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A Factor of F_{17}

By Gary B. Gostin

Abstract. A prime factor is given for F_{17}. The method of factoring and its machine implementation are given.

During the investigation reported here, a new factor of Fermat Number F_{17} was discovered. It is given in Table 1. The factor is significant, since F_{17} is the smallest Fermat number whose character was unknown, and since $17 = F_2$.

The method of factoring is similar to that of Hallyburton and Brillhart [1] and others [3], [4]. To determine whether a $d_k = k \cdot 2^{n+2} + 1$ divides F_n, the congruence $2^{2^n} \equiv -1 \pmod{d_k}$ is tested. This is done by beginning with the residue $r_i = 2^{32}$ for $i = 5$, and computing $r_i^2 \pmod{d_k}$ which becomes r_{i+1}. This operation is repeated $n - 5$ times. For any r_i, d_k divides F_i if $r_i \equiv -1 \pmod{d_k}$. Therefore this method tests to see if d_k divides any F_i for $5 < i \leqslant n$.

TABLE 1. *Factor of F_{17}*

n	Factor
17	$31065037602817 = 59251857 \cdot 2^{19} + 1$

This procedure was written in Compass assembly language for the CDC Cyber 175. The basic test, along with a provision to add a constant to d_k and to repeat the basic test, was written as a subroutine that can be called by Fortran. The subroutine can test up to 131,072 d_k's on a single call. The time to test a d_k is given in Table 2 for three Fermat Numbers.

TABLE 2. *Time to test a d_k for F_n*

n	time (microseconds)	# residue calculations
9	7.0	4
13	14.5	8
17	20.8	12

A computer generated sieve was considered for reducing the number of d_k's tested. It was rejected, however, because the cost (time plus memory) of rejecting a d_k, by using the sieve, was more than the cost of running the basic test with d_k. Instead the following method was used.

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If all d_k divisible by 3 or 5, or with k even, are crossed off a list of all d_k's, the pattern of divisors remaining is periodic with a period of thirty. Eight divisors out of each group of thirty will remain. Therefore a search of a sequence of d_k's can be broken down into eight searches, with $30 \cdot 2^{n+2}$ being added to the present divisor to get the next divisor in each test. Thus a sieve by 3 and 5 on a sequence of d_k's (k odd) can be done with no additional computer time. By consulting a table of primes, it is estimated that 60% of the composite d_k's are rejected by this sieve.

All the Fermat numbers from F_8 to F_{46} were examined. A limit of 2^{48} was placed on d_k, but $d_k = 2^{48} + 1$ was factored (thus rejected) by hand. The search limit for each Fermat Number is shown in Table 3. For each F_n, the limit on d_k includes odd values of k tried while examining F_n, and even values of k tried while examining $F_m > n$. Therefore in every case, all divisors up to the limit reported were covered. The total CPU execution time of this program was about seven hours.

TABLE 3. *Search limit for* F_n

n	Limit	
8	$d_k = 2^{41} + 1$	$k = 2^{31}$
9	$d_k = 2^{41} + 1$	$k = 2^{30}$
10	$d_k = 2^{42} + 1$	$k = 2^{30}$
11	$d_k = 2^{43} + 1$	$k = 2^{30}$
12	$d_k = 2^{45} + 1$	$k = 2^{31}$
13	$d_k = 2^{45} + 1$	$k = 2^{30}$
14 – 24	$d_k = 2^{47} + 1$	
25 – 46	$d_k = 2^{48} + 1$	

During the search, two factors for F_{12} and one for F_{11} were also found. These were found by S. Wagstaff at the University of Illinois to be products of smaller known factors. Wagstaff also mentioned that the cofactors of F_{11}, F_{12}, and F_{13} are composite.

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13354 Emily Road #136
Dallas, Texas 75240

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