Tables of Racah Coefficients*, translated by Prasenjit Basu, Pergamon Press, New York, 1965, xx + 319 pp., 26 cm. Price $15.00.

Racah coefficients occur in the quantum theory of angular momentum and may be briefly characterized as the matrix elements of invariant operators formed in the coupling of three tensor operators. These coefficients are of great importance in atomic and nuclear spectroscopy, in angular correlation theory, and in the quantum theory of angular momentum itself. The importance of the Racah coefficient may be judged from the fact that some 15 more or less extensive tabulations have appeared since 1952. The two most extensive recent tabulations (besides the compilation under review) are *The 3-j and 6-j Symbols*, by Rotenberg, et al., Technology Press, Cambridge, 1959, and *Tables of the Racah Coefficients*, Ishidzu, et al., Pan-Pacific Press, Tokyo, 1960 (English).

The present tabulation gives the Racah coefficient $W(abcd; ef)$ as 8-place decimal fractions, and has as its chief merit the extensive range of variables. The tables are divided into three sections:

1. $a, b, c, d$ half-integral $(\frac{1}{2}(1)\ \frac{17}{2})$; $e, f = 0(1)17$, (153 tables),
2. $a, b, c, d$ integral $1(1)9$; $e, f = 1(1)18$ (162 tables),
3. $a, c, e$ half-integral: $a, c = \frac{1}{2}(1)\ \frac{17}{2}$; $e$ half-integral $\frac{1}{2}(1)\ \frac{35}{2}$; $b, d, f$ integral $1(1)9$, (153 tables).

The present volume is essentially the Russian original with a translation of the preface (9 pages). The translation is not very smooth, but is quite adequate for the use of the tables. This tabulation suffers in comparison to earlier work (in particular, those mentioned above) in that it lacks both a discussion of the properties of the Racah coefficients as well as algebraic tables of the Racah coefficients (these are often more useful than numerical values in theoretical applications). It should be mentioned, too, that the Racah coefficients are square roots of rational numbers; consequently, tabulation as decimal fractions involves some loss of information (only the Ishidzu, et al. tabulation—of the recent work—is expressed in exact (nondecimal) form).

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**48[D].**—D. G. MARTIN, *Tables of* $(\sin^2 x)/x^2$ *to Six Decimal Places*, Report 4935, Atomic Energy Research Establishment, Harwell, England, 69 pp., 29 cm. Available from H. M. Stationery Office. Price 10s.

These unique tables were computed on an IBM 7030 system to expedite the calculation of the attenuation of a beam of long-wavelength neutrons passing through a solid containing a number of randomly oriented defects.

The format resembles that of the companion tables of $(\sin x)/x$, described in the preceding review. Values of $(\sin^2 x)/x^2$ are presented to 6D for $x = 0(0.001)25(0.01)100$, together with rounded first differences.

The critical comments on the typography in the companion tables of $(\sin x)/x$ apply equally to these tables.

J. W. W.

**49[D].**—D. G. MARTIN, *Tables of* $(\sin x)/x$ *to Six Decimal Places*, Report 4934, Atomic Energy Research Establishment, Harwell, England, 115 pp., 29 cm. Available from H. M. Stationery Office. Price 16s.

In an introduction to these extensive tables the author states that they were prepared on an IBM 7030 system to facilitate computation of the scattering of long-wavelength neutrons by defects in irradiated solids. He cites, as a further application, calculations of the diffraction of electrons and X-rays by polyatomic molecules in liquids, gases, and amorphous solids. Pertinent references to such applications are included in a short bibliography, which follows a concluding introductory paragraph describing the use of the tables.

These double-entry tables consist of 6D approximations to $(\sin x)/x$ for $x = 0(0.001)50(0.01)100$, together with first differences. As the author notes, the only comparable table is that of Reynolds [1], which gives 8D values for $x = 0(0.001)49.999$, *without* differences.

Unfortunately, the photographic reproduction of the computer sheets here has left much to be desired with respect to legibility; indeed, many entries contain figures that are completely, or almost completely, undecipherable. Apparently little effort was expended in assuring that these useful tables were printed in an acceptable manner.

J. W. W.

1. G. E. REYNOLDS, *Table of* $(\sin x)/x$, Technical Report 57-103, Air Force Cambridge Research Center, Cambridge, Massachusetts, 1957.

**50[D, E, H, L, P, X].**—KEITH A. SWITZER, *Tables of Roots of Certain Transcendental Equations Arising in Eigenfunction Expansions*, Circular 23, College of Engineering, Washington State University, Pullman, Washington, 1965, 61 pp., 28 cm.

The numerical tables herein were motivated by a need for a more extensive compilation than those already available of eigenvalues associated with boundary-value problems arising in analyses of heat transfer and of mechanical vibrations.

The first table (Table A) consists of 5D values of the first eight roots $S_n$ of the transcendental equation $C = S_n \tan S_n$, corresponding to $C = 0.001(0.001)0.1(0.01)1(0.1)10(1)100(10)400$.

In Table B there appear, to the same precision, the first eight roots of the equation $C + S_n \cot S_n = 0$, for $C = -0.999(0.001)-0.1(0.01)1(0.1)10(1)100(10)400$.

Finally, Table C presents, again to 5D, the first eight roots of the equation $C = S_n J_1(S_n)/J_0(S_n)$, for the same range of the parameter $C$ as in Table A.

The computational scheme followed in evaluating the tabular entries consisted of successive halving of the interval containing the desired root, starting with an increment of 0.1. This algorithm is written in ALGOL in this report, and the author states that the FORTRAN programs actually used to develop the tables may be obtained by communicating with him.

The most extensive previously published tables of such eigenvalues are those of Carslaw and Jaeger [1], which extend to 4D, and which the present author found inadequate for his purposes.

J. W. W.

**51[F, Z, X].**—*Revue Française de Traitement de l'Information*, Dunod, Éditeur, Paris, 1965, 158 pp., 22 cm. Price 12 f.

This little magazine has three articles in French, an algorithm section, and a bulletin describing computing activities in France. The three articles are:

(1) "Sur la résolution des programmes à solutions entières," M. Courtillot,

(2) "Algorithms d'analyse syntaxique pour langages 'Context-Free'," M. Basseur and J. Cohen and

(3) "Point de vue sur la programmation," G. Lettelier and J. Weber.

The last article, which is of wider interest than the other two, discusses a new language, MICMAC, for the CDC 3600. Before giving details of this language, the authors write an interesting discussion of various programming language entities and notions. Among these are compatibility, static vs. dynamic languages, assembly languages and algorithmic languages and notions such as conditional assembly and macro instructions. MICMAC itself appears to be a powerful macro language compiler with pseudo instructions for declaring arrays of various types. The authors claim that MICMAC facilitates the reprogramming problem by clarifying programs so that they can be easily understood.

The second article is the first part of a two part paper on context free syntactic analysis for compilers. After a section on theoretical aspects of the problem, different types of analyses and reviews of existing work on syntactic analysis, the authors develop the subject of top down analysis in fine detail and end part one of the paper with an ALGOL algorithm for such an analysis. The bibliography (mostly in English) on syntactic analysis and theory of languages is extensive and interesting in itself.

The first article, which is of comparatively restricted interest, describes a method which has already been the object of several notes published inside the Shell Company and has also been presented at the International Symposium on Mathematical Programming in London, 1964. If $f$ is a numerical concave function in $R^n$ and $g$ a concave function from $R^n$ into $R^m$, the method presented permits solution of programs of the type:

$$\max [f(x) \mid g(x) \geqq 0, x \in Z^n].$$

The method of all-integer-programming of R. Gomory is a special case of this method when $f$ and $g$ are linear with integer components.

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**52[G].**—Marvin Marcus & Henryk Minc, *Introduction to Linear Algebra*, The Macmillan Company, New York, 1965, x + 261 pp., 25 cm. Price $7.95.

This book is intended to provide material for a one-semester course at the sophomore or junior level. There are sixteen sections grouped into three chapters: the first chapter develops the elementary notions including inversion; the second deals with rank and with determinants; the third, on characteristic roots, gives the Schur triangularization theorem, which is used as a basis for a brief discussion of normal matrices, and concludes with a section on inequalities, including Gershgorin circles, ovals of Cassini, and the minimax theorems for Hermitian matrices. The normal forms of Jordan and of Frobenius are not included. There are a large number of exercises, with 47 pages of answers and solutions at the end.

The development is rigorous, and a number of auxiliary notions, such as rings and fields, permutation groups, and the like, are introduced, but subordinated to the main objective. The book should be quite teachable, and even suitable for independent reading by the novice.

A. S. H.

**53[G].**—Jean-Marie Souriau, *Calcul Linéaire*, Second edition (in two volumes), Presses Universitaires de France, Paris, 1964 and 1965, Volume I, xv + 259 pp., 19 cm. Price 27 f; Volume II, xv + 263 pp., 19 cm. Price 30 f.

The first volume bears the subtitle "Méthodes mathématiques de la physique," and in the preface it is stated that this is a work of pure mathematics, intended for all those who have to apply linear algebra.

The development is rigorous and abstract, but accompanied throughout by figures and illustrative material, and by numerous exercises. Solutions for the exercises are given at the end of Volume II. Inevitably the notations and special terminology become rather involved, but an unusually detailed index, together with a list of symbols, are of great assistance.

The first volume gives the general theory. An initial chapter of about forty pages develops the notion of sets, operators, inversion groups, and the like; linear spaces and linear operators come in chapter two of not quite fifty pages. Next come matrices, then dimensionality, then a chapter of about seventy pages on multilinear algebra with determinants, and finally a chapter on spectral properties of about thirty pages, that stops short of the Jordan normal form.

The second volume is given over to applications, beginning with the exponential and logarithm, but then going on to special spaces. Something over a hundred pages are devoted to "Euclidean and hermitian spaces": normal, hermitian, and anti-hermitian operators; orthogonality, unitary groups, the Lorentz group, to pick a few topics more or less at random. Finally there is a brief chapter on spaces of dimension 2, 3, and 4, and on spinors of Dirac.

There is very little direct reference to methods of computing. But the author is able to include quite a large amount of theory, and in general he succeeds very well in making intuitive his abstract approach.

A. S. H.

**54[K, L, M].**—T. Krishnan, *Table of Truncated Probits*, Indian Statistical Institute, Calcutta, 4 + 28 computer sheets, ms. deposited in UMT File.

This unpublished table is an elaboration of one appearing in a paper [1] by the

author, wherein the underlying theory and some applications are presented in detail.

The *truncated probit* is the value of $X$ (customarily increased by 5 to avoid negative values) that satisfies the equation

$$P + (1 - P)\varphi(K) = \varphi(X)$$

where $P$ and $K$ represent, respectively a specified probability and a given "standardized lower point of truncation" (the latter also augmented by 5 in the present tables). Here $\varphi(X)$ represents the integral

$$(2\pi)^{-1/2} \int_{-\infty}^{X} \exp(-u^2/2)\, du.$$

Values of the truncated probits are tabulated herein to 4D for $P = 0.01(0.01)0.99$, $K + 5 = 0.4(0.1)7.3$.

In private correspondence the author revealed that the table was prepared on an IBM 1401 system.

An introductory note sets forth the approximations published by Hastings [2] that were used here in the numerical evaluation of the error integral and its inverse, as required in the preparation of this unique and useful table.

J. W. W.

1. T. KRISHNAN, "Truncation in quantal assay," *Ann. Inst. Statist. Math.*, v. 17, 1965, pp. 211–223.
2. C. HASTINGS, J. T. HAYWARD & J. P. WONG, *Approximations for Digital Computers*, Princeton Univ. Press, Princeton, N. J., 1955.

**55[L].**—R. FRISCH-FAY, *Tables of Integrals of Fractional Order Bessel Functions*, UNICIV Report No. R-9, University of New South Wales, Kensington, N.S.W., Australia, June 1965, 13 pp., 23 cm. Copy deposited in UMT file.

In his introduction to these tables the author notes the lack of such tabulations except for those of the Fresnel integrals. With reference to integrals of Bessel functions of the first kind of positive integer order, he cites the unique tables of Knudsen [1].

The present tables give numerical values of $\int_0^z J_\nu(t)\, dt$ for $\pm\nu = \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$, and $\frac{5}{6}$ to 7D for $z = 0(0.1)6.3$ and to 5D for $z = 6.3(0.1)10$.

Standard power-series expansions formed the basis for the underlying calculations, which were performed on the UTECOM computer at the University of New South Wales.

References to previous tables of integrals of Bessel functions appear in a recent treatise by the reviewer [2].

Y. L. L.

1. H. L. KNUDSEN, *Bidrag Til Teorien For Antennesystemer Med Hel Eller Delvis Rotationssymmetri*, I Kommission Hos Teknisk Forlag, Copenhagen, 1953. [See *MTAC*, v. 7, 1953, pp. 244–245, RMT **1140**.]
2. Y. L. LUKE, *Integrals of Bessel Functions*, McGraw-Hill Book Company, New York, 1962, pp. 70–72. [See *Math. Comp.*, v. 17, 1963, pp. 318–320, RMT **51**.]

**56[L, X].**—FRANÇOISE MICHAUD. *Étude sur les représentations approchées des solutions de l'équation de Mathieu*, Centre National de la Recherche Scientifique, Institut Blaise Pascal, Paris, 1965, 108 + 10 unnumbered pp., 27 cm.

This pamphlet summarizes some of the known results relating to Mathieu's equation, $y'' + [b - s\cos^2 x]y = 0$, and the corresponding modified equation. Its main part is devoted to the characteristic values corresponding to real parameters, $s$. Several methods of generating these numbers are reviewed. No new methods are developed, but the author has used Bazley's method for self-adjoint operators to compute some additional values and compare them with the National Bureau of Standards tabular values. Her comments indicate that for $s = 100$ and the 8th characteristic value, 12 "intermediate problems" yielded 6 significant figures, and this accuracy could not be improved by further calculations.

This reviewer does not agree with her comment that the method of finding the roots of the classical continued fraction is "very tedious." Compared with this method, Bazley's method involves much more work, and its use is justified only as an exercise, to give insight into his techniques. His methods are of profound importance in more difficult problems, where other means are not available. As an exercise, then, the results now given have value.

The author includes two FORTRAN II codes; one for computing the trigonometric coefficients by the "method of G. Blanch." This code is useful only in the region where none of the trigonometric coefficients pass through zero. For example, there would be division by zero when generating $A_0/A_2$ (as she does) if 8 or 10 significant digits were carried and $s = 85.19452484$, order $r = 2$. The second code relates to one phase of Bazley's technique.

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**57[P, X].**—V. E. BENEŠ, *Mathematical Theory of Connecting Networks and Telephone Traffic*, Volume 17, Academic Press, Inc., New York, 1965, xiv + 319 pp., 24 cm. Price $12.00.

Various combinatorial problems associated with communication networks are the principal concern of this research monograph. Special emphasis is given to the arithmetic for the synthesis and design of connecting networks (such as those occurring in the telephone system of a central office), to the classical statistical mechanics of traffic analysis, and to the routing of signals in complex networks. The author's "thermodynamic model" of traffic flow in communication *networks* is sufficiently plausible as to lead the reviewer to conjecture that a "quantum model" exists for a related communication *field*.

Except for minor details, seven of the eight chapters coincide nearly word-for-word with the author's publications in the *Bell System Technical Journal* over the past ten years. The remaining chapter gives results by C. Clos on nonblocking connecting networks for telephone systems. This book, unlike others which have been "pitchforked" together and published, does not suffer from discontinuities between chapters, although one occasionally finds technical terms used before they

are defined or even delimited. It makes exceptionally good use of elementary lattice-theoretical ideas, switching theory, and number-theoretic notions.

Since there are no study problems, the compendium, by itself, would be unsuitable as a textbook. It should be particularly valuable to researchers interested in novel and practical communication networks or in queueing theory.

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**58[X].**—I. M. KHABAZA, *Numerical Analysis*, Pergamon Press Ltd., Oxford, England, 1965, xii + 239 pp., 20 cm. Price 25s. (Paperbound).

This book is an elementary approach to desk and computer calculation, yet it contains much advanced material (sometimes a bit too compressed for easy reading by the beginner). The basic approach, after a pair of chapters on digital computers and desk machines, is classical via many interpolation formulas derived by symbolic operator methods. The book also covers the topics of zeros of polynomials, solution of ordinary differential equations, and simultaneous linear algebraic equations including latent roots and vectors, orthogonal polynomials, and a bit on Gaussian quadrature. Thus it contains much of the usual material.

The material is well presented, has many practical remarks to aid the beginner, plus worked examples and exercises; thus it could serve as a text in a junior level course (except possibly for its occasional emphasis on desk machine methods which are passé with American students).

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**59[X, Z].**—*4-ème Congrès de Calcul et de Traitement de l'Information*, Dunod, Éditeur, Paris, 1965, 413 pp., 22 cm.

This is a collection in French of more than fifty separate short articles presented at the 4th Congress of the Association Français de Calcul et de Traitement de l'Information (AFCALTI) held April 21–April 24, 1964. The subjects range from a treatment of singular integral equations to the training of management engineers.

The book is parsed into three main sections—reports, communications and conferences. The Congress touched the areas of combinatorial analysis, compilers, numerical analysis, systems, boolean algebra, digital and analogue computers, integral equations, programming languages for administration, and boundary value problems and variational methods. This record of the Congress is touted as being addressed to management specialist as well as mathematician, logician as well as technician and computing center director as well as student.

Perhaps because of the extent of subject matter and audience, the treatment of topics tends to be rather superficial, and as in most collections of conference papers,

variable in calibre. There are, however, a few more deeply interesting articles such as one by P. Broise on dealing with tree structures in ALGOL.

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60[Z].—F. P. BROOKS, JR. & K. E. IVERSON, *Automatic Data Processing*, John Wiley & Sons, Inc., New York, 1963, xxv + 494 pp., 24 cm. Price $10.75.

It is an unfortunate situation that books written about such dynamic subjects as electronic computers are obsolescent even as they are appearing in print. (The same thing can be said about the computers themselves.) How much more so is a book based on a course given from 1954 on. This book has many interesting features, but it is difficult to determine for whom it is intended. If it is for a programmer, then why the detailed discussion of manual and punched-card equipment? If it is for a systems analyst, then why all the details of coding and computer organization? There are also some noticeable omissions here, specifically a treatment of programming languages. On the other hand, the material covered is treated well. The programming notation introduced is difficult, but rewarding, once you master it. In summary, in using this book, one must pick and choose, bearing in mind the great strides made in the field.

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61[Z].—ERIC A. WEISS, *Programming the IBM 1620: The Hands-on Approach*, McGraw-Hill Book Company, Inc., New York, 1965, viii + 299 pp., 24 cm. Price $7.50.

As is stated in the preface, this book is intended to be an introduction to computers and programming for high school and beginning college students. The reader becomes acquainted with IBM 1620 machine-language commands by typing short programs directly into the computer, using read-write, data-transmission, and branch instructions. After graduation from the typewriter to punched cards, other machine commands, along with their SPS mnemonic codes, are covered. The last five chapters are devoted to an introduction to FORTRAN. More complicated concepts such as functions and subroutines, indirect addressing, and SPS floating-point arithmetic are omitted. Readable, requiring a minimal mathematical background, covering only the basic commands, but explaining these thoroughly, this book affords a good fundamental understanding of computers for the young student who preferably has access to a 1620 system.

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**62[Z, P].**—WALTER HERBERT BURROWS, *Graphical Techniques for Engineering Computations*, Chemical Publishing Co., Inc., New York, 1965, xiii + 435 pp., 24 cm. Price $15.00.

It may seem strange to the digital computer expert that new books on graphical techniques are still being written. But it is not strange to the engineer who has always used graphs and slide rules, and who expects to use them indefinitely. Their accuracy is adequate for most of his problems; moreover, they are readily available, rapid, and economical. Even the more accurate and extensive data obtained from digital computers may be compressed into a simple graphical device for ready reference.

This book is a graph designer's handbook on the detailed methods of preparing accurate graphs and special slide rules, using drafting, photography, mechanical methods, and the use of special graph papers. Various methods of calculating with graphs are described, including differentiation, integration, and the solution of equations in two or more variables.

The largest part of the book is devoted to nomograms and alignment charts. A brief discussion of the general analytic procedures is given. The synthetic methods are given in great detail, using both straight scales and curved scales, uniform and irregular graduations. The use of the author's hyperbolic scales is emphasized. These enable simpler and more efficient constructions for many functions. Many practical examples are displayed, and these serve as suggestions for new problems that may be encountered by the designer. The possibilities of three-dimensional nomograms are discussed. The reduction of empirical tables and graphs to nomographic form is described and displayed.

Several of the earlier workers in the field are mentioned in the preface and in the test. However, the book could be improved by the addition of a selected bibliography from the vast literature of American and foreign sources.

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**63[Z, X].**—JOHN G. KEMENY, *Random Essays on Mathematics, Education and Computers*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964, ix + 163 pp., 22 cm. Price $4.95.

The title describes this little book very well. Written at various times between 1957 and 1963, the essays deal with such diverse topics as circuit-riding as a Mathematical Association lecturer, rigor vs. intuition in mathematics, education of the well-rounded man, and the computer topics listed below. The author chairs the Mathematics Department at Dartmouth College, and has taken the lead in changing the undergraduate mathematics curriculum. Dartmouth is also a leader in creating computer time-sharing systems for staff and students.

Except for occasional remarks, the material on computing consists of four essays:
Machines as Extensions of Human Brains,
Games of Life and Death,
A Library for 2000 A.D.,
Computing Center at a Liberal Arts College.

The author is a charming and convincing salesman for the Computer Revolution. He discusses the diagnosis of disease, the simulation of traffic, formula manipulation, war games, etc. very well at a popular level. His final essay (written in April, 1963) is a convincing case for remote consoles and time-sharing. The longest essay of the book, and the most daring forecast, is the one on future libraries, reprinted from Martin Greenberger (Editor), *Management and the Computer of the Future*, Wiley, New York, 1962. (Reviewed in *Math. Comp.*, v. 17, 1963, pp. 97–98). This is a proposal for largely replacing university research libraries by a national library of information, linked by communication networks to major centers of learning. If the proposal were taken literally, journal articles would normally be transmitted from a national center to each requesting scholar at the rate of 20 pages $\doteq$ ($4 \times 10^6$ bits) per second. The communication costs would be extremely high at present rates. The author observes that one way of reducing costs would be to have a central abstracting and searching service, with widely dispersed copies of the library material which is frequently read.

The book was a delight to read.

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