REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in braces are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

57[1, 12].—W. W. YOUDEN, *Computer Literature Bibliography*, Vol. 2, 1964–1967, National Bureau of Standards, Special Publication 309, 1968, 381 pages, 29 cm. Price $5.00. Order from Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402.

This is an extension of [1] and does for the years 1964–1967 what [1] did for the years 1946–1963. It is very similar in general format to [1]; the reader is referred to the longer review of this earlier volume for details. The literature listed is not all the literature on these subjects in those years, but that contained in 17 journals, 20 books, and 43 conference proceedings. In all, there are about 5200 references. This is a valuable reference book for libraries.

D.S.

1. W. W. YOUDEN, *ComputerLiterature Bibliography* 1946–1963, National Bureau of Standards, 1965. Reviewed in *Math. Comp.*, v. 19, 1965, p. 704, RMT **140**.

58[2.05, 2.10, 2.15, 2.20, 3, 4, 5, 6, 7, 8, 13.15].—LOUIS G. KELLY, *Handbook of Numerical Methods and Applications*, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1967, xiv + 354 pp., 24 cm. Price $14.50.

A compilation of numerical methods and selected topics of interest to scientists and engineers, the book is addressed to a wide computing clientele and should be useful to some for general orientation and references to source material. List of chapter headings: 1. Introduction, 2. Finite and divided differences, 3. Basic interpolating and approximating polynomials, 4. Differentiation and integration, 5. Curve fitting and data smoothing, 6. Nonlinear algebraic equations, 7. Matrix algebra and operations, 8. Linear algebraic equations, 9. Eigenvalues and eigenvectors, 10. Scaling of matrices, 11. Introduction to complex variables, 12. Introduction to the Laplace transform, 13. Difference equations, 14. Ordinary differential equations, 15. Transfer function computations, 16. Partial differential equations, 17. Harmonic analysis, 18. Special functions and integrals, 19. Sampled data and digital filtering, 20. Numerical solution of integral equations, 21. Numerical solution of vibration problems, 22. Padé approximation to a function, 23. Gram-Schmidt orthogonalization procedure, 24. Computer methods of function minimization, 25. Elementary Statistics.

W. G.

59[2.05, 3, 4, 5, 6, 7, 9, 10].—L. COLLATZ, G. MEINARDUS & H. UNGER, Editors, *Funktionalanalysis, Approximationstheorie, Numerische Mathematik*, Birkhäuser Verlag, Basel, 1967, 232 pp., 25 cm. Price: *sF* 29.00.

Nestled on a picturesque hillside near Oberwolfach, in the Black Forest, is the Mathematical Research Institute, which serves the German mathematical community, and its guests, as a meeting place and retreat. Frequent symposia are held in this congenial spot, and the present volume consists of summaries of the lectures delivered at two such gatherings in 1965.

The topic of the first collection is numerical problems in Approximation Theory, and it consists of 11 fairly full presentations, most of which deal with some aspect of Chebyshev approximation. The second symposium is entitled "Methods of Functional Analysis in Numerical Mathematics," and 16 talks are represented, some by brief abstracts.

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60[2.10].—BRUCE S. BERGER, ROBERT DANSON & ROBERT CARPENTER, **A.** *Tables of Zeros and Weights for Gauss-Laguerre Quadrature to* 24S *for* $N = 400, 500$, *and* 600, ms. of 4 typewritten pp. + 12 computer sheets (reduced), 28 cm. **B.** *Tables of Zeros and Weights for Gauss-Laguerre Quadrature to* 23S *for* $N = 700, 800$, *and* 900, ms. of 4 typewritten pp. + 18 computer sheets (reduced), 28 cm. Copies deposited in the UMT file; additional copies obtainable from Professor Berger, Department of Mechanical Engineering, The University of Maryland, College Park, Md. 20742.

These two manuscript tables (prepared in November 1968 and January 1969, respectively) represent an impressive extension of the authors' 24S table [1] of zeros and weights for Gauss-Laguerre quadrature corresponding to $N = 100, 150$, 200, and 300.

As in the preparation of the earlier table, the present tables were calculated on a CDC 6600 system, using double-precision floating-point operations accurate to approximately 30S. Moreover, the same over-all checks have been applied to the computed values.

The senior author has recently applied these extensive tables to calculations relating to a problem in acoustics [2].

J. W. W.

1. BRUCE S. BERGER & ROBERT DANSON, *Tables of Zeros and Weights for Gauss-Laguerre Quadrature*, ms. deposited in the UMT file. (See *Math. Comp.*, v. 22, 1968, pp. 458–459, RMT **40**.)
2. BRUCE S. BERGER, "Dynamic response of an infinite cylindrical shell in an acoustic medium," *J. Appl. Mech.*, v. 36, 1969, pp. 342–345.

61[2.10].—LEE M. HUBBELL and RALPH E. CHRISTOFFERSEN, *Tabulation of a New Set of Orthogonal Polynomials for Numerical Integration*, ms. of 8 typewritten pages & 18 typewritten pages of tables & 5 pages of figures, deposited in the UMT file.

The authors consider the orthogonal polynomials associated with the quadrature problem

$$(1) \qquad \int_1^\infty \frac{e^{-x}}{x^k} f(x)dx = \sum_{i=0}^{j} w_{i,j}^{(k)} f(x_{i,j}^{(k)}) + R_j^{(k)} f ,$$

where $R_j^{(k)} f = 0$ if $f(x)$ is a polynomial of degree $2j + 1$ or less. The abscissas $x_{i,m}^{(k)}$ are the zeros of a polynomial

$$\phi_m{}^{(k)}(x) = \sum_{j=0}^{m} a_{m,j}^{(k)} x^j \tag{2}$$

which is one of a set of orthogonal polynomials satisfying the orthonormal conditions

$$\int_1^\infty \frac{e^{-x}}{x^k}\, \phi_i{}^{(k)}(x)\phi_j{}^{(k)}(x)dx = \delta_{i,j}\,. \tag{3}$$

The method of calculation is described. As is well known such a calculation is highly unstable with respect to amplification of round-off error. An iterative refinement technique is used. The calculation is described for the more general case, associated with

$$\int_1^\infty \frac{e^{-\alpha x}}{x^k}\, f(x)dx\,.$$

The tabulation covers only the case $\alpha = 1$, and seems to have been carried out using double-precision (twenty significant figure) accuracy. No statement is made about the actual computer or the appropriate machine accuracy parameter.

The tabulated quantities include

$$a_{i,j}^{(k)},\ x_{i,j}^{(k)},\ w_{i,j}^{(k)},\ \tilde{\delta}_{i,j}^{(k)}\,, \quad i \leqq j \leqq 10, k = 1, 2, 3, 4, 5\,.$$

Here the first three are defined above. The fourth, $\tilde{\delta}_{i,j}^{(k)}$ are the actual approximations to $\delta_{i,j}$ obtained from (3) by expanding the polynomials according to (2) and using the known moments to evaluate the right-hand side of (3).

Finally, for $k = 1$ only, the approximations to the exact integral (1) using the quadrature rule on the right-hand side of (1) for each of the four functions

$$f(x) = e^{-x}, x^{18}, x \ln x \quad \text{and} \quad x \sin x$$

are given.

Unfortunately there is some vagueness about the precise accuracy to which these calculations were carried out, but the presumption is that double-precision function values were used. The reviewer is very happy to note that this sort of information is being brought to the attention of a possible user. But in many applications the user will have at his disposal only single-precision function values. He might like to know, for example, how his results will be affected if he uses single-precision weights and abscissas. It seems to the reviewer that a single element of information, the value of the machine accuracy parameter, is vital to obtain a complete picture.

The method described involves the prior calculation of the moments, a well-known source of error. Recently general procedures which do not involve the calculation of moments have been constructed by Gautschi [1], [2] and by Golub and Welsch [3].

J. N. L.

1. W. Gautschi, "Construction of Gauss-Christoffel Quadrature Formulas," *Math. Comp.*, v. 22, 1968, pp. 251–270.

2. W. Gautschi, "Algorithm 331: Gaussian Quadrature Formulas," *Comm. ACM*, v. 11, 1968, pp. 432–436.

3. G. H. Golub & J. M. Welsch, "Calculation of Gauss Quadrature Rules," *Math. Comp.*, v. 23, 1969, pp. 221–230.

62[2.10].—T. N. L. PATTERSON, "Gaussian Formula for the Calculation of Repeated Integrals," tables appearing in the microfiche section of this issue.

Abscissas and weights of the r-point Gaussian quadrature formula for the integral

$$(n-1)!\int_{-1}^{1} dx_1 \int_0^{x_1} dx_2 \int_0^{x_2} dx_3 \cdots \int_0^{x_{n-1}} f(x_n)dx_n \equiv \int_{-1}^{1} w(x)f(x)dx$$

are tabulated to 20 significant figures for $n = 2, 4$; $r = 2(2)16$ and $n = 3, 5$; $r = 2(1)16$. The resulting formula is exact if $f(x)$ is a polynomial of degree $2r - 1$ (or $2r$ when r is even). The weighting function is

$$w(x) = (-1)^n w(-x) = (1-x)^{n-1}, \qquad 0 < x \leqq 1 .$$

This has discontinuous even (odd) derivatives at $x = 0$.

A normal approach to such an integration might be to divide the interval into two sections and use either the Gauss-Legendre formula or better still the appropriate Gauss-Jacobi formula in each section separately. However, the existence of these tables does allow the interval $[-1, 1]$ to be treated as a whole.

J. N. L.

63[2.20, 7].—HENRY E. FETTIS & JAMES C. CASLIN, *More Zeros of Bessel Function Cross Products*, Report ARL 68–0209, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1968, v + 56 pp., 28 cm. [Released to the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.]

In this compact report the authors continue their previous 10D tabulation [1] of the roots of the equations (a) $J_0(\alpha)Y_0(k\alpha) = Y_0(\alpha)J_0(k\alpha)$, (b) $J_1(\alpha)Y_1(k\alpha) = Y_1(\alpha)J_1(k\alpha)$, and (c) $J_0(\alpha)Y_1(k\alpha) = Y_0(\alpha)J_1(k\alpha)$.

These new tables give the roots α_n and the corresponding normalized roots γ_n of all three equations, for $n = 5(1)10$ and $k = 0.001(0.001)0.3$. For equation (c) these roots are also tabulated corresponding to $k^{-1} = 0.001(0.001)0.3$.

The normalized roots are related to the others by the equation $\gamma_n = (1-k)\alpha_n/(n\pi)$ for (a) and (b), and by $\gamma_n = |k-1|\alpha_n/[(n-\frac{1}{2})\pi]$ for (c). The authors note the properties $\lim_{k\to 1}\gamma_n = 1$ (all n) and $\lim_{n\to\infty}\gamma_n = 1$ (all k).

For examples of applications of these tables, as well as details of their calculation, the user should consult the earlier report [1].

J. W. W.

1. HENRY E. FETTIS & JAMES C. CASLIN, *An Extended Table of Zeros of Cross Products of Bessel Functions*, Report ARL 66–0023, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1966. (See *Math. Comp.*, v. 21, 1967, pp. 507–508, RMT **64**.)

64[2.40, 7, 10].—JOHN RIORDAN, *Combinatorial Identities*, John Wiley & Sons, Inc., New York, 1968, xii + 256 pp., 23 cm. Price $15.00.

This volume deals in the main with identities involving the binomial coefficients. As is well known, binomial coefficients are the simplest combinatorial entities and arise quite naturally in a wide variety of combinatorial problems.

According to the preface, "the object of this book is to present identities in mathematical setting that provide areas of order and coherence." However, the author remarks that his "initial hope that some of this order and coherence would be acquired by the identities themselves now seems illusory, $\cdots$, the age-old dream of putting order in chaos is doomed to failure." Elsewhere in the preface, the author states that the tools of combinatorialists are "recurrence, generating functions and such transformations as the Vandermonde convolution; others, to my horror, use contour integrals, differential equations and other resources of mathematical analysis." An examination of the preface reveals a confusion which is reflected in the volume itself. The point is this: A good portion of the book leans heavily on well-known results for the Gaussian hypergeometric functions ${}_2F_1(a; b; c; z)$ and its natural extensions to hypergeometric functions of higher order, for example, the ${}_3F_2$. However, this well-known literature appears to have escaped the attention of the author. Thus, such references as the volumes by A. Erdélyi et al., *Higher Transcendental Functions*, Vols. 1–3, McGraw-Hill, 1953–1955, and E. D. Rainville, *Special Functions*, Macmillan Company, 1960, are not mentioned. Consequently, many results in the first two chapters and elsewhere that could be stated once and for all are proved anew.

In illustration, Example 4 on page 5, the Vandermonde convolution formula on page 8, Example 6 on page 13; and Example 2 on page 53 are all special cases of the ${}_2F_1$ of unit argument. Again, Example 7 on page 13 is a special case of a ${}_2F_1$ with argument equal to one-half which arises from a quadratic transformation formula. On page 15, Eq. (10) is a special case of a terminating Saalschützian ${}_3F_2$ or unit argument. The same is true of the last expansion on page 16. This is a rather interesting case as the author relates that in 1954 Turan observed that the formula occurs without proof in a book by the Chinese mathematician Le-Jen Shoo which is dated 1867. Riordan states that Turan's paper created a "wave of interest that is reflected in the bibliography." Saalschütz's work is dated 1890. It appears to us that the first two chapters could have been considerably shortened and made more useful on appeal to the theory of special functions.

Chapter I is titled 'Recurrence', and in the main is concerned with basic properties of series involving binomial coefficients and which are of the type ${}_2F_1$ and ${}_3F_2$ as noted above. The chapter also deals with Abel's generalization of the binomial formula and multinomial Abel identities. Chapter II uses the results of Chapter I to derive inverse relations based on the hypergeometric results already noted, for example,

$$a_n = \sum_{k=0}^{n} (-)^k \binom{n}{k} b_k \quad \text{implies} \quad b_n = \sum_{k=0}^{n} (-)^k \binom{n}{k} a_k ,$$

and vice versa. Corresponding results associated with the name of Abel are taken up in Chapter III. 'Generating Functions' is the subject of Chapter IV. As is perhaps well known the application of such relations is useful to derive difference, differential and other properties of transcendentals. In this connection, the references by A. Erdélyi et al., and Rainville noted above contain much valuable information.

Chapter V takes up partition polynomials which are also called Bell polynomials and inverses with applications to number theory. The forward difference operators $x\Delta$, Δx, the analogous backward difference operators, the central difference operator, and the derivative operators xD and Dx are studied in Chapter VI.

Each chapter contains numerous examples and problems for the reader. Undoubtedly, these should be useful for self-study and to locate specific examples needed in a wide variety of problems.

Y. L. L.

65[4, 7, 8, 11, 13].—Murray R. Spiegel, *Mathematical Handbook of Formulas and Tables*, McGraw-Hill Book Co., New York, 1968, x + 271 pp., 28 cm. Price $3.95 (paperbound).

This relatively inexpensive compilation of mathematical formulas and tables is a recent addition to the popular Schaum's Outline Series of books mainly in mathematics and engineering.

The book is divided into two main parts. Part I (Formulas) consists of 41 sections, of which 39 present a total of 2309 formulas (supplemented by diagrams and graphs) selected from a wide range of topics in such fields as algebra, geometry, trigonometry, analytic geometry, calculus, differential equations, vector analysis, Fourier series, Fourier and Laplace transforms, special functions (gamma, beta, Bessel, Legendre, elliptic, and others), and probability distributions. The first and last sections of Part I consist, respectively, of a table of 27 frequently used mathematical constants (given to from 10S to 25S) and a useful table of conversion factors.

Part II (Tables) consists of 52 numerical tables, preceded by a set of sample problems illustrating their use. These tables, which generally range in precision from 3S to 7S, cover the standard elementary functions as well as a large number of the higher mathematical functions, including the gamma function, Bessel functions, exponential integral, sine and cosine integrals, Legendre polynomials, elliptic integrals, and the error function. Also included are tables for the calculation of compound interest and annuities, and a small table of random numbers. An appended index of special symbols and notations and a general index have also been included.

Despite the existence of several errors (listed elsewhere in this issue), this reviewer considers this attractively arranged and clearly printed book to be a valuable addition to the ever-increasing number of such handbooks.

J. W. W.

66[3, 8].—Peter Lancaster, *Theory of Matrices*, Academic Press, New York, 1969, xii + 316 pp., 24 cm. Price $11.00.

This book differs considerably in the material presented from most books on matrices and linear algebra and deserves wide adoption, especially in courses intended for students majoring in other areas who are interested primarily in applications. Nevertheless, only a few sections are devoted to applications as such, and then only in terms of their mathematical formulation with no discussion of the physics itself. Thus there are sections on small vibrations, differential equations, and Markoff chains.

After a rather standard introduction in the first two chapters, the third discusses the Courant-Fischer and related theorems; the Smith canonical form and the Frobenius and Jordan normal forms are developed in the next chapter; the fifth chapter is devoted to functions of matrices, and the sixth to norms; then comes a chapter on perturbation theory, one on direct products and stability, and, finally, a chapter on nonnegative matrices.

The treatment is lucid, and the only prerequisites are elementary algebra and calculus. Only the real and complex fields are considered. There are a reasonable number of examples and exercises, and about three or four references per chapter for supplementary reading. The book provides an excellent background in the subject for prospective numerical analysts, as well as to the many nonmathematicians who need to use matrices in their work.

A. S. H.

67[7].—Irwin Roman, *Extrema of Derivatives of* $J_0(x)$, ms. of nine typewritten sheets (dated Jan. 1964), deposited in the UMT file.

This manuscript table gives to 10S the critical points below 100 and the corresponding extrema of the first four derivatives of the Bessel function $J_0(x)$. Accuracy to within two units in the final figure is claimed. Also tabulated are the (rational) values of these derivatives for zero argument.

A six-page introduction sets forth a detailed description of the method used in computing the tabular values on a desk calculator. In particular, formulas are listed relating the derivatives of $J_0(x)$ to linear combinations of $J_0(x)$ and $J_1(x)$, with coefficients expressed as polynomials in $1/x$. The values of these Bessel functions required in the computation of the table were obtained by interpolation in the Harvard tables [1].

A concluding page lists the six basic references cited in the introductory text.

As implied by the author, this unique table constitutes a natural supplement to published tables of extrema of $J_0(x)$, in particular, that compiled by the Mathematical Tables Committee of the British Association for the Advancement of Science [2].

J. W. W.

1. Harvard University, Computation Laboratory, *Annals*, v. 3: *Tables of the Bessel Functions of the First Kind of Orders Zero and One*, Harvard University Press, Cambridge, Mass., 1947. (See *MTAC*, v. 2, 1947, pp. 261–262, RMT **380**.)

2. British Association for the Advancement of Science, *Mathematical Tables*, v. 6: *Bessel Functions, Part I, Functions of Orders Zero and Unity*, Cambridge University Press, Cambridge, England, 1937. (See *MTAC*, v. 1, 1945, pp. 361–363, RMT **179**.)

68[7].—T. S. Murty & J. D. Taylor, *Zeros and Bend Points of the Legendre Function of the First Kind for Fractional Orders*, Oceanographic Research, Marine Sciences Branch, Department of Energy, Mines and Resources, Ottawa, Canada. Deposited in UMT file.

Let

$$P_\nu(x) = {}_2F_1\left(-\nu, 1 + \nu; \frac{1}{2}; \frac{1-x}{2}\right)$$

$$P_\nu(x_j) = 0\,, \qquad P_\nu'(y_j) = 0\,.$$

The following are tabulated:

$$P_\nu(x):\nu = -0.5(0.02)0.5, x = -1.0(0.01)1.0; 7\text{D},$$

$$P_\nu(x):\nu = -0.5(0.1)8.5, x = -1.0(0.02)1.0; 7\text{D},$$

$$x_j, y_j, P_\nu'(x_j), P_\nu(y_j), \nu = 0.1(0.1)8.5, 7\text{D}.$$

The values y_j are called bend points. Some related tables are (1) Gray, M. C., "Legendre functions of fractional order," *Quart. Appl. Math.*, v. 11, 1953, pp. 311–318, also *MTAC*, v. 8, 1954, p. 24; (2) Ben Daniel, D. J. and Carr, W. E., *Tables of Solutions of Legendre's Equation for Indices of Nonintegral Order*, University of California Lawrence Radiation Laboratory, Livermore, UCRL-5859, September, 1960 (or from Office of Technical Services, Wash., D. C.). See also *Math. Comp.*, v. 16, 1962, pp. 117–119; (3) Abramowitz, A. & Stegun, I. A., Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, AMS No. 55, U. S. Government Printing Office, 1964. See Chapter 8 and references given there. See also *Math. Comp.*, v. 19, 1965, pp. 147–149.

Y. L. L.

69[9].—Francis L. Miksa, *Table of Primitive Pythagorean Triangles, Arranged According to Increasing Areas*, ms. in five volumes comprising a total of 27 + 980 typewritten pp. (consecutively numbered), deposited in the UMT file.

The main table of this voluminous unpublished work gives in 980 pages the generators, sides, and areas of the 52,490 primitive Pythagorean triangles whose areas do not exceed 10^{10}, arranged according to increasing areas.

Running counts of these triangles are given for each page, and such counts are separately tabulated for areas less than 10^k at intervals of 10^{k-1}, for $k = 7(1)10$. The author develops a formula that provides an independent check on these counts.

A preliminary section (dated May 21, 1952) contains a listing of the sides and areas of all primitive Pythagorean triangles having equal areas less than 10^{10}. Included are 158 pairs and a single triple of such triangles. All equiareal triangles therein that are generated by a special formula attributed to Fermat are so indicated. Appended to this table is a list of the generators and sides of primitive triangles whose areas consist of the digits 1(1)9 or 0(1)9 appearing just once.

A foreword (dated November 11, 1961) to the main table presents the details of the underlying calculations, performed with the assistance of a 10-column desk calculator.

This elaborate census of primitive Pythagorean triangles according to areas may be considered as a companion to the manuscript tables of Anema [1] and the author [2] listing these triangles according to increasing perimeters.

It seemed appropriate here also to refer to recent pertinent computations by Jones [3] and Beiler [4] and to a book of the latter [5], wherein one finds many additional references.

J. W. W.

1. A. S. Anema, *Primitive Pythagorean Triangles with their Generators and their Perimeters, up to 119 992*, ms. in the UMT file. (See *MTAC*, v. 5, 1951, p. 28, UMT **111**.)

2. F. L. Miksa, *Table of Primitive Pythagorean Triangles with their Perimeters Arranged in Ascending Order from 119 992 to 499 998*, ms. in the UMT file. (See *MTAC*, v. 5, 1951, p. 232, UMT **133**.)

3. M. F. Jones, *Isoperimetric Right-Triangles*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, April 1967. (See *Math. Comp.*, v. 22, 1968, pp. 233–234, RMT **21**.)
4. Albert H. Beiler, *Consecutive Hypotenuses of Pythagorean Triangles*, ms. in the UMT file. (See *Math. Comp.*, v. 22, 1968, pp. 690–691, RMT **74**.)
5. Albert H. Beiler, *Recreations in the Theory of Numbers*, Dover, New York, 1964, Chapter XIV.

70[14].—COSRIMS, *The Mathematical Sciences: A Report*, National Academy of Sciences, Washington, D. C., 1968, xiv + 256 pp., 23 cm. Price $6.00.

A series of reports on major areas of science have been prepared and issued under the aegis of the Committee on Science and Public Policy of the National Academy of Sciences. COSRIMS, under the chairmanship of Lipman Bers, was responsible for the mathematical sciences (see the other reviews). This report sets forth their conclusions with respect to mathematical research in the U. S., its character, its importance, the current extent and sources of its support, and its needs. Some little space is devoted to an attempt to explain, in language understandable to the layman, what mathematics is all about, and how and where it is applied.

"Remarkably enough, it is impossible to predict which parts of mathematics will turn out to be important in other fields." This statement appears on page 8, and the theme recurs. "Cayley . . . believed that matrices, which he invented, would never be applied to anything useful (and was happy about it)" (page 49). "How fast ought society to expect the results of innovation to be transferred?" (page 215). However, the last half-page is entitled "The Nonutilitarian View."

The first chapter is entitled "Summary," and includes the "Recommendations." "The State of the Mathematical Sciences" attempts to describe in simple terms "Core Mathematics" and to illustrate applications. The third chapter takes up mathematical education, and the fourth, "Level and Forms of Support."

Altogether this is an impressive and authoritative statement of the place of mathematics in contemporary society.

A. S. H.

71[14].—COSRIMS, *The Mathematical Sciences: Undergraduate Education*, National Academy of Sciences, Washington, D. C., 1968, ix + 113 pp., 23 cm. Price $4.25.

COSRIMS, the Committee on Support of Research in the Mathematical Sciences, as one phase of its activities undertook to investigate the state of undergraduate education in mathematics. This is the report of the Panel on Undergraduate Education in Mathematics, made up of eleven members with John G. Kemeny as chairman. One member of the Panel, Henry Pollak, was from industry, all others from colleges and universities.

The general picture that develops can hardly come as a surprise to any mathematician, whether academic or industrial, pure or applied, but the documentation, and the array of facts and figures, is impressive. Over the past ten years the percentage of undergraduate students majoring in mathematics has increased from 1.5 to 4.0. Meanwhile the level of offerings has gone up in ways that are not easy to measure in quantitative terms. However, the "case histories" of eight colleges and universities over the past quarter century vividly illustrate these and other changes that have taken place.

Among the factors that have served to bring about these changes are the many new areas of application, the increasingly sophisticated demands made by traditional areas of application, and, as part of the same picture, the computer revolution. With increasing employment of mathematicians in government and industry, the colleges and universities, especially the smaller and less prestigious ones, are unable to maintain the quality of the staff, even where, in simpler days, it may have been adequate.

All this is spelled out in considerable detail in the report, and a series of recommendations are made. They are convincing enough to those already close to the problem. One can only hope that also others will be convinced.

A. S. H

72[14].—COSRIMS, *The Mathematical Sciences: A Collection of Essays*, The MIT Press, Cambridge, Mass., 1969, x + 271 pp., 24 cm. Price $8.95.

This volume is intended to supplement the main report of COSRIMS, also reviewed here, by developing more fully, still in the language of the layman, special branches of mathematics and special areas of application. It is in the form of twenty-two essays, mostly by mathematicians. A few titles and authors, selected almost at random, may give some notion of the depth and breadth: "The Social Sciences Call on Mathematics" (16 pages) by Kemeny; "Topology of Molecules" (15 pages) by Lederberg; "Non-Euclidean Geometry" (8 pages) by Coxeter; "Statistical Inference" (12 pages) by Kiefer; "Solving a Quadratic Equation on a Computer" (15 pages) by Forsythe; "Mathematical Linguistics" (7 pages) by Harris; "The Continuum Hypothesis" (9 pages) by Smullyan. Usually, but not always, the mathematical authors attempt to at least indicate some applications outside mathematics of the discipline they are discussing.

One could find fault, perhaps, with one or two of the essays, but taken as a whole this is a remarkable collection. It would seem that even one who is mathematically completely illiterate should be able to read it with both pleasure and profit.

A. S. H.