REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

1[2.05].—C. A. MICCHELLI & T. J. RIVLIN, Editors, *Optimal Estimation in Approximation Theory,* Plenum Press, New York, 1977, ix + 300 pp., 25½ cm.

The papers in this volume were presented at an International Symposium which was held in Freudenstadt, Federal Republic of Germany, September 27–29, 1976.

2[2.05.5, 7.20, 7.25, 7.30, 7.45, 7.55, 7.75].—YUDELL L. LUKE, *Algorithms for the Computation of Mathematical Functions,* Academic Press, New York, 1977, xiii + 284 pp., 23½ cm. Price $15.00.

The main purpose of this book is to provide computer programs for generating three types of approximations to special functions: expansions in Chebyshev polynomials, rational approximations of the Padé type (those on the main diagonal of the Padé table), and certain other rational approximations not of the Padé type. Much of the underlying mathematical machinery has been presented by the author on previous occasions [1], [2], and is summarized, once more, in the present volume. What has been added, now, are computer programs written in FORTRAN IV for the IBM 370/168, which generate the expansion coefficients and the coefficients in the rational approximations, and provide tests for their accuracy. The programs are applicable to rather wide classes of special functions. Thus, Chapters 4–9 provide programs for generating Chebyshev expansions for the Gauss hypergeometric function ${}_2F_1(a, b; c; z)$ and its confluent forms ${}_1F_1(a; c; z)$, ${}_0F_1(c; z)$, as well as for the hypergeometric functions ${}_1F_2(a; b, c; z)$ and certain G-functions. Some of the expansions are of the "ascending" type,

$$f(z) = \sum_{n=0}^{\infty} C_n(w)T_n^*(t),$$

where T_n^* is the (shifted) Chebyshev polynomial of degree n, and $z = tw$, $0 \leqslant t \leqslant 1$, with w a fixed complex number, while others are of the "descending" type,

$$f(z) = \sum_{n=0}^{\infty} G_n(w)T_n^*(1/t), \qquad z = tw,\ t > 1.$$

There is considerable discussion of the asymptotic behavior of the coefficients $C_n(w)$ and $G_n(w)$ as $n \longrightarrow \infty$, which ought to be helpful in determining the number of terms required for any given accuracy. Similarly, Chapters 13–21 provide programs for generating rational approximations for hypergeometric and confluent hypergeometric functions, including Bessel functions and ratios of Bessel functions. Here again, there is a detailed account of the asymptotic behavior of the error as the degree of the rational approximation tends to infinity. Additional programs, more of a utility nature, are included in Chapters 11 and 12. Some generate Chebyshev expansions for such functions as $g(z) = e^{-az}z^{-u-1}\int_0^z e^{at}t^u f(t)\,dt$, if one furnishes the expansion for f; others convert

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power series expansions into Chebyshev expansions. Programs that transform rational approximations into continued fraction form, on the other hand, are not included.

Contrary to what the title of the book might suggest, the algorithms provided here are not sufficiently complete and polished so as to be suitable for inclusion in a subroutine library. None of the algorithms, e.g., incorporates provisions for error control. Neither is there any discussion as to how different algorithms, valid in different (complementary) regions of the independent variable, are to be combined in order to produce efficient polyalgorithms. The material assembled in the book, nonetheless, may prove useful in constructing algorithms for computing special functions, particularly functions for which alternative methods are not readily available. It is to be expected, however, that the resulting algorithms are expensive in terms of computational effort, particularly if the generation of the coefficients is part of the algorithm. Of necessity, this will be the case if the coefficients are themselves functions of freely variable parameters.

W. G.

1. Y. L. LUKE, *The Special Functions and their Approximations,* vols. I, II, Academic Press, New York, 1969.

2. Y. L. LUKE, *Mathematical Functions and their Approximations,* Academic Press, New York, 1975.

3[5.15].—E. BOHL, L. COLLATZ & K. P. HADELER, Editors, *Numerik und Anwendungen von Eigenwertaufgaben und Verzweigungsproblemen,* International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, Switzerland, 1977, 218 pp., 24 cm. Price approximately sfr. 42.

This volume contains papers presented at a meeting organized by the editors. This meeting took place at the Mathematical Research Institute at Oberwolfach, Germany from November 14–20, 1976.

4[4].—W. G. SPOHN, *Table of Integral Cuboids and Their Generators,* 4 pp. + 45 pp. reduced size computer printout, deposited in the UMT file, 1978.

A list of 2472 integral cuboids with maximum edge less than 10^9 is presented. They are ordered by the shortest edge. Results from other tables [1], [2], [3], [4] are used, additional searches are made, formulas for families of solutions [2], [5] are evaluated, and new solutions are derived from known solutions. Thus with modest computer effort a great number of cuboids are found, using double precision on the IBM 360/91. Since no general formula is known, an extensive list of many of the smallest cases should be useful to researchers in the field.

One seeks solutions in positive integers to the three equations

$$x^2 + y^2 = w^2, \quad x^2 + z^2 = v^2, \quad y^2 + z^2 = u^2,$$

where x, y, z yield edges of the cuboid and u, v, w face diagonals. In the table the solutions are ordered with $x < y < z$. There are 13 columns of data, one for the entry number, three for the edges, three for the face diagonals, and six for the Kraitchik generators. There are two entries with shortest edge less than 10^2; 14, less

than 10^3; 56, less than 10^4; 177, less than 10^5; 566, less than 10^6; 1283, less than 10^7; 2140, less than 10^8. Since previous tables missed cases with shortest edge equal to 30, 156 or less, we cannot hope for any degree of completeness. No doubt a few cases less than 10^5 are missing and many cases less than 10^6.

An integral cuboid is called perfect, if the inner diagonals are also positive integers. This requires solutions to the fourth equation

$$x^2 + y^2 + t^2.$$

It is not known whether perfect cuboids exist. There are none in this table.

The 4-page text attached to the table gives a detailed comparison of the present table and those in [1]–[4].

AUTHOR'S SUMMARY

1. J. LEECH, UMT 12, "Five tables relating to rational cuboids," *Math. Comp.*, v. 32, 1978, pp. 657–659.

2. M. KRAITCHIK, *Théorie des Nombres*, t. III, *Analyse Diophantine et Applications aux Cuboïdes Rationnels*, Gauthier-Villars, Paris, 1947.

3. M. KRAITCHIK, *Sur les Cuboïdes Rationnels*, Proc. Internat. Congr. Math., vol. 2, North-Holland, Amsterdam, 1954, pp. 33–34.

4. M. LAL & W. J. BLUNDON, "Solutions of the Diophantine equations $x^2 + y^2 = l^2$, $y^2 + z^2 = m^2$, $z^2 + x^2 = n^2$," *Math. Comp.*, v. 20, 1966, pp. 144–147.

5. M. RIGNAUX, "Système $x^2 + y^2 = a^2$, $x^2 + z^2 = b^2$, $y^2 + z^2 = c^2$," *Intermédiare Math.*, v. 25, 1918, p. 127.

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