CORRIGENDA

D. M. GAY, "Modifying singular values: Existence of solutions to systems of nonlinear equations having a possibly singular Jacobian matrix," *Math. Comp.*, v. 31, 1977, pp. 962–973.

This note corrects an error pointed out by K. Tanabe [1978]. Theorem (5) of this paper should have been stated as:

(5) THEOREM. *If $F\colon \mathbf{R}^n \longrightarrow \mathbf{R}^n$ is continuous, then for each $x \in \mathbf{R}^n$ and $t_0 \in \mathbf{R}$ there exist $a \in [-\infty, t_0)$, $b \in (t_0, +\infty]$, and a continuously differentiable function x: $(a, b) \longrightarrow \mathbf{R}^n$ such that*

$$x(t_0) = x_0 \quad \textit{and} \tag{6a}$$

$$x'(t) = F(x(t)) \quad \textit{for all } t \in (a, b). \tag{6b}$$

If $\|F(x)\| \leqslant c$ for $\|x - x_0\| \leqslant d$, then $a < t_0 - d/c$ and $b > t_0 + d/c$. Moreover, if F is locally Lipschitz continuous, then the solution $x(t)$ of (6) is unique.

In [Gay, 1977] it was erroneously asserted that $a = -\infty$ and $b = +\infty$. This has no effect on the rest of the paper, except that the proof of Theorem (23) must be revised to show that $b = +\infty$ for the F of interest. The revised proof may be stated as follows:

Proof. Fix x_0. As already remarked, the existence of $x(t)$ on some interval $[0, b)$ follows easily from Theorems (13) and (5). We first show for $s, t \in [0, b)$ that

$$\|f(x(t))\| \leqslant \|f(x_0)\| e^{-\theta t} \qquad \textit{and} \tag{24.1}$$

$$\|x(s) - x(t)\| \leqslant [\|f(x_0)\|/(\theta\epsilon)]\, |e^{-\theta s} - e^{-\theta t}|. \tag{24.2}$$

Indeed, let $\phi(t) = \|f(x(t))\|^2$. Then $\phi'(t) = -2f^T J\hat{J}^+ f$, so (22) implies $\phi'(t) \leqslant -2\theta \|f(x(t))\|^2 = -2\theta\phi(t)$. Hence, $\psi(t) \equiv \ln \phi(t)$ has $\psi'(t) \leqslant -2\theta$, so (for $t \geqslant 0$)

$$\psi(t) = \psi(0) + \int_0^t \psi'(\tau)\, d\tau \leqslant \psi(0) - 2\theta t$$

and

$$\|f(x(t))\|^2 = \phi(t) = e^{\psi(t)} \leqslant \|f(x_0)\|^2 e^{-2\theta t},$$

which establishes (24.1). Because of (9a), we have

$$\|x'(t)\| = \|\hat{J}^+ f(x(t))\| \leqslant \|f(x(t))\|/\epsilon \leqslant (\|f(x_0)\|/\epsilon) e^{-\theta t},$$

whence

$$\|x(s) - x(t)\| = \left\| \int_s^t x'(\tau)\, d\tau \right\| \leqslant \left| \int_s^t \|x'(\tau)\|\, d\tau \right| \leqslant \frac{\|f(x_0)\|}{\epsilon} \left| \int_s^t e^{-\theta\tau} d\tau \right|,$$

which gives (24.2).

Now let $d = \|f(x_0)\|/(\theta\epsilon)$, $c = \max\{\|f(x)\|\colon x \in \overline{B}(x_0, d)\}/\epsilon$, and $b_0 = 0$. By (9a), (24.2), Theorem (5), and induction on k we find:

0025-5718/79/0000-0035/$01.50

$$b > b_k,$$
$$\|x(b_k) - x_0\| \leqslant [1 - \exp(-\theta b_k)]d,$$
$$\|\hat{J}^+ f(x)\| \leqslant c \quad \text{for } x \in \overline{B}(x(b_k), \exp(-\theta b_k)d),$$
$$b > b_{k+1} \equiv b_k + \exp(-\theta b_k)d/c = \frac{d}{c}(1 + e^{-\theta b_1} + e^{-\theta b_2} + \cdots + e^{-\theta b_k}).$$

From this it follows that $b = +\infty$, for if b were finite, then we would have $b > d(1 + ke^{-\theta b})/c$ for all k, which is impossible.

From (24.2) it follows that the sequence $x(t_1), x(t_2), x(t_3), \ldots$ is a Cauchy sequence for any choice of $t_1, t_2, \ldots$ with $\lim_{i\to\infty} t_i = +\infty$, whence $x^* = \lim_{t\to\infty} x(t)$ exists. By the continuity of f and (24.1), $f(x^*) = \lim_{t\to\infty} f(x(t)) = 0$. □

DAVID M. GAY

MIT Center for Computational Research in Economics and Management Science
575 Technology Square—9th Floor
Cambridge, Massachusetts 02139

K. TANABE, (1978), "Global analysis of continuous analogues of the Levenberg-Marquardt and Newton-Raphson methods for solving nonlinear equations." (Preprint.)

I. S. GRADSHTEYN & I. M. RYZHIK, *Table of Integrals, Series, and Products*, 4th ed., Academic Press, New York, 1965.

On p. 906 of MTE **428** (*Math. Comp.*, v. 22, 1968, pp. 903–907) listing corrections in this set of tables there appears a typographical error in the correction of Formula 8.521(4). The emended correction should read

$$-\frac{1}{\sqrt{(2ki\pi - z)^2 + x^2 + y^2}}\,.$$

MICHAEL ANDERSON

Department of Physics
The University of Michigan
Ann Arbor, Michigan 48109

REIJO ERNVALL & TAUNO METSÄNKYLÄ, "Cyclotomic invariants and E-irregular primes," *Math. Comp.*, v. 32, 1978, pp. 617–629.

On p. 619, the three first lines of the first table should read as follows:

x	π_B	π_E	π_{BE}	π_B/π	π_E/π	π_{BE}/π
2000	121	113	56	0.399	0.373	0.18
4000	218	213	91	0.396	0.387	0.17

REIJO ERNVALL

Department of Mathematics
University of Turku
SF-20500 Turku 50, Finland