REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

6 [2, 3, 4].—HANS J. STETTER, *Numerik für Informatiker – Computergerechte numerische Verfahren. Eine Einführung,* R. Oldenbourg Verlag, München, Wien, 1976, 149 pp., 24 cm. Price DM 19.80.

This is an introductory text on numerical methods addressed to students in computer science whose main interests lie outside the area of numerical computation. The author, therefore, makes a deliberate attempt to bring into focus the interfaces that exist between computer science and numerical analysis. The result is most noticeable in the three introductory chapters dealing with computer arithmetic, various sources of errors and error propagation, as well as in the concluding chapter on principles of numerical software development. The exposition, throughout, is concise and clear. Proofs are given only if they enhance the understanding of the subject. One year of calculus and some familiarity with the elements of linear algebra ought to be sufficient background for a profitable study of this booklet.

The chapter headings are as follows: 1. Introduction, 2. Computer arithmetic, 3. Error propagation, 4. Evaluation of functions, 5. Solution of equations, 6. Linear systems of equations, 7. Specification of functions through data, 8. Numerical integration and differentiation, 9. Ordinary differential equations, 10. Numerical software. Each chapter is followed by exercises, some of which involve projects to be carried out on the computer. Unfortunately, there is no index of any kind.

W. G.

7 [**3.00, 4.00**].—MARTIN GUTKNECHT, PETER HENRICI, PETER LÄUCHLI & HANS-RUDOLF SCHWARZ, *Heinz Rutishauser*: *Vorlesungen über numerische Mathematik,* Vols. 1 and 2, Birkhauser Verlag, Basel, 1976, 164 pp. and 228 pp. Price Vol. 1 Fr./DM 40; Vol. 2 Fr./DM 48.

When Heinz Rutishauser, a pioneer of computational mathematics, died at the age of 52 in 1970, he left updated notes of his lectures on Numerical Mathematics which he had intended to convert into a text book. With the aid of P. Henrici, P. Läuchli, and H. R. Schwarz, M. Gutknecht has managed to edit this material into two volumes which are strikingly uniform in form and style.

Those who have taught introductory courses in Numerical Mathematics will be delighted at first sight: numerous instructive examples and illustrations enhance a clearly and suggestively written text. There is the fine balance between mathematical and computational reasoning which is so essential, and the limitations of both aspects are exposed.

The first volume covers linear equations and inequalities, with special attention to positive-definite systems, nonlinear equations, optimization, interpolation, quadrature and approximation. Although most of the material is standard, the approach and the argumentation are often original; also the level of an introduction is maintained throughout without loss of understanding. Surprisingly, "condition" is not introduced as a concept although it is at the basis of many discussions.

The second volume is devoted to differential equations and eigenvalue problems. The treatment of ordinary initial value problems stresses the numerical mechanisms and even explains expontial fitting; that of ordinary boundary value problems includes shooting (called "artillery method"), differences and discrete variational methods. An exposition of the numerics of elliptic partial differential equations leads to iterative methods for linear systems (including conjugate gradients); norms and condition numbers of matrices appear at this late stage. An economic coding of the associated sparse matrices is suggested. In connection with heat conduction problems there is an interesting analysis of the feasibility of a systematic increase of the time step due to the elimination of high frequency error components. A short discussion of the one-dimensional wave equation completes the numerical analysis of differential equations. In just the same fashion, the section on eigenvalue problems guides the student to an understanding of the essential numerical aspects and methods without confusing him by unnecessary technicalities. Even the treatment of the shifts in the *LR*-method remains transparent and immediately convincing.

A most interesting appendix "An axiomatic basis for numerical computing and its application to the *QD*-algorithm" concludes the book. Rutishauser's approach is not to introduce intricate algebraic structures but simply to formulate as axioms those properties of common computer arithmetic which are necessary for a rigorous analysis of algorithm on a digital computer. The feasibility of his approach is shown by the proof of theorems on the performance of the *QD*-algorithm which has been introduced in the first part of the appendix. (The manuscript of this appendix was not completed when the author died.)

It is hoped that this book will see widespread use by the students for whom it has been intended and for whom it would furnish an exquisite introduction into the subject. In any case, the editors have set a worthy memorial to their friend H. Rutishauser.

H. J. S.

8[4.10.4, 5.10.3, 5.20.4].—J. T. ODEN & J. N. REDDY, *An Introduction to the Mathematical Theory of Finite Elements,* Wiley, New York, 1976, xii + 429 pp. Price \$21.95.

This book is devoted to the mathematical foundations of the Finite Element Method (F E.M.) and its application to the approximation of elliptic boundary value problems and time dependent partial differential equations.

Let us first describe briefly the content of each chapter: Chapter 1 is an introduction in which we find a brief history of the F.E.M., an outline of the following chapters and, finally, some of the mathematical notation to be used in the remaining part of the book.

The following eight chapters could be divided into two parts. Part I (Chapters 2, 3, 4, 5) contains the mathematical background for, and Part II (Chapters 6, 7, 8, 9) the theory of, the F.E.M. Each chapter has its own bibliography.

In Chapter 2 the authors define distributions on a domain Ω of $\mathbf{R}$, their derivatives, the convergence of a sequence of distributions, etc.; all these definitions are illustrated by various examples. Also distributional differential equations and the concept of fundamental solutions are briefly considered.

Chapter 3 is related to the theory of Sobolev spaces. Once the definition of such spaces has been given, the authors, following Sobolev [1], prove several properties of the Sobolev spaces, in particular various embedding theorems, when Ω has the so-called cone property.

In Chapter 4 the authors use the approach of Lions and Magenes [2] to define the the Sobolev spaces $H^s(\mathbf{R}^N)$, first for $s \in \mathbf{R}_+$, and then for $s < 0$; this approach which is now classical is based on the use of the Fourier transform. Then the authors prove a trace theorem for the functions of $H^m(\mathbf{R}^N_+)$ ($\mathbf{R}^N_+ = \{x \in \mathbf{R}^N \mid x = (x_1, x_2, \ldots, x_N), x_N > 0\}$) and study various properties of the trace operators (continuity, surjectivity, etc.). To define $H^s(\Omega)$ for $\Omega \neq \mathbf{R}^N$ and s arbitrary, the authors, following Lions and Magenes, loc. cit., use the theory of interpolation between Hilbert spaces. Then using the local mapping method and the above interpolation theory they prove trace theorems for $H^s(\Omega)$ with s not an integer.

Chapter 5 may be viewed as an introduction to the theory of elliptic boundary value problems. After some preliminary definitions and results the authors discuss results of existence, compatibility, uniqueness, for the general elliptic boundary value problem:

$$\begin{aligned} Au &= f \quad \text{in } \Omega, \\ B_k u &= g_k \quad \text{on } \partial\Omega,\ k = 0, 1, \ldots, m-1, \end{aligned}$$

where A is an elliptic differential operator of order m and where B_k is a differential boundary operator of order k.

The second part of the book is devoted to the theory of the F.E.M.

In Chapter 6 a definition of finite elements is given, and the authors discuss several concepts such as connectivity, local and global representation; these various topics are illustrated by various examples and figures. Then following Aubin [3], the authors define restriction, prolongation and projection operators and discuss with various examples the concepts of conjugate basis functions.

The next part of the chapter is related to classical concepts such as Lagrange families of finite elements, Serendipity elements, Hermite elements, conforming elements, isoparametric curved elements; all these finite elements are of polynomial type. Then there is a brief discussion of the Wachspress [4] rational finite elements. These concepts are illustrated with two- and three-dimensional examples and various figures. Then the authors, following several papers of Ciarlet and Raviart, prove, via the Bramble-Hilbert Lemma, interpolation error estimates in the Sobolev norms which can be applied to straight finite elements. The more complicated case of curved finite elements is briefly discussed at the end of the chapter.

In Chapter 7 the authors consider variational boundary value problems of elliptic type and give several examples of such problems. They then define the concept of coercive bilinear forms and they give several examples to illustrate it. In the following sections the authors introduce, following Babuška, the concept of weakly coercive bilinear forms and they discuss the treatment of nonhomogeneous boundary conditions viewed as constraints through the use of Lagrange multipliers. The authors conclude this chapter by proving a generalized Lax-Milgram theorem for variational problems related to weakly coercive bilinear forms. This theorem contains the classical Lax-Milgram theorem (for coercive bilinear forms) as a special case.

Chapter 8 is, from the finite element point of view, the most important part of the book since it describes the use of the finite elements studied in Chapter 6 to approximate the variational problems of Chapter 7. In the first sections there is a general study of Galerkin approximations, and a general formula for the approximation error is derived. Finite element approximations are then viewed as practical methods to implement the method of Galerkin. Finite element subspaces are built and various properties of the corresponding approximations are given, for example error estimates in weaker norms using the Aubin-Nitsche method. The so-called Inverse Property is also discussed. In Section 8.6 general error estimates in various norms are proved and the numerical results obtained from a simple one-dimensional Dirichlet problem are analyzed and compared with the theoretical predictions.

There is then a brief discussion of $L^\infty(\Omega)$ estimates of the approximation error.

The one-dimensional case is studied in detail but the results of Nitsche and Scott for higher dimensions are stated without proof (they were not yet available at the time the manuscript was completed), furthermore a large number of references is given which may be used by any reader interested by these rather difficult topics.

In the following sections the authors discuss the influence on the approximation error of the quadrature and data errors and also the effect of the approximation of the boundary by a simpler one (of polygonal type in many cases). In Section 8.9 the authors describe the H^{-1} finite element approximations, introduced by Rachford and Wheeler. In this method which is well suited for stiff one-dimensional boundary value problems, one uses different spaces for the approximate solution (piecewise linear, discontinuous, for instance) and for the test functions (piecewise cubic, C^1 if the approximate solution is as above).

To conclude Chapter 8 the authors describe the hybrid and mixed finite element methods which seem to be becoming more and more popular these days. They restrict their study to the approximation of second order elliptic problems, but many references related to fourth order problems are given. Several kinds of mixed and/or hybrid approximations are considered and a priori error estimates are obtained.

Chapter 9, which is the final one of the book, may be viewed as an introduction to the approximation of time dependent problems.

The authors study first the effect of the space discretization on time dependent problems of diffusion type (for example, the heat equation). Assuming that there is no time discretization they obtain a system of ordinary differential equations in a variational form (semidiscrete L^2 Galerkin approximations) for which an error estimate in the $L^2(\Omega)$ norm is given at any time (relation (9.19)). To study the effect of the full discretization (time and space) the authors use a semigroup approach and obtain L^2 estimates of the approximation error.

In Section 9.6 the authors describe the full discretization of the standard wave equation by the ordinary explicit scheme. Stability conditions and errors estimates are given.

To conclude this chapter, the authors discuss briefly the approximation of some first order hyperbolic equations using the Laplace transform to obtain error estimates. This concludes the descriptive part of this review.

From a more critical point of view we would like to make four major observations concerning this book:

(1) We think that students or engineers with a modest mathematical preparation may find this book difficult since its mathematical level is fairly high.

(2) The methodology used in this book in studying the Sobolev spaces, follows the Soviet school approach. In particular, the embedding theorems are not proved for $C^k(\overline{\Omega})$, but for less standard spaces which we think are less useful than the $C^k(\overline{\Omega})$ spaces in the study of the convergence properties of the F.E.M. in many linear and nonlinear boundary value problems. In this direction we think that the Nečas [5] approach is better suited to the study of the mathematical properties of the F.E.M.

(3) From a practical point of view and thinking of the possible users, we regret that Theorem 6.8 of page 279, which is in fact the main result of the book, has not been illustrated by several examples to link it with the Lagrange and Hermite finite elements described previously.

(4) We think that Chapter 9 is too theoretical, for its small number of pages. In our opinion it would have been preferable to describe more schemes (multistep, Runge-Kutta, Newmark, Wilson, etc.), give the basic properties of them and just indicate the order of the approximation in the more standard cases.

During our review we noticed some minor mistakes or ambiguities:

P. 67: In Theorem 3.3,(iii) the relation

$$\int_\Omega D^\alpha uv\,dx = (-1)^{|\alpha|} \int_\Omega uD^\alpha v\,dw$$

holds for all $v \in W_q^m(\Omega)$.

Pp. 107–109: In Example 4.5, the Sobolev spaces $H^s(\Omega)$, s nonintegral, have not been defined yet.

P. 110: The dual space of $H^m(\Omega)$ is not contained in $H^{-m}(\Omega)$.

P. 113:

$$D_n^j u = \frac{\partial^j u}{\partial n^j} \equiv (-1)^j \frac{\partial^j u}{\partial t^j} = (-1)^j D_t^j u.$$

On lines 7 and 8 the normal derivatives of u and v of order $j < m$ match for any extension v of u.

P. 142: (4.106) (resp. (1.110)) are not true $\forall u \in H^m(\Omega)$ (resp. $\forall u \in H^s(\Omega)$).

P. 179: Example 5.20: Since $\partial\Omega = \{0, 1\}$, the notation $H^{s-3/2}(\partial\Omega)$ may be confusing for a beginner.

P. 319: The conventional Lax-Milgram theorem also applies to nonsymmetric strongly coercive bilinear forms.

P. 347: Example 8.4; since a piecewise linear function is not in H^2 in general, we do not understand the penultimate line of p. 347.

P. 370: We think that $L_2(P) = (L^2(\Omega))^2$.

P. 391: In these kinds of problems it is very important to specify to which spaces the initial value u_0 and the right-hand side f belong.

P. 396: If A is defined by (9.2) it would be important to specify U in this case, since A is obviously not bounded from $H^m(\Omega)$ into $H^m(\Omega)$.

To conclude our review we would like to say that J. T. Oden and J. N. Reddy have performed a considerable task in writing a mathematical introduction to the F.E.M. They have tried to make it as self-contained as possible and usable by beginners and people with a modest mathematical education, and have written a book with the following qualities:

. The general plan is remarkably well conceived.

. It gives an introduction to the mathematical foundations of the F.E.M. (as the title indicates).

. It is quite adequate for self-study for a mathematically oriented reader.

. It may be used as a text for an advanced Numerical Analysis course.

. It contains an extensive up-to-date bibliography.

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1. S. L. SOBOLEV, *Applications of Functional Analysis in Mathematical Physics*, Transl. Math. Monographs, Vol. 7, Amer. Math. Soc., Providence, R. I., 1963.

2. J. L. LIONS & E. MAGENES, *Non Homogeneous Boundary Value Problems and Applications*, Vol. 1, Springer-Verlag, New York, 1972.

3. J. P. AUBIN, *Approximation of Elliptic Boundary-Value Problems*, Wiley-Interscience, New York, 1972.

4. E. L. WACHPRESS, *A Rational Finite Element Basis*, Academic Press, New York, 1975.

5. J. NEČAS, *Les Méthodes Directes en Théorie des Équations Elliptiques*, Masson, Paris 1967.

9[5.10, 5.20].—V. G. SIGILLITO, *Explicit A Priori Inequalities With Applications to Boundary Value Problems*, Pitman Publishing, Ltd., London, 1977, 103 pp., 24½ cm. Price £5.50.

In the late 1950's and early 1960's considerable effort was devoted to the derivation of explicit a priori inequalities with the idea of using them to compute upper and lower bounds in various types of boundary and initial-boundary value problems. During that same period the computers became more and more sophisticated and finite difference and finite element methods for the handling of such problems developed so rapidly that the a priori methods saw little or no use. The present monograph is, to the author's knowledge, the first publication devoted to this topic. The monograph is written for the user in the sense that the author makes no attempt to derive the most general result for the most general problem. He rather puts across the ideas and methods, considering a number of basic problems, deriving the desired explicit a priori inequalities, and clearly indicating how these methods may be extended to more general problems.

The author states that his purpose in writing the monograph is three fold: (1) to bring together into a single volume a number of scattered results that are not widely known in the hope that they may become better known, (2) to illustrate a method of computing approximate solutions (with computable error bounds) based on a priori inequalities and (3) to indicate techniques used in deriving the inequalities. The first three sections provide introductory and background material. The next four sections are devoted to the development of explicit a priori inequalities which provide norm (primarily L_2) bounds for solutions of (i) Second order elliptic problems, (ii) Second order parabolic problems, (iii) Pseudoparabolic problems, and (iv) Fourth order elliptic problems. This is followed by one section on pointwise bounds, another on the use of a priori inequalities in the estimation of eigenvalues and a final section on numerical examples.

The monograph of Sigillito is quite readable, written in such a way that the average user of partial differential equations can understand the material and apply it. The examples are enlightening and the practical suggestions made at the end of the final section should be helpful.

L. P.

10[7.05, 10.30].—JEFFREY SHALLIT, *Table of Bell Numbers to* $B(400)$, Princeton University, Princeton, N. J., 1977, ms of 1 typewritten page + 59 computer sheets (reduced) deposited in the UMT file.

This clearly printed and attractively arranged table of the first 400 Bell numbers considerably extends that of Levine and Dalton [1], to which the author refers in a brief introduction.

The calculation of the present table was performed on an IBM 370/158 system, using the algorithm suggested by Becker [2] and a computer program written in APL.

J. W. W.

1. J. LEVINE & R. E. DALTON, "Minimum periods, modulo p, of first-order Bell exponential integers," *Math. Comp.*, v. 16, 1962, pp. 416–423.

2. H. W. BECKER, "Solution of Problem E 461", *Amer. Math. Monthly*, v. 48, 1941, pp. 701–703.

11[9].—REIJO ERNVALL, "E-irregular primes and related tables," 22 sheets of computer output deposited in the UMT file, University of Turku, Finland, September 1976.

These tables were computed in connection with the work [3].

The first column lists all the primes from 5 to 10000. The stars in the second column indicate the E-irregular ones. In the third column one finds the primitive root r, for which either r or $r - p$ is the least in its absolute value. These primitive roots have been checked from [5].

The next column gives the residue class mod 4 of the prime. It is known that $E_{p-1} \equiv 0$ or 2 mod p if $p \equiv 1$ or 3 mod 4, respectively. In the fifth column E_{p-1}/p mod p is given in the case $p \equiv 1$ mod 4. It turns out that in our range E_{p-1} never vanishes mod p^2. Cf. [3, Theorem 3].

In the next column there is the value of the Fermat quotient q_2 for those primes p that are either congruent to 1 mod 4 or E-irregular. This was printed in order to check our computations and was compared with the tables of [4]. Our value of q_2 was different from that of [4] for eleven primes, namely 2437, 4049, 4733, 4969, 5689, 6113, 6997, 7121, 7321, 8089, and 8093. A comparison with [1] and [2] showed that in these cases q_2 is incorrectly given in [4].

Similarly, for the primes p congruent to 1 mod 4 or E-irregular, we computed the value mod p of the sum

$$-6 \sum_{k=1}^{(p-1)/2} (2k-1)^2 q_{2k-1}$$

which is given in the next column. This value is always 1, as it should be.

The last three columns are associated with the E-irregular primes. First, the indices $2n$ ($2n \leqslant p - 3$) are given for which $E_{2n} \equiv 0$ mod p, i.e. the pair (p, $2n$) is E-irregular. The last two columns give the values of E_{2n}/p and $(E_{2n+p-1} - E_{2n})/2p$ mod p. We observe that in our range E_{2n} and $E_{2n+p-1} - E_{2n}$ never vanish mod p^2. Cf. [3, Theorem 5].

AUTHOR'S SUMMARY

1. N. G. W. H. BEEGER, "On a new case of the congruence $2^{p-1} \equiv 1 \pmod{p^2}$," *Messenger of Math.*, v. 51, 1922, pp. 149–150. Jbuch **48**, 1154.

2. N. G. W. H. BEEGER, "On the congruence $2^{p-1} \equiv 1 \pmod{p^2}$ and Fermat's last theorem," *Messenger of Math.*, v. 55, 1925/26, pp. 17–26. Jbuch **51**, 127.

3. R. ERNVALL & T. METSÄNKYLÄ, "Cyclotomic invariants and E-irregular primes," *Math. Comp.*, v. 32, 1978, pp. 617–629.

4. R. HAUSSNER, "Reste von $2^{p-1} - 1$ nach dem Teiler p^2 für alle Primzahlen bis 10009," *Arch. Math. Naturvid.*, v. 39, 1925, 17 pp. Jbuch **51**, 128.

5. A. E. WESTERN & J. C. P. MILLER, *Tables of Indices and Primitive Roots*, Roy. Soc. Math. Tables, vol. 9, Cambridge Univ. Press, London, 1968. MR **39** #7792.

12[9].—JOHN LEECH, *Five Tables Relating to Rational Cuboids*, 46 sheets of computer output deposited in the UMT file, University of Stirling, Scotland, January 1977.

A *perfect rational cuboid* is a rectangular parallelepiped whose three edges, three face diagonals and body diagonal all have integer lengths. None is known. The present tables relate to cuboids of which six of these seven lengths are integers. For a general discussion see [5].

1. *Body diagonal irrational.* The dimensions satisfy

$$x_2^2 + x_3^2 = y_1^2, \quad x_3^2 + x_1^2 = y_2^2, \quad x_1^2 + x_2^2 = y_3^2.$$

Table 1 lists 769 solutions of the equation

$$\frac{a_1^2 - b_1^2}{2a_1b_1} \cdot \frac{a_2^2 - b_2^2}{2a_2b_2} = \frac{a_3^2 - b_3^2}{2a_3b_3},$$

in which each pair of integers a_i, b_i are of opposite parity with $a_i > b_i > 0$. The table is complete for solutions in which two of the a_i do not exceed 376. To each solution of this equation there correspond two cuboids, with

$$\frac{x_1}{x_2}, \frac{x_2}{x_3} = \frac{a_1^2 - b_1^2}{2a_1b_1}, \frac{a_2^2 - b_2^2}{2a_2b_2}$$

in respective or reverse order. Dimensions not exceeding 10^6 are given to facilitate comparison with published lists. These previous tables are those of Kraitchik [1], which lists dimensions and generators for 241 cuboids whose odd dimension does not exceed 10^6, Kraitchik [2], which supplements this list with 18 cuboids whose odd dimension does not exceed 10^5, and Lal and Blundon [3], which lists cuboids corresponding to $a_1, a_3 \leqslant 70$ (but often only one of each pair). Table 2 reproduces Kraitchik's cuboids as the original publications are not readily available. For the results of comparisons see the Table Errata in this issue.

2. *One face diagonal irrational.* The dimensions satisfy

$$x_1^2 + x_2^2 = y_3^2, \quad x_1^2 + x_3^2 = y_2^2, \quad x_1^2 + x_2^2 + x_3^2 = z^2.$$

Solutions are in cycles of five [4], [5]. Table 3 lists 560 cycles of five integer pairs a_i, b_i, satisfying the equation

$$\frac{a_{i-1}^2 - b_{i-1}^2}{2a_{i-1}b_{i-1}} \cdot \frac{a_{i+1}^2 - b_{i+1}^2}{2a_{i+1}b_{i+1}} = \frac{a_i^2 + b_i^2}{2a_ib_i},$$

where the subscripts are cyclically reduced modulo 5 and $a_i > b_i > 0$. The table is complete for cycles in which two of the a_i do not exceed 376. To each cycle there correspond five cuboids, having

$$\frac{x_2}{x_1} = \frac{a_i^2 - b_i^2}{2a_ib_i}, \quad \frac{x_3}{x_1} = \frac{a_{i+1}^2 - b_{i+1}^2}{2a_{i+1}b_{i+1}}.$$

Dimensions are not listed. An asterisk is placed between a_i, b_i and a_{i+1}, b_{i+1} in each solution satisfying the additional condition that $(a_ia_{i+1})^2 + (b_ib_{i+1})^2$ and $(a_ib_{i+1})^2 + (a_{i+1}b_i)^2$ are both perfect squares (their product is always square). This table extends the short table of 35 cycles in [4]. Table 4 lists the dimensions of the 130 cuboids with $z < 250000$, with the corresponding cycles of generators.

3. *One edge irrational.* The dimensions satisfy

$$x_1^2 + x_2^2 = y_3^2, \quad x_1^2 + y_1^2 = x_2^2 + y_2^2 = t + y_3^2 = z^2,$$

where t is the square of the irrational edge. The generators are integers a_1, b_1, a_2, b_2, α, β such that

$$\frac{z}{x_1} = \frac{a_1^2 + b_1^2}{2a_1b_1}, \quad \frac{z}{x_2} = \frac{a_2^2 + b_2^2}{2a_2b_2}, \quad \frac{x_2}{x_1} = \frac{\alpha^2 - \beta^2}{2\alpha\beta};$$

satisfying

$$\frac{a_1^2 + b_1^2}{2a_1b_1} \cdot \frac{2a_2b_2}{a_2^2 + b_2^2} = \frac{\alpha^2 - \beta^2}{2\alpha\beta}.$$

Since any ratio x_1/x_2 can occur in solutions [5], it is of less interest to list solutions according to their generators. Table 5 lists the generators and dimensions of the 160 solutions with $z < 250000$. Of these 78 have $t > 0$ and correspond to real cuboids; the other 82 have $t < 0$. There are no previous tables.

AUTHOR'S SUMMARY

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3. M. LAL & W. J. BLUNDON, "Solutions of the Diophantine equations $x^2 + y^2 = l^2$, $y^2 + z^2 = m^2$, $z^2 + x^2 = n^2$," *Math. Comp.*, v. 20, 1966, pp. 144–147.

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