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A Note on the Integral $\int_0^\infty t^{2\alpha-1}(1 + t^2)^{1-\alpha-\beta}J_\nu(x\sqrt{1 + t^2})\,dt$

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Abstract. The integral

$$I_\nu^{\alpha,\beta}(x) = \int_0^\infty t^{2\alpha-1}(1 + t^2)^{1-\alpha-\beta}J_\nu(x\sqrt{1 + t^2})\,dt$$

is expressed in terms of Bessel and related functions for various values of the parameters by summing the hypergeometric series representation given by Schmidt.

In a recent paper [1] Schmidt has studied the integral in the title of this note, which he denotes $I_\nu^{\alpha,\beta}(x)$, by expressing it in terms of the hypergeometric series, ${}_1F_2$. In addition, he presented simple expressions for $I_\nu^{1/2,1}$, $I_\nu^{1/2,1/2}$ and $I_\nu^{\alpha,\nu/2+1-\alpha}$ in terms of Bessel functions. The purpose of this note is to augment the results in [1] by a number of simple formulas obtained by summing the hypergeometric series. For brevity the rather messy derivations have been omitted. The notation is that of [2].

$$I_0^{1,1/2}(x) = \frac{1}{x} - J_0(x) + \frac{\pi}{2}\,[J_0(x)\mathbf{H}_1(x) - J_1(x)\mathbf{H}_0(x)],$$

$$I_1^{1,1/2}(x) = J_0(x)/x, \quad I_1^{1/2,1}(x) = x^{-1}\sin x, \quad I_0^{1/2,1/2}(x) = x^{-1}\cos x,$$

$$I_{1/2}^{\alpha,5/2-2\alpha}(1) = -\sqrt{\frac{\pi}{8}}\,[J_{3/2-2\alpha}(1/2)Y_{1/2}(1/2) + Y_{3/2-2\alpha}(1/2)J_{1/2}(1/2)], \qquad 1/2 < \alpha < 1,$$

$$I_0^{1/2,3/2}(1) = -\,\mathrm{si}(1),$$

$$I_\nu^{(\nu-1/2),(3/2-(1/2)\nu)}(x) = \frac{\Gamma(\nu - 1/2)}{2^{\nu+1}\sqrt{\pi}}\,x^{-\nu}\sin x, \qquad \frac{1}{2} < \nu < \frac{5}{2},$$

$$I_\nu^{(\nu+1/2),(1/2-(1/2)\nu)}(x) = \frac{\Gamma(\nu + 1/2)}{2^{\nu}\sqrt{\pi}}\,x^{-\nu-1}\cos x, \qquad -1/2 < \nu < 1/2,$$

$$I_\nu^{(1/2-\nu),((3/2)\nu+1)}(x) = -\,\frac{\sqrt{\pi}}{2^{\nu+1}}\,\Gamma(1/2 - \nu)x^\nu J_\nu(\tfrac{1}{2}x)Y_\nu(\tfrac{1}{2}x), \qquad |\nu| < 1/2.$$

$$I_\nu^{1,1/4}(x) = x^{-1/2}\{J_{\nu-1}(x)S_{1/2,\nu}(x) + (\tfrac{1}{2} - \nu)J_\nu(x)S_{-1/2,\nu-1}(x)\}.$$

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This list is not exhaustive and can be extended considerably by use of the relations

$$(x/2\nu)[I_{\nu-1}^{\alpha,\beta}(x) + I_{\nu+1}^{\alpha,\beta}(x)] = I_\nu^{\alpha,\beta+1/2}(x),$$

$$x^{-\nu}[x^\nu I_\nu^{\alpha,\beta}(x)]' = I_{\nu-1}^{\alpha,\beta-1/2}(x),$$

$$x^\nu[x^{-\nu}I_\nu^{\alpha,\beta}(x)]' = -I_{\nu+1}^{\alpha,\beta-1/2}(x).$$

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1. P. W. SCHMIDT, *Math. Comp.*, v. 32, 1978, pp. 265–269.

2. Y. L. LUKE, *The Special Functions and Their Approximations*, Academic Press, New York, 1969.