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Some New Primes of the Form $k \cdot 2^n + 1$

By G. Matthew and H. C. Williams

Abstract. All primes of the form $k \cdot 2^n + 1$, k odd, for $9 \leqslant k \leqslant 99$, $512 \leqslant n \leqslant 1000$ and for $101 \leqslant k \leqslant 129$, $1 \leqslant n \leqslant 1000$ are determined and factors are found for the Fermat numbers F_{744} and F_{556}.

Recently Hallyburton and Brillhart [1] found, by means of a computer search, a new factor of each of the Fermat numbers F_{12} and F_{13}. In this note we present two new factors of Fermat numbers which were found by using the method of Robinson [2]. In [2] Robinson tabulated all primes of the form $k \cdot 2^n + 1$ for $k = 3,7$, $1 \leqslant n \leqslant 1279$, for $k = 5$, $1 \leqslant n \leqslant 2004$, and for odd k such that $9 \leqslant k \leqslant 99$, $1 \leqslant n \leqslant 511$. In this note we extend his table to include all primes of the form $k \cdot 2^n + 1$ for $9 \leqslant k \leqslant 129$, $1 \leqslant n \leqslant 1000$. These results are presented in Tables 1 and 2.

TABLE 1
Primes of the form $k \cdot 2^n + 1$, k odd, for $9 \leqslant k \leqslant 99$, $512 \leqslant n \leqslant 1000$

k	values of n	k	values of n
9	663, 782	63	626, 693, 741, 768
13	1000	65	553
15	517, 522, 654, 900	67	598, 726, 870
17	747	69	515, 842
21	899	71	705
23	649	75	675, 831, 984
25	554, 664, 740, 748	77	559, 655, 667
33	525, 726, 828	79	538, 970
35	663	81	539, 577, 592, 711, 809, 852
37	712	85	624
39	518, 818, 865	87	518, 602
43	778	89	589, 711
45	801	91	696
47	583	93	686
49	594	95	533, 621, 661, 753, 993
51	695, 825	97	652, 722
53	857	99	631, 894
57	719		

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TABLE 2

Primes of the form $k \cdot 2^n + 1$, *k odd, for* $101 \leqslant k \leqslant 129$, $1 \leqslant n \leqslant 1000$

k	values of n
101	3, 9, 17, 21, 27, 39, 45, 47, 71, 95, 117, 123, 143, 173, 387, 389, 513, 633, 827, 971
103	16, 18, 30, 40, 58, 138, 250, 616, 622, 736
105	1, 2, 5, 7, 8, 12, 14, 23, 27, 33, 38, 49, 61, 62, 85, 93, 94, 107, 155, 182, 215, 273, 382, 392, 413, 434, 490
107	3, 7, 23, 27, 291, 303, 311, 479, 567
109	6, 14, 58, 62, 318
111	1, 4, 28, 32, 44, 47, 71, 128, 137, 193, 676
113	1, 5, 13, 33, 145, 365, 409, 509, 553, 673, 733, 961
115	2, 12, 20, 26, 42, 114, 228, 396, 456, 482
117	3, 4, 6, 10, 16, 30, 36, 91, 94, 156, 382, 454, 643, 867
119	1, 3, 7, 13, 21, 23, 45, 63, 553
121	8, 12, 44, 84, 96, 228, 264, 320, 732, 788
123	6, 8, 17, 21, 29, 32, 46, 57, 69, 128, 141, 268, 333, 476, 742, 832
125	1, 5, 7, 17, 25, 35, 67, 281, 331, 491, 581, 941
127	2, 12, 18, 24, 54, 72, 114, 180, 214, 504, 558, 964
129	3, 5, 21, 27, 59, 75, 111, 287, 414, 786, 966

Each of the new primes found was tested as a possible divisor of a Fermat number and only the two following factors were found:

$$17 \cdot 2^{747} + 1 \mid F_{744}, \qquad 127 \cdot 2^{558} + 1 \mid F_{556}.$$

Department of Computer Science
University of Manitoba
Winnipeg, Manitoba R3T 2N2, Canada

1. JOHN C. HALLYBURTON, JR. & JOHN BRILLHART, "Two new factors of Fermat numbers," *Math. Comp.*, v. 29, 1975, pp. 109–112. MR 51 # 5460.

2. RAPHAEL M. ROBINSON, "A report on primes of the form $k \cdot 2^n + 1$ and on factors of Fermat numbers," *Proc. Amer. Math. Soc.*, v. 9, 1958, pp. 673–681. MR 20 # 3097.