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The Orders of the Known Simple Groups as Far as One Trillion

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Abstract. In this paper an update of L. E. Dickson's 1901 list of the orders of the known simple groups up to one billion is given. On the microfiche supplement is a complete list of all of the orders of the known simple groups as far as one trillion together with the Lie notation for these groups.

In 1901 L. E. Dickson [3, p. 309] gave a list of the orders of all the known simple groups as far as one million and a partial list of those up to one billion (the groups $\mathrm{PSL}_2(q)$ for $q > 113$ were omitted). Since then no complete list updating Dickson's has appeared in print despite the fact that several new discoveries in this range have been made and much of Dickson's notation is no longer in use.

In Table 1 we bring Dickson's list up to date and, on the microfiche supplement, give a complete list of all the orders of the known nonabelian simple groups as far as one trillion in the Lie notation used by Gorenstein [5, p. 491]. The combined efforts of Marshall Hall, Jr. [6], [7] and Beisiegel and Stingl [1] now show that Table 1 includes all possible simple groups of order up to one million. Table 2 extends Table 1 with the known simple groups of order up to one billion, excluding the groups $\mathrm{PSL}_2(p^n)$. The interested reader is referred to the recent paper by Gallian [4] for a detailed historical account of the search for finite simple groups.

Our notation is as follows: $\mathrm{PSL}_n(q)$ is the projective special linear group; $\mathrm{Sp}_{2n}(q)$ is the symplectic group; $\mathrm{U}_n(q)$ is the unitary group; $\mathrm{Alt}(n)$ is the alternating group; $\mathrm{Sz}(q)$ is the Suzuki group; $\mathrm{G}_2(q)$ is the Chevalley group of type G_2; $\mathrm{D}_4(q)$ is the Chevalley group of type D_4; ${}^2\mathrm{D}_4(q)$ is the twisted orthogonal group of type D_4; ${}^3\mathrm{D}_4(q)$ is the triality twisted group of type D_4. See [2] for complete details on these groups.

Standard notation is used for the sporadic groups: M_i is the group of permutations on i symbols discovered by Mathieu, Ja is the Janko group, HaJ is the Hall-Janko group, HiS is the Higman-Sims group, HJM is the Hall-Janko-McKay group, McL is the McLaughlin group, HHM is the Held-Higman-McKay group, Rud is the Rudvalis group, Suz is the Suzuki group, O'Nan is the O'Nan group, and Co_3 is the Conway .3 group. A brief discussion of these groups can also be found in [2, Chapter 16] and a complete list of all 26 known sporadic simple groups is given in [4].

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TABLE 1

The orders of all simple groups up to one million

Order	*Group type*
$60 = 2^2 \cdot 3 \cdot 5$	$PSL_2(4) = PSL_2(5) = Alt(5)$
$168 = 2^3 \cdot 3 \cdot 7$	$PSL_2(7) = PSL_3(2)$
$360 = 2^3 \cdot 3^2 \cdot 5$	$PSL_2(9) = Alt(6)$
$504 = 2^3 \cdot 3^2 \cdot 7$	$PSL_2(8)$
$660 = 2^2 \cdot 3 \cdot 5 \cdot 11$	$PSL_2(11)$
$1\,092 = 2^2 \cdot 3 \cdot 7 \cdot 13$	$PSL_2(13)$
$2\,448 = 2^4 \cdot 3^2 \cdot 17$	$PSL_2(17)$
$2\,520 = 2^3 \cdot 3^2 \cdot 5 \cdot 7$	$Alt(7)$
$3\,420 = 2^2 \cdot 3^2 \cdot 5 \cdot 19$	$PSL_2(19)$
$4\,080 = 2^4 \cdot 3 \cdot 5 \cdot 17$	$PSL_2(16)$
$5\,616 = 2^4 \cdot 3^3 \cdot 13$	$PSL_3(3)$
$6\,048 = 2^5 \cdot 3^3 \cdot 7$	$U_3(3)$
$6\,072 = 2^3 \cdot 3 \cdot 11 \cdot 23$	$PSL_2(23)$
$7\,800 = 2^3 \cdot 3 \cdot 5^2 \cdot 13$	$PSL_2(25)$
$7\,920 = 2^4 \cdot 3^2 \cdot 5 \cdot 11$	M_{11}
$9\,828 = 2^2 \cdot 3^3 \cdot 7 \cdot 13$	$PSL_2(27)$
$12\,180 = 2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 29$	$PSL_2(29)$
$14\,880 = 2^5 \cdot 3 \cdot 5 \cdot 31$	$PSL_2(31)$
$20\,160 = 2^6 \cdot 3^2 \cdot 5 \cdot 7$	$PSL_4(2) = Alt(8)$
$20\,160 = 2^6 \cdot 3^2 \cdot 5 \cdot 7$	$PSL_3(4)$
$25\,308 = 2^2 \cdot 3^2 \cdot 19 \cdot 37$	$PSL_2(37)$
$25\,920 = 2^6 \cdot 3^4 \cdot 5$	$Sp_4(3) = U_4(2)$
$29\,120 = 2^6 \cdot 5 \cdot 7 \cdot 13$	$Sz(8)$
$32\,736 = 2^5 \cdot 3 \cdot 11 \cdot 31$	$PSL_2(32)$
$34\,440 = 2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 41$	$PSL_2(41)$
$39\,732 = 2^2 \cdot 3 \cdot 7 \cdot 11 \cdot 43$	$PSL_2(43)$
$51\,888 = 2^4 \cdot 3 \cdot 23 \cdot 47$	$PSL_2(47)$
$58\,800 = 2^4 \cdot 3 \cdot 5^2 \cdot 7^2$	$PSL_2(49)$
$62\,400 = 2^6 \cdot 3 \cdot 5^2 \cdot 13$	$U_3(4)$
$74\,412 = 2^2 \cdot 3^3 \cdot 13 \cdot 53$	$PSL_2(53)$
$95\,040 = 2^6 \cdot 3^3 \cdot 5 \cdot 11$	M_{12}
$102\,660 = 2^2 \cdot 3 \cdot 5 \cdot 29 \cdot 59$	$PSL_2(59)$
$113\,460 = 2^2 \cdot 3 \cdot 5 \cdot 31 \cdot 61$	$PSL_2(61)$
$126\,000 = 2^4 \cdot 3^2 \cdot 5^3 \cdot 7$	$U_3(5)$

$150\,348 = 2^2 \cdot 3 \cdot 11 \cdot 17 \cdot 67$	$PSL_2(67)$
$175\,560 = 2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 19$	Ja
$178\,920 = 2^3 \cdot 3^2 \cdot 5 \cdot 7 \cdot 71$	$PSL_2(71)$
$181\,440 = 2^6 \cdot 3^4 \cdot 5 \cdot 7$	Alt(9)
$194\,472 = 2^3 \cdot 3^2 \cdot 37 \cdot 73$	$PSL_2(73)$
$246\,480 = 2^4 \cdot 3 \cdot 5 \cdot 13 \cdot 79$	$PSL_2(79)$
$262\,080 = 2^6 \cdot 3^2 \cdot 5 \cdot 7 \cdot 13$	$PSL_2(64)$
$265\,680 = 2^4 \cdot 3^4 \cdot 5 \cdot 41$	$PSL_2(81)$
$285\,852 = 2^2 \cdot 3 \cdot 7 \cdot 41 \cdot 83$	$PSL_2(83)$
$352\,440 = 2^3 \cdot 3^2 \cdot 5 \cdot 11 \cdot 89$	$PSL_2(89)$
$372\,000 = 2^5 \cdot 3 \cdot 5^3 \cdot 31$	$PSL_3(5)$
$443\,520 = 2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	M_{22}
$456\,288 = 2^5 \cdot 3 \cdot 7^2 \cdot 97$	$PSL_2(97)$
$515\,100 = 2^2 \cdot 3 \cdot 5^2 \cdot 17 \cdot 101$	$PSL_2(101)$
$546\,312 = 2^3 \cdot 3 \cdot 13 \cdot 17 \cdot 103$	$PSL_2(103)$
$604\,800 = 2^7 \cdot 3^3 \cdot 5^2 \cdot 7$	Ha J
$612\,468 = 2^2 \cdot 3^3 \cdot 53 \cdot 107$	$PSL_2(107)$
$647\,460 = 2^2 \cdot 3^3 \cdot 5 \cdot 11 \cdot 109$	$PSL_2(109)$
$721\,392 = 2^4 \cdot 3 \cdot 7 \cdot 19 \cdot 113$	$PSL_2(113)$
$885\,720 = 2^3 \cdot 3 \cdot 5 \cdot 11^2 \cdot 61$	$PSL_2(121)$
$976\,500 = 2^2 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 31$	$PSL_2(125)$
$979\,200 = 2^8 \cdot 3^2 \cdot 5^2 \cdot 17$	$Sp_4(4)$

TABLE 2

The orders of the known simple groups between one million and one billion excluding $PSL_2(p^n)$

Order	*Group type*
$1\,451\,520 = 2^9 \cdot 3^4 \cdot 5 \cdot 7$	$Sp_6(2)$
$1\,814\,400 = 2^7 \cdot 3^4 \cdot 5^2 \cdot 7$	Alt(10)
$1\,876\,896 = 2^5 \cdot 3^2 \cdot 7^3 \cdot 19$	$PSL_3(7)$
$3\,265\,920 = 2^7 \cdot 3^6 \cdot 5 \cdot 7$	$U_4(3)$
$4\,245\,696 = 2^6 \cdot 3^6 \cdot 7 \cdot 13$	$G_2(3)$
$4\,680\,000 = 2^6 \cdot 3^2 \cdot 5^4 \cdot 13$	$Sp_4(5)$
$5\,515\,776 = 2^9 \cdot 3^4 \cdot 7 \cdot 19$	$U_3(8)$
$5\,663\,616 = 2^7 \cdot 3 \cdot 7^3 \cdot 43$	$U_3(7)$
$6\,065\,280 = 2^7 \cdot 3^6 \cdot 5 \cdot 13$	$PSL_4(3)$
$9\,999\,360 = 2^{10} \cdot 3^2 \cdot 5 \cdot 7 \cdot 31$	$PSL_5(2)$

$10\,200\,960 = 2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	M_{23}
$13\,685\,760 = 2^{10} \cdot 3^5 \cdot 5 \cdot 11$	$U_5(2)$
$16\,482\,816 = 2^9 \cdot 3^2 \cdot 7^2 \cdot 73$	$PSL_3(8)$
$19\,958\,400 = 2^7 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11$	Alt(11)
$32\,537\,600 = 2^{10} \cdot 5^2 \cdot 31 \cdot 41$	Sz(32)
$42\,456\,960 = 2^7 \cdot 3^6 \cdot 5 \cdot 7 \cdot 13$	$PSL_3(9)$
$42\,573\,600 = 2^5 \cdot 3^6 \cdot 5^2 \cdot 73$	$U_3(9)$
$44\,352\,000 = 2^9 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 11$	HiS
$50\,232\,960 = 2^7 \cdot 3^5 \cdot 5 \cdot 17 \cdot 19$	HJM
$70\,915\,680 = 2^5 \cdot 3^2 \cdot 5 \cdot 11^3 \cdot 37$	$U_3(11)$
$138\,297\,600 = 2^8 \cdot 3^2 \cdot 5^2 \cdot 7^4$	$Sp_4(7)$
$174\,182\,400 = 2^{12} \cdot 3^5 \cdot 5^2 \cdot 7$	$D_4(2)$
$197\,406\,720 = 2^{12} \cdot 3^4 \cdot 5 \cdot 7 \cdot 17$	${}^2D_4(2)$
$211\,341\,312 = 2^{12} \cdot 3^4 \cdot 7^2 \cdot 13$	${}^3D_4(2)$
$212\,427\,600 = 2^4 \cdot 3 \cdot 5^2 \cdot 7 \cdot 11^3 \cdot 19$	$PSL_3(11)$
$239\,500\,800 = 2^9 \cdot 3^5 \cdot 5^2 \cdot 7 \cdot 11$	Alt(12)
$244\,823\,040 = 2^{10} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 23$	M_{24}
$251\,596\,800 = 2^{12} \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 13$	$G_2(4)$
$270\,178\,272 = 2^5 \cdot 3^2 \cdot 7 \cdot 13^3 \cdot 61$	$PSL_3(13)$
$811\,273\,008 = 2^4 \cdot 3 \cdot 7^2 \cdot 13^3 \cdot 157$	$U_3(13)$
$898\,128\,000 = 2^7 \cdot 3^6 \cdot 5^3 \cdot 7 \cdot 11$	McL
$987\,033\,600 = 2^{12} \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 17$	$PSL_4(4)$

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