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Some Factorizations of $10^n \pm 1$

By I. O. Angell and H. J. Godwin

Abstract. Factorizations of $10^n + 1$ and/or $10^n - 1$ are given for a number of values of n.

The factorizations of $10^n \pm 1$ given in Tables 1* and 2 supplement those given by Riesel [4]. Small factors were found by computing $10^n \pm 1$ modulo m, where m runs through the terms of an arithmetic progression determined by Fermat's theorem (e.g. the factors of $10^{34} + 1$, except for 101, are of the form $68k + 1$). One minute's running time on the University of London's CDC 6600 sufficed to try about one million such factors. Larger factors were obtained by the continued fraction method (see, e.g. Knuth [1]). Primality of factors was tested by Lehmer's method [2]. The factorizations of $N - 1$ required for this were all sufficiently simple to make it unnecessary to reproduce them. In these cases small factors were obtained by division by successive odd numbers. The factor 2028119 of $10^{37} - 1$ is due to Ondrejka [3].

The smallest number of the form $10^n \pm 1$ so far unfactorized is $10^{41} + 1$, which is 11 times a 40-digit composite number with no factor less than 92134955.

TABLE 1

n	$(10^n - 1)/9$
31	$2791 \cdot 6943319 \cdot 57336415063790604359$
33	$3 \cdot 37 \cdot 67 \cdot 21649 \cdot 513239 \cdot 1344628210313298373$
37	$2028119 \cdot 247629013 \cdot 2212394296770203368013$
39	$3 \cdot 37 \cdot 53 \cdot 79 \cdot 265371653 \cdot 900900900900990990990991$
41	$83 \cdot 1231 \cdot 538987 \cdot 201763709900322803748657942361$
43	$173 \cdot 1527791 \cdot 1963506722254397 \cdot 2140992015395526641$
45	$3^2 \cdot 31 \cdot 37 \cdot 41 \cdot 271 \cdot 238681 \cdot 333667 \cdot 2906161 \cdot 4185502830133110721$

**Editorial note.* Two editors of *Math. Comp.*, and other investigators interested in such problems, were aware that John Brillhart had completely factored

$$(10^p - 1)/9$$

for $p = 31, 37, 41, 43$, and some larger values some time ago, but he had not published them. His factorizations agree exactly with the corresponding four entries in Table 1.

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TABLE 2

n	$10^n + 1$
22	89·101·1052788969·1056689261
26	101·521·1900381976777332243781
28	73·137·7841·127522001020150503761
29	11·59·154083204930662557781201849
33	$7\cdot 11^2\cdot 13\cdot 23\cdot 4093\cdot 8779\cdot 599144041\cdot 183411838171$
34	101·28559389·1491383821·2324557465671829
35	11·9091·909091·4147571·265212793249617641
37	11·7253·422650073734453·296557347313446299
38	101·722817036322379041·1369778187490592461
39	$7\cdot 11\cdot 13^2\cdot 157\cdot 859\cdot 6397\cdot 216451\cdot 1058313049\cdot 388847808493$
40	17·5070721·5882353·19721061166646717498359681
42	29·101·281·9901·226549·121499449·4458192223320340849
43	11·57009401·2182600451·7306116556571817748755241
44	73·137·617·16205834846012967584927082656402106953
45	7·11·13·19·211·241·2161·9091·29611·52579·3762091·8985695684401
48	97·353·449·641·1409·69857·206209·66554101249·75118313082913
49	11·197·909091·5076141624365532994918781726395939035533

Department of Statistics and Computer Science
Royal Holloway College
Englefield Green
Surrey, TW20 0EX, England

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4. H. Riesel, *En Bok om Primtal* [*A Book on Prime Numbers*], Studentlitteratur, Lund, 1968. (Swedish) MR **42** #4507.