Tables of Reductions of Symmetrized Inner Products
("Inner Plethysms") of Ordinary Irreducible
Representations of Symmetric Groups

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Abstract. Decompositions of symmetrized inner products $[\alpha] \boxtimes [\beta]$ of ordinary
irreducible representations $[\alpha]$ of symmetric groups $S_n$ and $[\beta]$ of $S_m$ were eval-
uated on a CDC 6400. Tables were obtained for $2 \leq n \leq 10$ and $2 \leq m \leq 5$ as
well as for $m = 6$ and $2 \leq n \leq 7$.

In [3] R. C. King published tables of reductions of symmetrized inner products
$[\alpha] \boxtimes [\beta]$ which he calls inner plethysms, of ordinary irreducible representations $[\alpha]$ of $S_n$ and $[\beta]$ of $S_m$, where $n = 4$ and $m \leq 5$, $n = 5$ and $m \leq 4$, $n = 6$ and $m \leq 3$.*

He obtained the decomposition by restricting certain representations $[\beta]$ of the
general linear group $GL_n$ to symmetric subgroups.

Such decompositions can be obtained directly by evaluating the character of
$[\alpha] \boxtimes [\beta]$ which is

$$
\chi^{[\alpha] \boxtimes [\beta]}(g) = \frac{1}{|S_m|} \sum_{\pi \in S_m} \chi^{[\beta]}(\pi) \prod_{k=1}^{n} \xi^{[\alpha]}(g^k)^{a_k(\pi)},
$$

where $g \in S_n$ and $a_k(\pi)$ denotes the number of cyclic factors of length $k$ in $\pi \in S_m$, $1 \leq k \leq m$.

For this formula see [1], [2], and [4, p. 74]. The evaluation was carried out
with the aid of a computer (CDC 6400 RWTH Aachen) by using the program described
in [1], in double-precision arithmetic. The characters of the products $[\alpha] \boxtimes [\beta]$ were
then decomposed into their irreducible constituents via orthogonality relations by using
the character table of $S_n$.

Tables were thus obtained of the reductions of the symmetrized inner products
of the ordinary irreducible representations of the symmetric groups $S_2$ up to $S_{10}$ with
the ordinary irreducible representations of $S_2$ up to $S_5$ and of the characters of $S_2$
up to $S_7$ with those of $S_6$. These tables appear on the microfiche card in this issue.

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