REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

64[A–E, X, Z].—L. A. Lyusternik, O. A. Chervonenkis & A. R. Yanpol'skii, *Handbook for Computing Elementary Functions*, translated by G. J. Tee, Pergamon Press, New York, 1965, xiii + 251 pp., 23 cm. Price $10.00.

This handbook is a translation with corrections of *Matematicheskii analiz-Vychisleniye elementarnykh funktsii*, published in Moscow in 1963. It is Volume 76 in the International Series of Monographs in Pure and Applied Mathematics.

The authors point out in the preface the previous lack of any comprehensive collection of computational formulas for the elementary functions. As examples of earlier handbooks containing such formulas they cite specifically the collection of polynomial and rational approximations developed by Hastings and his coworkers [1] and the tables of Ryshik & Gradstein [2], which contain a large amount of material on the representation of functions by power series.

Included in the introduction to the present book is a concise discussion of the several ways of representing mathematical functions; namely, power series, series of orthogonal polynomials, continued fractions, interpolation formulas, best approximations (by polynomials and otherwise), iterative sequences, and differential equations. The great impetus given the search for numerical algorithms by the advent of electronic digital computers is noted by the authors.

The body of this book is divided into four chapters, followed by two appendices and a bibliography of 81 publications, together with a supplementary list of references for the appendices.

The first chapter deals with polynomials, rational functions, and power functions. The various methods of evaluating polynomials such as those of Horner, John Todd, Y. L. Ketkov, and V. Y. Pan are discussed. For the elementary rational functions there are presented power series expansions, expansions in Chebyshev polynomials, infinite products, and iterative formulas. Corresponding to power functions we find similar information, supplemented by continued fraction expansions, Padé approximations, and approximations by linear functions. Throughout the book virtually every formula is followed by a reference to the item in the bibliography that gives its source.

The second chapter is devoted to exponential and logarithmic functions. General information is supplied on each of these functions, followed in each case by the various types of expansions and approximations considered in the preceding chapter. A minor omission noted here is the source reference of iterative process 2.7, 2° on p. 75, which should be item 74 in the bibliography. An innovation is the inclusion of rational approximations to binary logarithms.

The trigonometric and hyperbolic functions and their inverses are treated in similar detail in the third chapter. Here we also find expansions of certain of the trigonometric functions in series of elementary rational functions. The numerical values of the coefficients in the expansion of the first four trigonometric functions (and the inverse of the first three) in series of Chebyshev and Legendre polynomials are tabulated to from 11 to 24 decimal places. Similar tables are presented for the

coefficients of the best polynomial approximations (in the sense of Chebyshev) to certain trigonometric functions and their inverses. This information was extracted from the publications of Hastings [1] and of succeeding workers in the field of polynomial approximation, to which specific reference is made in the bibliography.

Chapter IV consists of brief descriptions of algorithms used for computing elementary functions on several Soviet computers, namely, Strela, BESM, M-2, M-3, and Ural.

The first appendix consists of an exposition of the definitions, mathematical properties, and various expansions of the gudermannian, harmonic polynomials, the hypergeometric function, and orthogonal polynomials (including those of Legendre, Chebyshev, Laguerre, and Hermite). This appendix is concluded with the tabulation to 6D of the zeros of the following polynomials: $P_n(x)$, $n = 1(1)40$; $L_n(x)$, $n = 1(1)15$; $H_n(x)$, $n = 1(1)20$; and $h_n(x)$, $n = 1(1)22$.

The second appendix consists exclusively of pertinent mathematical tables. Table 1, entitled Coefficients of Certain Series, gives for $n = 1(1)10$: n^{-1} and $\sum_{h=1}^{n} k^{-1}$ to 5D; exact values of $n!$, $(2n-1)!!$, $(2n)!!$ and their reciprocals to from 5 to 11D; $n!/(2n-1)!!$, $2^n n!/(2n+1)!!$, $(2n-1)!!/2^n n!$, all to 5D; $(2n-1)!!/2^n n!(2n+1)$, 6–7D; $(2n-1)!!/2^{n+1}(n+1)!$, 5–7D; and $(2n-1)!!/2^{n+1}(n+1)!\cdot(2n+3)$, 6–8D. Table 2 gives to at most 8S the binomial coefficients $\binom{n}{m}$ for $n = 1(1)50$, while Table 3 gives the exact values of these coefficients $\binom{\nu}{m}$ for $m = 1(1)6$, $\pm\nu = \frac{1}{2}(1)\frac{7}{2}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{1}{4}(\frac{1}{2})\frac{5}{4}$, and $-\nu = 1(1)5$. Table 4 gives sums $\sum_{k=1}^{n} k^r$ for $r = 1(1)5$, $n = 1(1)50$. In Table 5 the gudermannian, $\mathrm{gd}(x)$, is given to 5D for $x = 0(0.01)5.99$ and to 6D for $x = 6(0.01)9$. The inverse gudermannian $\arg \mathrm{gd}(x)$ is given in Table 6 to 5D for $x = 0(0.01)1.57$, and to 3D for $x = 1.47\cdot(0.001)1.57$. Table 7 consists of 4D values of $P_n(x)$ for $n = 2(1)7$, $x = 0(0.01)1$. The normalized Laguerre polynomials $(1/n!)L_n(x)$ are tabulated to 4D in Table 8, corresponding to $n = 2(1)7$, $x = 0(0.1)10(0.2)20$. Finally, in Table 9 there are listed 4D values of the Hermite polynomials $(-1)^n h_n(x)$ for $n = 2(1)6$, $x = 0(0.01)4$.

In summary, this handbook constitutes the most complete compilation of formulas extant for the computation of the elementary mathematical functions, attractively arranged in a very convenient and accessible form. It can be recommended as a valuable accession to the libraries of all individuals and laboratories whose work involves numerical mathematics.

J. W. W.

1. C. Hastings, J. T. Hayward & J. P. Wong, *Approximations for Digital Computers*, Princeton Univ. Press, Princeton, N. J., 1955. See *MTAC*, v. 9, 1955, pp. 121–123, RMT **56**.
2. I. M. Ryshik & I. S. Gradstein, *Tables of Series, Products, and Integrals*, VEB Deutscher Verlag der Wissenschaften, Berlin, 1957. See *Math. Comp.*, v. 14, 1960, pp. 381–382, RMT **69**.

65[C, D].—L. A. Lyusternik, Editor, *Ten-Decimal Tables of the Logarithms of Complex Numbers and for the Transformation from Cartesian to Polar Coordinates*, Pergamon Press, New York, 1965, ix + 110 pp., 25 cm. Price $7.50.

This set of tables, constituting Volume 33 of the Mathematical Tables Series of Pergamon Press, is a reprint, with a translation by D. E. Brown, of *Desi͡atiznachnye tablit͡sy logarifmov kompleksnykh chisel i perekhoda ot Dekartovykh koordinat k*

poliarnym, originally published in Moscow in 1952 by the Academy of Sciences of the USSR and reviewed in this journal [1].

The following four functions are herein tabulated to 10D, each at interval 0.001, together with first and second differences:

$$\ln x,\ 1 \leqq x < 10; \qquad \tfrac{1}{2}\ln(1 + x^2),\ 0 \leqq x \leqq 1;$$

$$\arctan x,\ 0 \leqq x \leqq 1; \quad \text{and} \quad (1 + x^2)^{1/2}, \qquad 0 \leqq x \leqq 1.$$

A supplementary loose sheet contains: a 4D table of $x(1 - x)/2$ for $x = 0(0.001)0.5$ to facilitate quadratic interpolation; a 12D table of $\ln 10^n$ for $n = 1(1)25$; and 10D values of $\ln(-1)$ and $\ln i$.

Regrettably no attempt appears to have been made to correct in this reprint the 16 known errors [2] in the original tables. This unfortunately common practice of reprinting mathematical tables without proper attention to previously published errata cannot be condoned.

A further adverse criticism is the complete absence of any bibliographic references. A good list of such references is to be found in the fundamental double-entry conversion tables [3] of the Royal Society.

It seems appropriate here to point out that the first 90 pages of the total of 110 pages comprising the present tables are devoted to the tabulation of $\ln x$, which has been adequately tabulated to 16D—however, without differences—for an interval of 10^{-4} in the argument over the same range in the well-known NBS tables [4].

Despite these defects, the present tables constitute one of the most useful working tables for conversion from rectangular to polar coordinates.

J. W. W.

1. *MTAC*, v. 8, 1954, p. 149, RMT **1206.**
2. *MTAC*, v. 11, 1957, pp. 125–126, MTE **253.**
3. E. H. Neville, *Rectangular-Polar Conversion Tables*, Royal Society Mathematical Tables, v. 2, Cambridge Univ. Press, Cambridge, 1956. (*MTAC*, v. 11, 1957, p. 23, RMT **3.**)
4. NBS Applied Mathematics Series, No. 31, *Table of Natural Logarithms for Arguments between Zero and Five to Sixteen Decimal Places*, U. S. Government Printing Office, Washington, D. C., 1953. (*MTAC*, v. 8, 1954, p. 76, RMT **1167.**) NBS Applied Mathematics Series, No. 53, *Table of Natural Logarithms for Arguments between Five and Ten to Sixteen Decimal Places*, U. S. Government Printing Office, Washington, D. C., 1958. (*MTAC*, v. 12, 1958, pp. 220–221, RMT **86.**)

66[D].—W. K. Gardiner & E. B. Wright, *Five-Figure Table of the Functions* $1/(1 - \tan\theta)$ *and* $1/(1 - \cot\theta)$ *for the Range* $-90°(1')90°$, NRL Report 6362, U.S. Navy Research Laboratory, Washington, D.C., 1965, 50 pp., 26 cm. Price $2.00. Copies available from Clearinghouse for Federal Scientific and Technical Information (CFSTI), 5285 Port Royal Road, Springfield, Virginia 22151.

According to the introduction, this table was prepared to facilitate the application of the method of Ivory [1] to determine the Seebeck coefficients of various sample materials with respect to given thermocouple materials.

In their prefatory remarks the authors state that the tabular entries were obtained by rounding to 5S the corresponding results obtained on an IBM 1620 computer, using a word length of 13 decimal digits. Errors of transcription were minimized by printing the tables from punched-card computer output, followed by

photographic reproduction. The maximum error in any entry is claimed to be less than 0.50005 in units of the least significant figure.

The tabular data are arranged quadrantally for each sexagesimal minute in floating-point format, the values for each successive pair of degrees appearing on a single page. Thus, the tabulated values of the functions $1/(1 \pm \tan \theta)$ are read from the top to the bottom of each page, while those of the functions $1/(1 \pm \cot \theta)$ are read in the reverse direction, using the indicated complementary angles. In this manner the range of argument stated in the title is covered.

The table user is further assisted by a composite graph of the tabulated functions, which immediately precedes the tables.

This unique table should be a useful addition to the extensive tabular literature devoted to the trigonometric functions.

J. W. W.

1. J. E. Ivory, "Rapid method for measuring Seebeck coefficient as ΔT approaches zero," *Rev. Sci. Instr.*, v. 33, 1962, pp. 992–993.

67[D, P, R].—L. S. Khrenov, *Tables for Computing Elevations in Topographic Levelling*, translated by D. E. Brown, Pergamon Press, New York, 1964, vii + 200 pp., 26 cm. Price $10.00.

This translation of *Tablitsy dlya vyehisleniya prevyshnii*, originally published in Moscow, constitutes Volume 31 of the Pergamon Preess Mathematical Tables Series.

We are informed in the Preface that topographic levelling is used to determine elevations when the horizontal distances between points to be levelled are known either through direct measurement or by trigonometric calculations.

The main table (Table I) permits elevations h ($= d \tan \alpha$) in meters to be read off directly to 2 decimal places for the angle of elevation $\alpha = 1'(1')5°50'$ and the distance $d = 1(1)350$ meters. Flexibility in the use of the table is attained by virtue of the fact that d (in meters) and α (in minutes of arc) can be interchanged, with a resultant error in h not exceeding 0.047 m., as demonstrated on p. 193, in the section entitled "Theoretical Basis and Description of Tables I and II."

Furthermore, this range in d and in α can be doubled, as the author points out, by use of the formula $h = h' + \Delta h$, where $h' = 2d \tan \alpha/2$ and $\Delta h = \frac{1}{4} d\alpha^3 \sin^3 1'$. The necessary correction, Δh, is included in the right margins of Table I.

Table II simply consists of values of $1000 \tan \alpha$ to 5 significant figures for $\alpha = 11°(1')31°$.

Table III (Slope corrections ΔD for distances measured by tape) gives $\Delta D = 2D \sin^2 \alpha/2$, $D = 40(10)90, 200, 300$, 3 dec.; $D = 1000$, 2 dec.; $\alpha = 0°(10')30°$.

The following table (Reductions to horizontal of slopes measured by tape) gives $D \cos \alpha$, $D = 60(10)90, 400, 500$, 2 dec.; $D = 100(100)300$, 3 dec.; $\alpha = 1°(1°)30°$.

For the same range in α, Table V (Corrections D for slopes of distances measured by range-finder) gives $D \sin^2 \alpha$ to 1 dec. for $D = 100(100)900$.

The same ranges in α and D and the same precision appear in Table VI (Reductions to horizontal of slope distances measured by range-finder) which tabulates $D \cos^2 \alpha$.

Table VII (Corrections for refraction and curvature of the earth) gives integer values of d (in meters) corresponding to $f = 0.01(0.01)1.68$, also in meters.

The final table (Horizontal distances and gradients) consists of $\tan \alpha$, 5 dec.; $\cot \alpha$, 5 fig.; $h \cot \alpha$, 2 dec. for $h = 0.5$, 1, and 1 dec. for $h = 2$, 2.5, 5, 10, 20; $\alpha = 0°30'(30')10°(1°)30°$.

The user is well advised to study the illustrative examples in the use of the tables, which appear on pp. 196–198.

A supplementary loose sheet lists 25 known typographical errors in these tables.

This convenient set of tables should materially expedite the calculations involved in topographic levelling.

J. W. W.

68[F].—N. G. W. H. Beeger, *Tafel van den kleinsten factor der getallen van* 999 999 000–1000 119 120 *die niet deelbaar zijn door* 2, 3, 5, ms. of 57 pp. (unnumbered) deposited in the UMT File.

In accordance with the bequest of the late Dr. Beeger this factor table, together with the one described in the next review, has been placed in the file of unpublished mathematical tables that is maintained by this journal.

The format is that devised by L. Poletti in his *Neocribrum* and used subsequently by Dr. Beeger in his factor table for the eleventh million [1]. Accordingly, we find in the present table the least prime factor of all integers not divisible by 2, 3, or 5 in the range of the 120,120 numbers designated in the title.

The details of the construction of this factor table are set forth in English on a carefully handwritten introductory page.

Each page of the manuscript is devoted to the factors of numbers prime to 30 over an interval of 2310 consecutive integers, and the number of primes is subtotaled for each such interval and for each member of the reduced residue class modulo 30. The grand total of all primes listed is 5775.

Comparison of these data with the table of primes for the thousandth million by Baker & Gruenberger [2] revealed complete agreement in the 63 entries common to the two tables.

Information on the inside title page shows that Dr. Beeger compiled the present table between 19 December 1937 and 18 June 1938. It represents an impressive accomplishment for this well-known expert in the art of factoring large numbers.

J. W. W.

1. N. G. W. H. Beeger, *Table of the Least Factor of the Numbers that are not Divisible by* 2, 3, 5, *of the Eleventh Million*, ms. in UMT file. See *MTAC*, v. 10, 1956, pp. 36–37, RMT **5**. For a brief description of the *Neocribrum*, see *MTAC*, v. 4, 1950, pp. 145–146, RMT **768**.

2. C. L. Baker & F. J. Gruenberger, *Primes in the Thousandth Million*, deposited in UMT file. See *MTAC*, v. 12, 1958, p. 226, RMT **89**.

69[F].—N. G. W. H. Beeger, *Tafel van den kleinsten factor der getallen* 61 621 560–61 711 650 *die niet door* 2, 3, 5 *deelbaar zijn*, ms. of 54 pp. (unnumbered) deposited in UMT file.

This manuscript table consists of three fascicles, each giving the least prime factor of integers relatively prime to 30 over an interval of 30,030 consecutive numbers within the range stated in the title.

The format is identical with that used in other factor tables by the author (see the preceding review).

The explanatory text (in Italian) consists of four printed pages prepared by L. Poletti in 1921 to accompany the printed fascicles constituting his "Neocribrum."

Cumulative counts of the primes belonging to each member of the reduced residue class modulo 30 are shown on each page. The total number of primes in the table is given as 5005.

Dr. Beeger compiled the first fascicle between 1 January 1928 and 4 May 1929; the remainder of this unique table was completed on 1 January 1933.

Both this manuscript and the one described in the preceding review are listed in the *Guide* [1] of D. H. Lehmer.

J. W. W.

1. D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, National Research Council Bulletin 105, National Academy of Sciences, Washington, D. C., 1941 (reprinted 1961), pp. 39 and 86.

70[L, M].—L. N. OSIPOVA & S. A. TUMARKIN, *Tables for the Computation of Toroidal Shells*, P. Noordhoff, Ltd., Groningen, The Netherlands, 1965, 126 pp., 27 cm. Price $7.00.

This is a translation by Morris D. Friedman of *Tablitsy dlya rasheta toroobraznykh obolochek*, which was published by Akad Nauk SSSR in 1963 and previously reviewed in this journal (*Math. Comp.*, v. 18, 1964, pp. 677–678, RMT **94**). The highly decorative dust jacket of this translation shows a sea shell, which the publisher presumably associates with the subject matter.

J. W. W.

71[L, M, K].—I: F. D. MURNAGHAN & J. W. WRENCH, JR., *The Converging Factor for the Exponential Integral*, DTMB Report 1535, David Taylor Model Basin, Washington, 1963, ii + 103 pp., 26 cm.

II: F. D. MURNAGHAN, *Evaluation of the Probability Integral to High Precision*, DTMB Report 1861, David Taylor Model Basin, Washington, 1965, ii + 128 pp., 26 cm.

These two reports, herein referred to as I and II, concern the calculation, to high precision, of converging factors (c.f.'s) for the following functions:

$$\text{(A) } Ei(x) = \int_{-\infty}^{x} (e^t/t)\, dt, \qquad \text{(B) } -Ei(-x) = \int_{x}^{\infty} (e^{-t}/t)\, dt,$$

$$\text{(C) } T(x^{1/2}) = \frac{1}{2}\int_{x}^{\infty} e^{-t}t^{-1/2}\, dt.$$

Functions (A) and (B), the exponential integrals of positive and negative arguments, are treated in I; function (C), which is related to the probability integral, in II.

For a function $f(x)$ with asymptotic expansion $\sum_{r=0}^{\infty} a_r x^{-r}$, the c.f., $C_n(x)$ is given by

$$f(x) = \sum_{r=0}^{n-1} a_r x^{-r} + a_n x^{-n} C_n(x).$$

The c.f.'s $C_n(n + 1)$ and $C_n(n + \frac{1}{2})$ are tabulated to 45D in Case (A) and to 48–50D in Case (B), for $n = 4$ or $5(1)20$. In Case (C), $C_n(n + \frac{1}{2})$ and $C_n(n)$ are given to 63D for $n = 0(1)64$ and $n = 2(1)64$, respectively. In addition, auxiliary tables are presented which permit the evaluation to comparable accuracy of a c.f. when the argument is not an integer or half an integer. These take the form of tables of coefficients in the Taylor series for $C_n(n + 1 + h)$ (Cases (A) and (B)) or $C_n(n + \frac{1}{2} + h)$ (Case (C)). The same series, whose coefficients can be generated by recurrence, were in fact used to compute the key values $C_n(n + 1)$ (or $C_n(n + \frac{1}{2})$) for successively decreasing integer values of n. We note that when x is small a very large number of terms is needed; thus in Case (C), 326 terms in the expansion of $C_1(1.5 + h)$ are significant to 63D. A variant of the method, in which this difficulty is avoided by use of a variable interval in x, has been described by the reviewer [2] and applied to Case (B).

A starting value $C_n(n + 1)$ or $C_n(n + \frac{1}{2})$, for some large n, can be calculated from an expansion in inverse powers of n. The so-called Airey asymptotic series, applicable in Cases (B) and (C), is extended in II from the 23 terms listed by Airey [1] to as many as 67 terms. Various generalizations are also treated. In Case (A), 21 coefficients (of which only 4 were available previously) are derived by ingenious use of recurrence relations. As a by-product, 20 coefficients in Stirling's asymptotic series for the Gamma function are also obtained, 13 more than had previously been published. (The computational utility of the extended Stirling's series in the evaluation of factorials of integers has been noted in a recent review [3].) Furthermore, in I function (A) is tabulated to 44S for $x = 6(1)20$, and function (B) to 45D for $x = 6(1)21$.

These reports undoubtedly constitute an important contribution to the art of calculating special functions to high precision.

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1. J. R. Airey, "The 'converging factor' in asymptotic series and the calculation of Bessel, Laguerre and other functions," *Philos. Mag.*, (7), v. 24, 1937, pp. 521–552.
2. G. F. Miller, *Tables of Generalized Exponential Integrals*, Math. Tab. Nat. Phys. Lab., v. 3, 1960. (See *Math. Comp.*, v. 15, 1961, pp. 213–214, RMT **49**.)
3. See RMT **40**, *Math. Comp.*, v. 18, 1964, p. 326.

72[P].—James L. Marshall, *Introduction to Signal Theory*, International Textbook Co., Scranton, Pa., 1965, xv + 254 p., 24 cm. Price $9.00.

Professor Marshall's well-written monograph on the transformation of signals is intended to be an elementary textbook for advanced undergraduate students in the physical and engineering sciences or in applied mathematics. Following tradition, prime emphasis is given to electric systems rather than to nuclear, mechanical, chemical, or thermal processes. Graduated problems, together with some solutions, accompany each of the ten nonintroductory chapters of this handy book.

The author gives the rudiments of such standard topics as polynomial and trigonometric approximations to wave forms, use of Fourier and LaPlace transforms for the analysis of linear systems, complex Fourier series, reciprocal spreading relations, Parseval's theorem, and partial-fraction expansions of rational functions. A novel

feature of the book is a brief discussion of almost-periodic signals in the sense of H. Bohr [1], who is also known for work on Dirichlet series. Special attention is given to *finite*-series representations with *incommensurable* periods, although most of the results hold for infinite-series representations, too. Further, Parseval's theorem for almost-periodic functions is mentioned. Probability theory is considered in order to discuss the engineering uses of correlation functions and the notion of information. Finally, some basic nonlinear processes are discussed including such subjects as modulation and detection. A discussion or mention of Wiener's idea [2] of using shot noise as a standard signal for probing nonlinear systems would have added to the value of the monograph. Finally, if the author had treated Thévenin's theorem or Norton's theorem, the analysis of various interesting systems with a single nonlinear element would have been amenable.

The book should be especially useful for supplementary reading by undergraduates in electric science.

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1. H. BOHR, *Almost Periodic Functions*, Chelsea, New York, 1947.
2. N. WIENER, *Nonlinear Problems in Random Theory*, Technology Press and Wiley, New York, 1958.

73[P, X].—PATRICK BILLINGSLEY, *Ergodic Theory and Information*, John Wiley and Sons, Inc., New York, 1965, xiii + 193 pp., 23 cm. Price $8.50.

This beautiful little book, which grew out of a series of lectures given by the author in 1963, centers about the Kolmogorov-Sinai theory of entropy of measure-preserving transformations.

The book begins with a discussion of the ergodic theorem, whose significance is illustrated by a well-chosen battery of examples drawn from analysis and from probability theory; the notion of ergodicity is explained; criteria for ergodicity are derived, and applied to various of the examples considered. A succinct proof of the ergodic theorem by the method of Riesz follows. Various more subtle examples of measure-preserving transformations are then discussed, including the standard measure-preserving transformation associated with the continued fraction expansion of real numbers lying in the unit interval, whose analysis, in a truly elegant section of 10 pages, yields numerous interesting measure-theoretic results concerning this expansion.

The second main, and the central, chapter of the book begins with a general discussion of the problem of isomorphism for measure-preserving transformations, following upon which the entropy invariant is defined, its properties established, and, after a technical section on separable measure spaces, its invariance proved. The nonisomorphism of certain measure-preserving shift transformations follows. An additional section discusses the spectral type of a measure-preserving transformation as another isomorphism invariant, and exhibits transformations of different entropies with identical spectral types. A final section in this chapter surveys some additional results and describes a number of open problems.

Next follows a chapter of a preparatory character which sets forth the basic

properties of the notion of conditional probability and expectation, and which concludes with a discussion of the convergence properties of conditional expectations. The results obtained are then used to derive the convergence properties of the entropy function, the Shannon-McMillan-Brieman theorem on convergence almost everywhere of the averaged logarithm of a conditional probability, and the equipartition property of ergodic shifts. A final section of the fourth chapter presents some interesting results of Eggleston and the author, relating the entropy of certain shift transformations to the Hausdorff measure of Cantor-like perfect sets on the real axis.

In the fifth and final chapter, the general theory is applied to the concrete problems of coding and information transmission, out of which the theory originally developed. A noiseless channel is modelled as a measurable transformation in a clear and convincing way; the proof that the entropy of a source is a lower bound for the capacity of a channel capable of transmitting the source without loss of information follows almost immediately. The converse result, i.e., the existence of a channel of capacity equal to the source entropy capable of transmitting the source, is reduced to a hypothesis concerning standard shifts. The results are then extended to noisy channels. Finally, the abstract existence-of-a-channel result is reformulated in concrete terms as a statement about the existence of block codes for the transmission of data from a given source, which is, of course, the form in which the result is of greatest interest to the practicing communications engineer.

The exposition is consistently crisp, succinct, unpretentious, and maximally clear. Professor Billingsley's work may be recommended not only to mathematicians wishing to become familiar with the interesting advances in ergodic theory on which he reports, and to communications scientists desiring insight into the theoretical foundations of information-rate theory, but also to potential authors, who will see in it an inspiring example of what the short survey monograph can be.

D. S.

74[P, X].—Raymond W. Southworth & Samuel L. Deleeuw, *Digital Computation and Numerical Methods*, McGraw-Hill Book Co., Inc., New York, 1965, xiv + 508 pp., 23 cm. Price $11.75.

According to the preface, "The aims of this course are (1) to introduce the student to numerical methods, as applied to the analysis and solution of engineering problems and (2) to develop enough facility in the programming of computers to allow him to solve problems on a digital computer." With this purpose in view the authors have divided the book into two main sections: a programming section comprising about a third of the book and a numerical analysis section comprising the remaining two thirds. Most chapters end with illustrative examples taken from various fields of engineering in addition to a large selection of problems for the student.

The section of the book devoted to programming covers flow charting and FORTRAN (FORTRAN IV) programming. The book also contains a chapter on machine language programming which appears to be too terse to eliminate the students' "black box" view of the computer. The chapter could have been deleted without damage to the book as a whole.

The scope of the numerical analysis section may be indicated by simply listing

the chapter headings: Rounding and Truncation Errors, Roots of Equations, Simultaneous Linear Equations, Interpolation, Numerical Differentiation and Integration, Taylor's Series, Numerical Solution of Ordinary Differential Equations, and Empirical Formulas and Approximation. In general, the technique the authors have adopted is to describe a numerical analysis procedure and then to follow this procedure with a flow chart of a possible algorithm. The actual mathematics is seldom justified except in the most intuitive way.

On the whole, the book seems to be well written and much of the included material would be of value to the beginning engineering students.

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75[W].—CHARLES CHRISTENSON, *Strategic Aspects of Competitive Bidding for Corporate Securities*, Division of Research, Harvard Business School, Boston, Mass., 1965, x + 116 pp., 26 cm. Price $6.00.

A syndicate preparing a bid to underwrite a bond issue must usually specify three quantities in order to prepare a responsive, and responsible, bid. The quantities are the offering price which will be charged the public, the coupon rate which the issuer must pay during the life of the bond, and the proceeds to the issuer which will be paid by the syndicate for the securities.

This monograph provides a complete analysis of the interplay of forces between the syndicate and its competitors as well as of the factors which affect the relationship of members of the syndicate to each other and to the public.

The scope and setting of the problem is described in detail with the help of a case study. The exposition is sufficient to set the stage for any neophyte to the bond market if he has any degree of mathematical sophistication.

The pricing problem is considered in a chapter devoted to this decision process, but greatest attention is paid to the bidding problem, consisting of setting the coupon rate and the proceeds. The bidding problem is approached via game theory in one chapter, assuming perfect rationality among competitors who seek to adopt a strategy to maximize their return. In a Bayes approach, to which another chapter is devoted, statistical decision theory is employed to make a choice under uncertainty. Considerable new and novel material is to be found in these sections of the book.

Another interesting and, to the financial world, novel finding is one that suggests it may be more profitable to the syndicate to hold the bonds in inventory for a while instead of seeking an immediate total disposal.

It would be interesting to apply some of the author's precepts to a recent British underwriting in which the bid on some Imperial Chemical Industries bonds was so high—and hence disadvantageous to ICI—that the public subscription ran to many times the available number of securities.

As one of the "Studies in Managerial Economics," the book is attractively printed with a complete bibliography, but without an index. Misprints noted were few: "Player 1" should be "Player 0" on the sixth line from the bottom of p. 58; the equation reference in the text in the middle of p. 73 should be to "(6-1)" rather than

"(5-1)"; no difficulty will be found in reading "manager" for "manaser" on the fifth line of p. 112.

The author does not claim to have written the definitive work; avenues for further research are suggested. Other approaches have been proposed in the technical literature. Recently, Cohen and Hammer (*Management Science* 12:1, pp. 68–82) proposed a linear programming formulation of the scheduling interest coupons. In the meantime, this tour-de-force will serve as the most complete overall analysis of the problem available.

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76[W, Z].—JAMES MARTIN, *Programming Real-Time Computer Systems*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965, xii + 386 pp., 24 cm. Price $11.75.

This reviewer objects to the current ambiguous use of the term "real-time," which is continued in this book. A distinction ought to be made, once and for all, between rapid systems response for comfort and convenience, and response required by the physical process being monitored or controlled. The most dramatic contrast between these two different kinds of systems requirements, both called "real-time" by the author, may be found on p. 22: "It may be installed to give speedier action . . . for example, bank customers queueing to draw cash in their lunch hour, or two airplanes on a possible collision course."

The reviewer also found somewhat misleading the use of the term "programming" in the title of the book.

The first third of this book discusses, on a very elementary level, the advantages and disadvantages of "real-time" systems, their history, and some examples of their implementation.

Beginning in the middle of the book, the author proceeds to more serious discussions. He addresses himself to techniques found essential or useful, with enough detail to satisfy managers of programming teams or of computer installations.

In the final third of the book, Mr. Martin draws on extensive experience with the sample systems described to present a convincing chronicle of the many pitfalls that await the naive traveler down the path of multiprogrammed, multiprocess, on-line system design.

The book is highly recommended for those in management contemplating the use, creation, or installation of new systems in the true "real-time," the "pseudo-real-time," or other multi-access, quick-response modes of operation.

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77[X].—V. K. SAUL'YEV, *Integration of Equations of Parabolic Type by the Method of Nets*, The Macmillan Company, New York, 1964, xvii + 346 pp., 23 cm. Price $12.00.

This is a translation by G. E. Tee of the original Russian monograph, which

appeared in 1960. Written for the user rather than the specialist, it provides a well documented and fairly complete survey of finite-difference methods for the numerical solution of parabolic partial differential equations. The notation and style are classical and no highly specialized mathematical knowledge is required for its reading. Although it is certainly a practical book, it is definitely not a cookbook. However, in order to make the book accessible to a wide range of scientists and engineers, the author does avoid a rigorous mathematical formulation and has omitted the details of many proofs.

The book is divided into two parts. Part I is devoted to the construction, stability and convergence of various difference schemes for parabolic operators. Included are all of the classical difference schemes and several recent ones, many of which have appeared before only in the Russian literature. The second part describes methods for the practical solution of systems of equations arising from the implicit parabolic difference equations considered in Part I. Included are direct methods, simple and block iterative methods, variational methods, and Chebyshev semi-iterative methods.

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78[X].—FRANK G. LETHER, *Abscissas for Chebyshev Quadrature*, Computer Center, University of Utah, Salt Lake City, Utah. Ms. of six typewritten pages deposited in the UMT File.

Values of the abscissas for the Chebyshev quadrature formula with unit weight function are herein tabulated to 95D for $n = 2(1)7$ and 9. The author in his introduction cites Bernstein's theorem [1] that for no other values of n greater than 1 are all the abscissas real.

The abscissas were calculated as the zeros of the associated polynomials by means of the Newton algorithm, using as initial values the 10D approximations of Salzer [2]. The underlying computations were carried to 100D on an IBM 7044, using multiple-precision arithmetic.

The careful overall checks applied to the final results inspire confidence in the accuracy of these extended tabular values.

J. W. W.

1. S. N. BERNSTEIN, "Sur les formules de quadrature de Cotes et Tchebycheff," *Dokl. Akad. Nauk SSSR*, v. 14, 1937, pp. 323–326.

2. H. E. SALZER, "Tables for facilitating the use of Chebyshev's quadrature formula," *J. Math. Phys.*, v. 26, 1947, pp. 191–194.

79[X].—WALTER JENNINGS, *First Course in Numerical Methods*, The Macmillan Company, New York, 1964, xiv and 233 pp. 24 cm. Price $7.50.

The purpose of this book is to serve as an introduction to Numerical Analysis for an undergraduate student of science or engineering, presupposing only the calculus and differential equations. According to the author, the book is intended to present Numerical Analysis "in breadth rather than depth, without being superficial," to lay an adequate groundwork for the study of the more sophisticated problems of Numerical Analysis, and to motivate students to continue their studies

in this field. By presenting a not overly long and very readable discussion centered about the basic problems of approximation, solutions of polynomial equations and systems of equations, and the numerical solution of ordinary differential equations, the author attains all of his objectives.

The book consists of 22 relatively short chapters. The first twelve chapters are concerned with approximation of functions and the solution of equations; included herein are discussions of Chebyshev and Legendre polynomials, approximation in the square integral sense, interpolation, and the methods of Newton-Raphson and Bernoulli. The next five chapters deal with numerical differentiation and integration. In the last five chapters are introduced some methods for the solution of ordinary differential equations, systems of linear equations, matrix inversion and eigenvalue problems. At the end of the book is a set of five appendices, containing statements of some basic results of calculus and linear algebra, to which the student may refer.

The main emphasis of the book is on the application of the methods presented to specific problems. The text is studded with numerical examples, most of which are well suited for a desk calculator; in addition each chapter contains a nice assortment of problems (and answers), as well as a bibliography containing, to a large measure, references to recent literature which are suitable for a good undergraduate student.

The main shortcomings of the book lie in the presence of a number of misprints and an occasional tendency to examine some ideas too briefly.

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80[X].—S. B. NORKIN, Editor, *The Elements of Computational Mathematics*, Pergamon Press, New York, 1965, xiii + 192 pp., 21 cm. Price $6.00.

This is an excellent and unobtrusive translation by G. J. Tee of a short, elementary textbook on computational mathematics. The Russian original (1960) was designed for use by correspondence students taking basic courses in higher mathematics, and for engineering students as a supplementary course in computational techniques.

It consists of seven chapters: on computation with approximate numbers by I. A. Zhabin, on the construction of tables by M. I. Rozental′, on the approximate solution of equations by D. P. Polozkov, on systems of linear equations by Kh. R. Suleimanova, on interpolation polynomials by S. B. Norkin, on the approximate computation of integrals by R. Ya. Berri, and on the approximate integration of ordinary differential equations again by S. B. Norkin.

The contents of the chapters were considered by the authors jointly, and the finished product was reviewed by the faculties of a number of institutions of higher learning in Moscow.

As one might expect, the standard of presentation is quite exceptionally high: the material is carefully selected, the exercises strategically placed, and the successive chapters disciplined into a balanced and harmonious whole.

If one feels called upon to offer adverse criticism, it is that the publication of this book would not have been out of place fifty years ago. Certainly all the methods dealt with are older than this, and there is a preoccupation with such topics as the

growth of errors in a difference table, work sheets for use when solving linear equations, and various other matters which recall the sort of hand-computing drudgery that most of us would prefer to forget.

It is not assumed that the student will have access to a digital computer. Moreover the authors state: "The use of fast machines for the comparatively small calculations most often arising in engineering and industry is not advantageous, and sometimes it is actually inconvenient."

Clearly the authors have a very good idea of the public to which they address their book. Moreover they have a far more intimate knowledge of the sort of computing facilities generally available in the Soviet Union than do most of the readers of this review. When preparing their book, the authors made certain assumptions and acted consistently upon them. However, any person in the United States who made similar assumptions and produced a similar textbook would, in the opinion of the reviewer, be old-fashioned.

The book is pleasantly produced.

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81[X].—PATRICIA C. STAMPER, *Table of Gregory Coefficients*, The Johns Hopkins University, Applied Physics Laboratory, Silver Spring, Md. Ms. of two typewritten pages deposited in the UMT File.

Herein are tabulated in floating-point form to 15S (generally unrounded) the first 50 coefficients of the Gregory integration formula, computed in double precision on an IBM 7094 by use of the recurrence formula

$$G_n = \sum_{i=1}^{n} (-1)^{i+1} G_{n-i}/(i+1) + (-1)^{n+1} n/2(n+1)(n+2)$$

with $G_0 = 0$. This yields $G_1 = \frac{1}{12}$, $G_2 = -\frac{1}{24}$, $\cdots$, in contradistinction to the values $G_1 = \frac{1}{2}$, $G_2 = -\frac{1}{12}$, $G_3 = \frac{1}{24}$, with reversed signs, which are found from the conventional recurrence relation for these numbers, which has $(-1)^{n+1}/(n+1)$ for the second term on the right.

It seems appropriate to note here that the first 20 of these numbers have been computed in rational form by Lowan and Salzer [1]. Furthermore, their asymptotic character has been most recently investigated by Davis [2], who refers to them as "logarithmic numbers" because of their identification with the coefficients in the Maclaurin series for $x/\ln(1+x)$.

The present manuscript table appears to be the most extensive one of these coefficients extant.

J. W. W.

1. A. N. LOWAN & H. E. SALZER, "Tables of coefficients in numerical integration formulae," *J. Math. Phys.*, v. 22, 1943, pp. 49–50.
2. H. T. DAVIS, "The approximation of logarithmic numbers," *Amer. Math. Monthly*, v. 64, 1957, pp. 11–18.

EDITORIAL NOTE: It may be noted that no table of these coefficients, which are quite important (at least for small n), appears in the celebrated, and otherwise quite complete, NBS *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables.*

82[Z].—CLARENCE B. GERMAIN, *Programming the IBM 1620*, Second Edition, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965, 191 pp., 28 cm. Price $4.95 (Paperbound).

This publication presents a comprehensive treatment of all aspects of programming the IBM 1620 and apart from some of the exercises, no knowledge of college mathematics is assumed. Chapter I serves as an introduction to the uninitiated. Chapter II presents a subset of machine instructions. A description of the operation of the machine in Chapter III is followed by a general discussion of programming (flowcharts, round-off errors, etc.) and an introduction to FORTRAN in Chapters IV and V respectively.

The author permits the student to communicate with the machine as soon as a minimum of instruction is presented. Upon completion of Chapter V, it is possible to program and run simple problems utilizing either absolute or FORTRAN coding. Subsequently, in Chapters VI and VII additional FORTRAN statements are discussed; in Chapters VIII and IX, additional machine instructions (including address modification) are presented; advanced operating techniques are discussed in Chapter X. The remaining chapters and appendices discuss disk storage, SPS (assembly language), and miscellaneous odds and ends. The problem sets at the end of each chapter are quite challenging and make a complete check of the level of competence reached. Numerous examples are given to clarify the functioning of the hardware for various instructions (TFL, FSL, etc.).

Two criticisms should be made at this point. One concerns the substantial number of misprints which abound throughout the text. In addition, there exists no glossary of terms, a deficiency shared by other manuals of this type. However, on the whole, the book is well organized and complete.

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83[Z].—JAMES T. GOLDEN, *FORTRAN IV—Programming and Computing*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965, 270 pp., 26 cm. Price $6.00 (Paperbound).

"This book is written as a college-level introduction to computing and programming in FORTRAN IV. Its two major objectives are to develop the reader's ability to generate algorithms and to guide him in creating strategies for problem solving on a digital computer. This book may be used as either a self-study text or as a text on programming to supplement a numerical methods course. . . ." This quotation (written by the scientific marketing manager of IBM Corporation) is taken from the book.

During the last few months a number of books, having the same scope and intention, have appeared. One hesitates to say that this is the best of them. In venturing this opinion I do not wish to impugn the author's technical competence: he clearly has a vast experience in programming, and all that he has to say upon general matters is well considered and eminently wise. But the balance which is struck in this book has clearly been chosen in such a way that the computational exercises serve only as a vehicle to promote the programming language. No normally con-

stituted college student will be prepared to digest such a large and detailed text devoted to a subject which represents a relatively minor part of his curriculum. Furthermore the text is at the moment quite unsuitable for use in conjunction with a course of lectures. It would, in the reviewer's opinion, have to be reorganized considerably for this purpose.

Perhaps the most appropriate rôle which might be allocated to this book is that of a text and reference work for relatively junior staff serving a computing installation which uses FORTRAN IV as its principal machine language.

The book is rather shoddily put together; for example, in the reviewer's copy two pages are interchanged.

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84[Z, X].—INSTITUTO GULBENKIAN DE CIENCIA, Centro de Calculo Cientifico, *Sistema de Programacao, Fortran 2*, Lisbon, 1964, 150 pp., 25 cm. Price Escudos 65 (Paperbound).

In the recent past, English literature has been graced by a number of texts which expound the programming language FORTRAN and show, by means of worked examples, how this language may be used in scientific computation. We are now offered such a book in Portuguese.

The book is brilliantly successful. In the first part (seventy-eight pages) we are introduced to the elements of the language and shown the various constructions of which it is capable; in the second part (seventy pages) some thirteen complete programs are given, together with specimen numerical results. In order to impart to the reader some idea of the scope of this text, let me list the subjects of some of these programs: Euler's transformation of a series, integration by Simpson's rule, the Runge-Kutta method, inversion of matrices by Jordan's method, Gauss-Seidel iteration, largest eigenvalue of a matrix, summing a Chebyshev series by Clenshaw's method, curve fitting by means of orthogonal polynomials, calculation of the gamma-function, and Bairstow's method for finding the roots of a polynomial.

The entire book makes a most pleasing impression. The material is elegantly subdivided, the examples are skillfully placed, and the exposition has been handled with a competence of the highest order.

Undoubtedly this book will do much to promote scientific computation in Portuguese speaking countries.

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